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国外物理名著系列 28

（影印版）

Mathematica for Theoretical Physics:

Electrodynamics, Quantum Mechanics, General Relativity, and Fractals

(2nd Edition)

理论物理中的 Mathematica

——电动力学，量子力学，广义相对论和分形

（第二版）

G. Baumann

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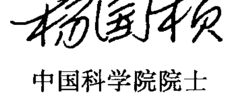
国外物理名著系列序言

对于国内的物理学工作者和青年学生来讲，研读国外优秀的物理学著作是系统掌握物理学知识的一个重要手段。但是，在国内并不能及时、方便地买到国外的图书，且国外图书不菲的价格往往令国内的读者却步，因此，把国外的优秀物理原著引进到国内，让国内的读者能够方便地以较低的价格购买是一项意义深远的工作，将有助于国内物理学工作者和青年学生掌握国际物理学的前沿知识，进而推动我国物理学科研和教学的发展。

为了满足国内读者对国外优秀物理学著作的需求，科学出版社启动了引进国外优秀著作的工作，出版社的这一举措得到了国内物理学界的积极响应和支持，很快成立了专家委员会，开展了选题的推荐和筛选工作，在出版社初选的书单基础上确定了第一批引进的项目，这些图书几乎涉及了近代物理学的所有领域，既有阐述学科基本理论的经典名著，也有反映某一学科专题前沿的专著。在选择图书时，专家委员会遵循了以下原则：基础理论方面的图书强调“经典”，选择了那些经得起时间检验、对物理学的发展产生重要影响、现在还不“过时”的著作（如狄拉克的《量子力学原理》）。反映物理学某一领域进展的著作强调“前沿”和“热点”，根据国内物理学研究发展的实际情况，选择了能够体现相关学科最新进展，对有关方向的科研人员和研究生有重要参考价值的图书。这些图书都是最新版的，多数图书都是 2000 年以后出版的，还有相当一部分是当年出版的新书。因此，这套丛书具有权威性、前瞻性和应用性强的特点。由于国外出版社的要求，科学出版社对部分图书进行了少量的翻译和注释（主要是目录标题和练习题），但这并不会影响图书“原汁原味”的感觉，可能还会方便国内读者的阅读和理解。

“他山之石，可以攻玉”，希望这套丛书的出版能够为国内物理学工作者和青年学生的工作和学习提供参考，也希望国内更多专家参与到这一工作中来，推荐更多的好书。



中国科学院院士

中国物理学会理事长

Preface

As physicists, mathematicians or engineers, we are all involved with mathematical calculations in our everyday work. Most of the laborious, complicated, and time-consuming calculations have to be done over and over again if we want to check the validity of our assumptions and derive new phenomena from changing models. Even in the age of computers, we often use paper and pencil to do our calculations. However, computer programs like *Mathematica* have revolutionized our working methods. *Mathematica* not only supports popular numerical calculations but also enables us to do exact analytical calculations by computer. Once we know the analytical representations of physical phenomena, we are able to use *Mathematica* to create graphical representations of these relations. Days of calculations by hand have shrunk to minutes by using *Mathematica*. Results can be verified within a few seconds, a task that took hours if not days in the past.

The present text uses *Mathematica* as a tool to discuss and to solve examples from physics. The intention of this book is to demonstrate the usefulness of *Mathematica* in everyday applications. We will not give a complete description of its syntax but demonstrate by examples the use of its language. In particular, we show how this modern tool is used to solve classical problems.

This second edition of *Mathematica in Theoretical Physics* seeks to prevent the objectives and emphasis of the previous edition. It is extended to include a full course in classical mechanics, new examples in quantum mechanics, and measurement methods for fractals. In addition, there is an extension of the fractal's chapter by a fractional calculus. The additional material and examples enlarged the text so much that we decided to divide the book in two volumes. The first volume covers classical mechanics and nonlinear dynamics. The second volume starts with electrodynamics, adds quantum mechanics and general relativity, and ends with fractals. Because of the inclusion of new materials, it was necessary to restructure the text. The main differences are concerned with the chapter on nonlinear dynamics. This chapter discusses mainly classical field theory and, thus, it was appropriate to locate it in line with the classical mechanics chapter.

