数学科学英语

周之铭　蔡克聚 编

MATHEMATICAL SCIENCES ENGLISH

中山大学出版社

责任编辑：李文
封面设计：朱琳

数学科学英语

周之铭　蔡克聚 编

*

中山大学出版社出版发行
广东省新华书店经销
中山大学印刷厂印刷

*

787×1092毫米　16开本　21.25印张　50万字
1991年3月第1版　1991年3月第1次印刷
印数：1—3000册
ISBN7—306—00307—0

H.18　定价：5.85元

本教材力图达到二个目标：一为基本要求，即通过学习使学生掌握一些最基本的专业词汇和数学书刊中常见的短语和句子，以提高阅读专业书刊的能力，同时学会初步运用英语陈述和回答数学问题；二是通过一些内容广泛，体裁多样的课文，扩大学生知识面，拓广视野和提高学习兴趣，使学生能运用英语这一语言，多渠道去获取多方面的专业信息。

根据这些目标，本书在选材时注意了内容的广泛性和体裁的多样性。在内容方面既涉及基础数学、应用数学、计算机科学，也涉及数学与生物学、社会科学、经济学、管理学的关系以及数学在这些方面的应用前景等；课文既介绍了古典基础学科，也反映了一些新兴学科；既有一般的教科书内容，也包含诸如数学家传略，数学领域里的争论以至有关数学研究和写作的一些精辟论述等。在体裁方面，有教科书体裁，序言式体裁，专论式体裁以及采访，报告体裁等。课文内容的广泛性与体裁的多样性使本书的适应面较广，避免了同类书籍过分局限于教科书内容那种单一体裁的单调性，有利于提高学生的阅读能力和学习兴趣。

本书分两部分。每一部分除正课文外均附有补充阅读课文。第一部分侧重于教科书内容，目的在于使学生能在较短时间内掌握一定数量的专业词汇和常见句型，有利于帮助学生尽快阅读原版教科书。这一部份课文与补充阅读课文基本上呈一一对应关系。教师可灵活选择讲授内容，不应受正课文与补充课文这一区分的限制。为了掌握较全面的基础专业词汇，对这一部分未于讲授的课文，学生应尽力挤出时间自学。第二部分内容较广泛，除几课有关学科介绍外，从语言角度看，难度稍大些。但正是这一部分内容包含的信息量大，涉及的知识面广。一些文章很有启发性，某些段落和句子很美，还包含一些哲理性内容，可读性甚强。这一部分内容，不像教科书内容那样呆板，对教师来说有较大的发挥余地，我们的教学实践证明，如处理得当，这一部分内容对学生是有较强的吸引力的，同时对学生英语水平和阅读能力的提高也是大有裨益的。

本书的每一正课文都附有习题和少量注释。我们在选编习题时作了如下考虑：一是通过习题帮助学生巩固已学的专业名词，同时补充一些在课文中没有出现的基本词汇；二是通过习题使学生有步骤地掌握常见的数学用语和句型，训练学生逐步学会运用英语表达数学内容；三是通过双向翻译去提高学生的阅读、理解和表达能力。本书较少安排针对课文提问并要求回答的那类练习。

本书各课所配练习分量有多有少，教师可酌情增减分量。

本书还附有几个有一定价值的附录材料。这些材料不是孤立的，而是与课文、习题和注释有一定联系的。因此读者在做练习之前，最好先参阅有关的附录资料，同时在阅读课文时，注意学习与附录材料有关或相似的常见句型以达到更好掌握的目的。

本书虽经编者 3 年多的教学实践并经过反复修改补充才撰写而成，但不论在课文的选编，习题的安排以及注释方面，不妥以至错误之处在所难免，诚恳希望读者指正，以便进一步修改。

编　者

1991 年 3 月

Content

PART IA *TEXT*

PART ⅠB *SUPPLEMENTARY READING MATERIAL*

PART IA

LESSON ONE

The Real Number System

The Real–Number System

The real–number system is a collection of mathematical objects, called real numbers, which acquire mathematical life by virtue of certain fundamental principles, or rules, that we adopt. The situation is somewhat similar to a game, like chess, for example. The chess system, or game, is a collection of objects, called chess pieces, which acquire life by virtue of the rules of the game, that is, the principles that are adopted to define allowable moves for the pieces and the way in which they may interact.

Our working experience with numbers has provided us all with some familiarity with the principles that govern the real–number system. However, to establish a common ground of understanding and avoid certain errors that have become very common, we shall explicitly state and illustrate many of these principles.

The real number system includes such numbers as −27, −2, 2 / 3,... It is worthy of note that positive numbers, 1 / 2, 1, for examples, are sometimes expressed as +(1 / 2), +1. The plus sign,"+", used here does not express the operation of addition, but is rather part of the symbolism for the numbers themselves. Similarly, the minus sign, "−", used in expressing such numbers as −(1 / 2), −1, is part of the symbolism for these numbers.

Within the real number system, numbers of various kinds are identified and named. The numbers 1,2,3,4,... which are used in the counting process, are called natural numbers. The natural numbers, together with −1,−2,−3,−4,... and zero, are called integers. Since 1,2,3,4,... are greater than 0, they are also called positive integers;−1,−2,−3,−4,... are less than 0, and for this reason are called negative integers. A real number is said to be a rational number if it can be expressed as the ratio of two integers, where the denominator is not zero. The integers are included among the rational numbers since any integer can be expressed as the ratio of the integer itself and one. A real number that cannot be expressed as the ratio of two integers is said to be an irrational number.

One of the basic properties of the real–number system is that any two real numbers can be compared for size. If a and b are real numbers, we write $a<b$ to signify that a is less than b. Another way of saying the same thing is to write $b>a$, which is read " b is greater than a ".

Geometrically, real numbers are identified with points on a straight line. We choose a straight line, and an initial point of reference called the origin. To the origin we assign the number zero. By marking off the unit of length in both directions from the origin, we assign positive integers to marked–off points in one direction (by convention, to the right of the origin) and negative integers to marked–off points in the other direction. By following through in terms of the chosen unit of length, a real number is attached to each point on the number line, and each point on the number line has attached to it one number.

Geometrically, in terms of our number line, to say that $a < b$ is to say that a is to the left of b ; $b > a$ means that b is to the right of a .

Properties of Addition and Multiplication

Addition and multiplication are primary operations on real numbers. Most, if not all, of the basic properties of these operations are familiar to us from experience.

(a) Closure property of addition and multiplication.

Whenever two real numbers are added or multiplied, we obtain a real number as the result. That is, performing the operations of addition and multiplication leaves us within the real–number system.

(b) Commutative property of addition and multiplication.

The order in which two real numbers are added or multiplied does not affect the result obtained. That is, if a and b are any two real numbers, then we have (i) $a + b = b + a$ and (ii) $ab = ba$. Such a property is called a commutative property. Thus, addition and multiplication of real numbers are commutative operations.

(c) Associative property of addition and multiplication.

Parentheses, brackets, and the like, we recall, are used in algebra to group together whatever terms are within them. Thus 2+(3+4) means that 2 is to be added to the sum of 3 and 4 yielding 2+7 = 9 whereas (2+3)+4 means the sum of 2 and 3 is to be added to 4 yielding also 9. Similarly, 2 • (3 • 4) yields 2 • (12) = 24 whereas (2 • 3) • 4 yields the same end result by the route 6 • 4 = 24. That such is the case in general is the content of the associative property of addition and multiplication of real numbers.

(d) Distributive property of multiplication over addition.

We know that 2 • (3+4) = 2 • 7 = 14 and that 2 • 3 +2 • 4 = 14, thus 2 • (3+4) = 2 • 3 + 2 • 4. That such is the case in general for all real numbers is the content of the distributive property of multiplication over addition, more simply called the distributive property.

Substraction and Division

The numbers zero and one. The following are the basic properties of the numbers zero and one:

(a) There is a unique real number, called zero and denoted by 0, with the property that $a + 0 = 0 + a$, where a is any real number.

There is a unique real number, different from zero, called one and denoted by 1, with the property that $a \cdot 1 = 1 \cdot a = a$, where a is any real number.

(b) If a is any real number, then there is a unique real number x, called the additive inverse of a, or negative of a, with the property that $a + x = x + a = 0$. If a is any nonzero real number, then there is a unique real number y, called the multiplicative inverse of a, or reciprocal of a, with the property that $ay = ya = 1$.

The concept of the negative of a number should not be confused with the concept of a negative number; they are not the same. "Negative of" means "additive inverse of". On the other hand, a "negative number" is a number that is less than zero.

The multiplicative inverse of a is often represented by the symbol $1/a$ or a^{-1}. Note that since the product of any number y and 0 is 0, 0 cannot have a multiplicative inverse. Thus 1 / 0 does not exist.

Now substraction is defined in terms of addition in the following way.

If a and b are any two real numbers, then the difference $a-b$ is defined by $a-b=c$ where c is such that $b+c=a$ or $c=a+(-b)$. That is, to substract b from a means to add the negative of b (additive inverse of b) to a.