The text contains a large number of examples that are solvable using *Mathematica*. The defined functions and packages are available on CD accompanying each of the two volumes. The names of the files on the CD carry the names of their respective chapters. Chapter 1 comments on the basic properties of *Mathematica* using examples from different fields of physics. Chapter 2 demonstrates the use of *Mathematica* in a step-by-step procedure applied to mechanical problems. Chapter 2 contains a one-term lecture in mechanics. It starts with the basic definitions, goes on with Newton's mechanics, discusses the Lagrange and Hamilton representation of mechanics, and ends with the rigid body motion. We show how *Mathematica* is used to simplify our work and to support and derive solutions for specific problems. In Chapter 3, we examine nonlinear phenomena of the Korteweg–de Vries equation. We demonstrate that *Mathematica* is an appropriate tool to derive numerical and analytical solutions even for nonlinear equations of motion. The second volume starts with Chapter 4, discussing problems of electrostatics and the motion of ions in an electromagnetic field. We further introduce *Mathematica* functions that are closely related to the theoretical considerations of the selected problems. In Chapter 5, we discuss problems of quantum mechanics. We examine the dynamics of a free particle by the example of the time-dependent Schrödinger equation and study one-dimensional eigenvalue problems using the analytic and numeric capabilities of *Mathematica*. Problems of general relativity are discussed in Chapter 6. Most standard books on Einstein's theory discuss the phenomena of general relativity by using approximations. With *Mathematica*, general relativity effects like the shift of the perihelion can be tracked with precision. Finally, the last chapter, Chapter 7, uses computer algebra to represent fractals and gives an introduction to the spatial renormalization theory. In addition, we present the basics of fractional calculus approaching fractals from the analytic side. This approach is supported by a package, FractionalCalculus, which is not included in this project. The package is available by request from the author. Exercises with which *Mathematica* can be used for modified applications. Chapters 2–7 include at the end some exercises allowing the reader to carry out his own experiments with the book.

Acknowledgments Since the first printing of this text, many people made valuable contributions and gave excellent input. Because the number of responses are so numerous, I give my thanks to all who contributed by remarks and enhancements to the text. Concerning the historical pictures used in the text, I acknowledge the support of the http://www-gapdcs.st-and.ac.uk/~history/ webserver of the University of St Andrews, Scotland. My special thanks go to Norbert Südland, who made the package FractionalCalculus available for this text. I'm also indebted to Hans Kölsch and Virginia Lipscy, Springer-Verlag New York Physics editorial. Finally, the author deeply appreciates the understanding and support of his wife, Carin, and daughter, Andrea, during the preparation of the book.

Cairo, Spring 2005

Gerd Baumann

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4 Electrodynamics

4.1 Introduction

This chapter is concerned with electric fields and charges encountered in different systems. Electricity is an ancient phenomenon already known by the Greeks. The experimental and theoretical basis of the current understanding of electrodynamical phenomena was established by two men: Michael Farady, the self-trained experimenter, and James Clerk Maxwell, the theoretician. The work of both were based on extensive material and knowledge by Coulomb. Farady, originally, a bookbinder, was most interested in electricity. His inquisitiveness in gaining knowledge on electrical phenomena made it possible to obtain an assistantship in Davy's lab. Farady (see Figure 4.1.1) was one of the greatest experimenters ever. In the course of his experiments, he discovered that a suspended magnet would revolve around a current bearing-wire. This observation led him to propose that magnetism is a circular force. He invented the dynamo in 1821, with which a large amount of our current electricity is generated. In 1831, he discovered electromagnetic induction. One of his most important contributions to physics in 1845 was his development of the concept of a field to describe magnetic and electric forces.

Figure 4.1.1. Michael Faraday: born September 22, 1791; died August 25, 1867.