Division is defined in terms of multiplication in the following way.

If a and b are any real numbers, where $b \neq 0$, then $a \div b$ is defined by $a \div b = a \cdot (1/b) = a \cdot b^{-1}$. That is, to divide a by b means to multiply a by the multiplicative inverse (reciprocal) of b. The quotient $a \div b$ is also expressed by the fraction symbol a/b.

Vocabulary

real number 实数
the real number system 实数系
collection 集体,总体
object 对象,目的
principle 原理,规则
adopt 采用
define 定义(动词)
definition 定义(名词)
establish 建立
explicit 清晰的,明显的
illustrate 说明;表明
positive 正的
express 表达
plus 加
sign 记号,符号;正负号
operation 运算,操作
addition 加法
multiplication 乘法
substraction 减法
division 除法
sum 和,总数
procuct 乘积
negative 负的
rational number 有理数
ratio 比,比率
denominator 分母
numerator 分子
irrational number 无理数
signify 表示
geometrical 几何的
straight line 直线
initial point 初始点
point of reference 参考点
origin 原点
assign 指定
unit 单位
property 性质
closure property 封闭性质
commutative 交换的
associative 结合的
parentheses 圆括号
brackets 括号
algebra 代数
yield 产生,给出

difference 差,差分	term 术语,项
quotient 商	distributive 分配的
symbolism 符号系统	unique 唯一的
minus 减	additive inverse 加法逆元素
identify 使同一	multiplicative inverse 乘法逆元素
count 计数	reciprocal 倒数;互逆
natural number 自然数	concept 概念
zero 零	fraction 分数
integer 整数	arithmetic 算术的
greater than 大于	solution 解,解法
less than 小于	even 偶的
be equal to 等于	odd 奇的
arbitrary 任意的	square 平方
absolute value 绝对值	square root 平方根
cube 立方	induction 归纳法

Notes

1. Our working experience with numbers has provided us all with some familiarity with the principles that govern the real-number system.

   意思是:我们对数的实际工作经验使我们大家对支配着实数系的各原则早已有了某些熟悉,这里 working 作"实际工作的"解, govern 作"支配"解.

2. The plus sign, "+", used here does not express the operation of addition, but is rather part of the symbolism for the numbers themselves.

   意思是:这里的正符号"+"不是表示加法运算,而是数本身的符号系统的一部分.

3. A real number is said to be a rational number if it can be expressed as the ratio of two integers, where denominator is not zero.

   这是定义数学术语的一种形式. 下面是另一种定义数学术语的形式:

   A matrix is called a square matrix if the number of its rows equals the number of its columns.

   这里 is called 与 is said to be 可以互用,注意 is called 后面一般不加 to be 而 is said 后面一般要加.

4. A real number that cannot be expressed as the ratio of two integers is said to be an irrational number.

与注 3. 比较，这里用定语从句界定术语。

5. There is a unique real number, called zero and denoted by 0, with the property that $a+0=0+a$, where a is any real number.

意思是：存在唯一的一个实数，叫做零并记为 0，具有性质 $a+0=0+a$，这里(其中) a 是任一实数。

1) 这里 called 和 denoted 都是过去分词，与后面的词组成分词短语，修饰 number.

2) with the property 是前置词短语，修饰 number.

3) 注意本句和注 3. 中 where 的用法，一般当需要附加说明句子中某一对象时可用此结构。

Exercise

I. Turn the following arithmetic expressions into English:

i) $3+(-2)=1$ ii) $2+3(-4)=-10$

iii) $\sqrt[3]{-125}=-5$ iv) $\sqrt{9}=3$

v) $2/5-1/6=7/30$

II. Fill in each blank the missing mathematical term to make the following sentences complete.

i) The __________ of two real numbers of unlike signs is negative.

ii) An integer n is called ________ if $n=2m$ for some integer m.

iii) A solution to the equation $x^n=c$ is called the n th __________ of c.

iv) If x is a real number, then the _________ of x is a nonnegative real number denoted by $|x|$ and defined as follows

$$|x|=\begin{cases} x & \text{if } x\geq 0 \\ -x & \text{if } x<0 \end{cases}$$

III. Translate the following exercises into Chinese:

i) If x is an arbitrary real number, prove that there is exactly one integer n such that $x<n<x+1$.

ii) Prove that there is no rational number whose square is 2.

iii) Given positive real numbers $a_1, a_2, a_3, \ldots$ such that $a_n < ca_{n-1}$ for all $n > 2$, where c is a fixed positive number, use induction to prove that $a_n < c^{n-1}a_1$, for all $n > 1$.

iv) Determine all positive integers n for which $2^n < n$!

IV. Translate the followin_ passage into Chinese:

There are many ways to introduce the real number system. One popular method is to begin with the positive integers 1,2,3,... and use them as building blocks to construct a more comprehensive system having the properties desired. Briefly, the idea of this method is to take the positive integers as undefined concepts, state some axioms concerning them, and then use the positive integers to build a larger system consisting of the positive rational numbers. The positive rational numbers, in turn, may then be used as basis for constructing the positive irrational numbers. The final step is the introduction of the negative numbers and zero. The most difficult part of the whole process is the transition from the rational numbers to the irrational numbers.

V. Translate the following theorems into English:

1. 定理 A: 每一非负实数有唯一一个非负平方根.

2. 定理 B: 若 $x > 0$, y 是任意一实数, 则存在一正整数 n 使得 $nx > y$.

VI. 1. Try to show the structure of the set of real numbers graphically.

2. List and state the laws that operations of addition and multiplication of real numbers obey.

LESSON TWO

Basic Concepts of the Theory of Sets

In discussing any branch of mathematics, be it analysis, algebra, or geometry, it is helpful to use the notation and terminology of set theory. This subject, which was developed by Boole and Cantor in the latter part of the 19th century, has had a profound influence on the development of mathematics in the 20th century. It has unified many seemingly disconnected ideas and has helped to reduce many mathematical concepts to their logical foundations in an elegant and systematic way.A thorough treatment of the theory of sets would require a lengthy discussion which we regard as outside the scope of this book. Fortunately, the basic noticns are few in number, and it is possible to develop a working knowledge of the methods and ideas of set theory through an informal discussion. Actually, we shall discuss not so much a new theory as an agreement about the precise terminology that we wish to apply to more or less familiar ideas.

In mathematics, the word "set" is used to represent a collection of objects viewed as a single entity. The collections called to mind by such nouns as "flock", "tribe", "crowd", "team", are all examples of sets. The individual objects in the collection are called elements or members of the set, and they are said to belong to or to be contained in the set. The set in turn, is said to contain or be composed of its elements.

We shall be interested primarily in sets of mathematical objects: sets of numbers, sets of curves, sets of geometric figures, and so on. In many applications it is convenient to deal with sets in which nothing special is assumed about the nature of the individual objects in the collection.These are called abstract sets. Abstract set theory has been developed to deal with such collections of arbitrary objects, and from this generality the theory derives its power.

NOTATIONS. Sets usually are denoted by capital letters: A , B , C ,..., X , Y , Z ; elements are designated by lower–case letters: a, b, c ,..., x, y, z . We use the special notation

$$x \in S$$

to mean that " x is an element of S " or " x belongs to S ". If x does not belong to S, we write $x \notin S$. When convenient, we shall designate sets by displaying the elements in braces; for example, the set of positive even integers less than 10 is denoted by the symbol $\{2,4,6,8\}$ whereas the set of all positive even integers is displayed as $\{2,4,6,\cdots\}$, the dots taking the place of "and so on".

The first basic concept that relates one set to another is equality of sets:

DEFINITION OF SET EQUALITY Two sets A and B are said to be equal (or identical) if they consist of exactly the same elements, in which case we write $A = B$. If one of the sets contains an element not in the other, we say the sets are unequal and we write $A \neq B$.

SUBSETS. From a given set S we may form new sets, called subsets of S. For example, the set consisting of those positive integers less than 10 which are divisible by 4 (the set $\{4,8\}$) is a subset of the set of all even integers less than 10. In general, we have the following definition.

DEFINITION OF A SUBSET. A set A is said to be a subset of a set B, and we write

$$A \subseteq B$$

whenever every element of A also belongs to B. We also say that A is contained in B or B contains A. The relation is referred to as set inclusion.

The statement $A \subseteq B$ does not rule out the possibility that $B \subseteq A$. In fact,we may have both $A \subseteq B$ and $B \subseteq A$, but this happens only if A and B have the same elements. In other words, $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.

This theorem is an immediate consequence of the foregoing definitions of equality and inclusion. If $A \subseteq B$ but $A \neq B$, then we say that A is a proper subset of B; we indicate this by writing $A \subset B$.

In all our applications of set theory, we have a fixed set S given in advance, and we are concerned only with subsets of this given set. The underlying set S may vary from one application to another; it will be referred to as the universal set of each particular discourse.