Maxwell (see Figure 4.1.2) started out by writing a paper entitled "On Faraday's Lines of Force" (1856), in which he translated Faraday's theories into mathematical form. This description of Faraday's findings by means of mathematics presented the lines of force as imaginary tubes containing an incompressible fluid. In 1861, he published the paper "On Physical Lines of Force" in which he treated the lines of force as real entities. Finally, in 1865, he published a purely mathematical theory known as "On a Dynamical Theory of the Electromagnetic Field". The equations derived by Maxwell and published in "A Treaties on Electricity and Magnetism" (1873) are still valid and a source of basic laws for engineering as well as physics.

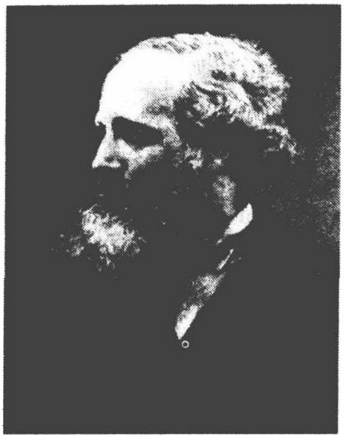

Figure 4.1.2. James Clerk Maxwell: born June 13, 1831; died November 5, 1879.

The aim of this chapter is to introduce basic phenomena and basic solution procedures for electric fields. The material discussed is a collection of examples. It is far from being complete by considering the huge diversity of electromagnetic phenomena. However, the examples discussed demonstrate how symbolic computations can be used to derive solutions for electromagnetic problems.

This chapter is organized as follows: Section 4.2 contains material on point charges. The exampl discuss the electric field of an assembly of discrete charges distributed in space. In Section 4.3, a standard boundary problem from electrostatics is examined to solve Poisson's equation for an angular segment. The dynamical interaction of electric fields and charged particles in a Penning trap is discussed in Section 4.4.

4.2 Potential and Electric Fields of Discrete Charge Distributions

In electrostatic problems, we often need to determine the potential and the electric fields for a certain charge distribution. The basic equation of electrostatics is Gauss' law. From this fundamental relation connecting the charge density with the electric field, the potential of the field can be derived. We can state Gauss' law in differential form by

$$\operatorname{div} \vec{E} = 4\pi\rho(\vec{r}). \qquad (4.2.1)$$

If we introduce the potential Φ by $\vec{E} = -\operatorname{grad}\Phi$, we can rewrite Eq. (4.2.1) for a given charge distribution ρ in the form of a Poisson equation

$$\Delta\Phi = -4\pi\rho \qquad (4.2.2)$$

where ρ denotes the charge distribution. To obtain solutions of Eq. (4.2..2), we can use the Green's function formalism to derive a particular solution. The Green's function $G(\vec{r}, \vec{r}')$ itself has to satisfy a Poisson equation where the continuous charge density is replaced by Dirac's delta function $\Delta_r G(\vec{r}, \vec{r}') = -4\pi\delta(\vec{r} - \vec{r}')$. The potential Φ is then given by

$$\Phi(\vec{r}) = \int_V G(\vec{r}, \vec{r}')\,\rho(\vec{r}\,')\,d^3 r'. \qquad (4.2.3)$$

In addition, we assume that the boundary condition $G|_V = 0$ is satisfied on the surface of volume V. If the space in which our charges are located is infinitely extended, the Green's function is given by

$$G(\vec{r}, \vec{r}\,') = \frac{1}{|\vec{r} - \vec{r}\,'|} \qquad (4.2.4)$$

The solution of the Poisson equation (4.2.3) becomes

$$\Phi(\vec{r}) = \int \frac{\rho(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|}\,d^3 r'. \qquad (4.2.5)$$

Our aim is to examine the potential and the electric fields of a discrete charge distribution. The charges are characterized by a strength q_i and are located at certain positions $\vec{r}_i$. The charge density of such a distribution is given by

$$\rho(\vec{r}) = \sum_{i=1}^{N} q_i\,\delta(\vec{r}_i). \qquad (4.2.6)$$