

Z427/1033(2009)-(38)



NUAA2010055236

Z427
1033 (2009) - (38)

经济与管理学院

教师



2010055236

(38)

| 序号 | 姓名 | 职称 | 单位 | 论文题目 | 刊物、会议名称 | 年、卷、期 | 类别 |
|-----|-----|-----|-----|---|---|---------------------------------------|-----|
| 73. | 陈晔 | 教授 | 091 | A Strategic Classification Support System for Brownfield Redevelopment | Environmental Modelling and Software | 2009 年 24 卷 pp. 647-654 | SCI |
| 74. | 陈晔 | 教授 | 091 | Using A Benchmark in Case-Based Multiple Criteria Ranking | IEEE Transactions on Systems, Man, and Cybernetics, Part A | 2009 年 39 卷 2 期 pp. 358-368 | SCI |
| 75. | 陈晔 | 教授 | 091 | A DEA-TOPSIS Method for Multiple Criteria Decision Analysis in Emergency Management | Journal of Systems Science and Systems Engineering | 2009 年 18 卷 4 期 pp. 489-507 | SCI |
| 76. | 陈晔 | 教授 | 091 | Strategic Performance Comparison of Provinces in China | 2009 年 IEEE International Conference on Systems, Man and Cybernetics 会议 | pp. 2321-2326 | EI |
| 77. | 徐海燕 | 教授 | 091 | Matrix Representation of Solution Concepts in Multiple Decision Maker Graph Models | IEEE Transactions on Systems, Man, and Cybernetics-Part A: Systems and Humans | 2009 年 39 卷 1 期 pp. 96-108 | SCI |
| 78. | 徐海燕 | 教授 | 091 | A Matrix-based Approach to Searching Colored Paths in a Weighted Colored Multidigraph | Applied Mathematics and Computation | 2009 年 215 卷 1 期 pp. 353-366, | SCI |
| 79. | 徐海燕 | 教授 | 091 | Multiple Levels of Preference in Interactive Strategic Decisions | Discrete Applied Mathematics | 2009 年 157 卷 15 期 pp. 3300-3313 | SCI |
| 80. | 徐海燕 | 教授 | 091 | A Matrix Approach to Status Quo Analysis in the Graph Model for Conflict Resolution | Applied Mathematics and Computation | 2009 年 212 卷 2 期 pp. 470-480 | SCI |
| 81. | 徐海燕 | 教授 | 091 | An Algebraic Approach to Calculating Stabilities in the Graph Model with Strength of Preference | Proceedings 2009 IEEE International Conference on Systems, Man and Cybernetics | 2009.pp 1539-1544 | EI |
| 82. | 罗正军 | 讲师 | 092 | 高校文科强化班培养模式的探索与实践 | 南航学报（社科版） | 2009.11.1 | |
| 83. | 张卓 | 教授 | 093 | A measurement of Organizational Complexity and Its Impact on Quality Economics---A Grey Perspective | 2009 IEEE SMC International Conference on Grey Systems and Intelligent Services | 2009 | |
| 84. | 党耀国 | 教授 | 091 | An Approach of the GM(1,1) Model Based on Linear Transformation | The Journal Of Grey System | 2009: 21 (3) | |
| 85. | 党耀国 | 教授 | 091 | The Optimization Method of Objective Weight in Grey Situation Decision | 2009 IEEE ICGSIS | 2009 | |
| 86. | 党耀国 | 教授 | 091 | Construction of New Weakening Buffer Operators Based on New Information and Their Applications | IEEE SMC 2009 | 2009 | |
| 87. | 肖龙阶 | 副教授 | 096 | 人民币汇率变动影响中国贸易条件机制研究 | 经济问题 | 2009.2 | |
| 88. | 周德群 | 教授 | 091 | 对加强能源软科学研究的思考 | 中国科学基金 | 2009 年 1 期 | |
| 89. | 张钦 | 副教授 | 091 | 创新型企业的评价指标体系和方法研究 | 科学管理研究 | 2009.27.6 | |

| | | | | | | | |
|-----|-----|-----|-----|--|--|-----------------|--|
| 90. | 张钦 | 副教授 | 091 | 国防科技工业创新能力的实证分析 | 统计与决策 | 2009.8 | |
| 91. | 张钦 | 副教授 | 091 | 基于投影寻踪模型的我国国防科技工业科研创新能力实证研究 | 科学管理研究 | 2009.27.3 | |
| 92. | 张钦 | 副教授 | 091 | 物流配送延迟的一个干扰管理模型 | 统计与决策 | 2008.26.10 | |
| 93. | 菅利荣 | 教授 | 091 | 基于优势粗糙集的建设项目过程评价 | 管理科学工程 | 2009年 18卷 5期 | |
| 94. | 菅利荣 | 教授 | 091 | A Hybrid approach of grey rough set and probabilistic neural network to uncertain decision | IEEE International conference on grey systems and intelligent services | 2009.10 | |
| 95. | 李南 | 教授 | 091 | 团队中枢节点的效率模型 | 数学的实践与认识 | 2009, 39, 3 | |
| 96. | 李南 | 教授 | 091 | 改革开放 30 年与女性创新人才培养 | 《改革开放 30 年与女性创新人才培养》论文集 | 上海大学出版社, 2009.5 | |
| 97. | 李敏 | 副教授 | 093 | 营销学课程探究式教学模式的探索与实践 | 理论探讨 | 2009年6期 | |
| 98. | 李敏 | 副教授 | 093 | 人情主义与中国式组织文化的主要特征 | 领导科学 | 2009年5期 | |
| 99. | 李敏 | 副教授 | 093 | 论中国组织文化的人情主义特色 | 江海学刊, 2009年第十三届世界管理论坛暨东方管理论坛会议论文集 | 2009年 | |
| 100 | 李敏 | 副教授 | 093 | 服务企业基于客户感知价值的顾客满意及客户价值研究 | 统计与决策 | 2009年4期 | |
| 101 | 余臻 | 讲师 | 091 | 基于协同标注的多用户社会化学习研究 | 企业经济 | 2009.12 | |
| 102 | 余臻 | 讲师 | 091 | 社会化标注系统中用户需求偏好的一种获取方法 | 南航学报 | 2009.41.1 | |
| 103 | 吴和成 | 教授 | 091 | 净现金流量的一种经验估计方法 | 运筹与管理 | 2009.18.1 | |
| 104 | 吴和成 | 教授 | 091 | 随机投入产出模型控制方程研究 | 浙江大学学报(理学版) | 2009.36.4 | |
| 105 | 彭灿 | 教授 | 093 | 技术能力、创新战略与创新绩效的关系研究 | 科研管理 | 2009年30卷2期 | |
| 106 | 彭灿 | 教授 | 093 | 突破性创新的资产基础与面向突破性创新的联盟战略 | 研究与发展管理 | 2009年21卷3期 | |
| 107 | 陈洪转 | 副教授 | 091 | 基于群决策 DEA 的农村水利投入产出研究 | 河海大学学报(自然版) | 2009.2 | |
| 108 | 陈洪转 | 副教授 | 091 | Study on Grey Evolutionary Game of "Industry-University-Institute" Cooperative Innovation | GSIS2009 | 2009 | |
| 109 | 王建玲 | 副教授 | 093 | The status quo and supply prediction for Suzhou Sci-tech innovative talents | SMC2009.IEEE International Coneference | 2009 | |
| 110 | 王建玲 | 副教授 | 093 | The Gap Prediction for Sci-tch Innovative Talents of Jiangsu Province | IEEE International conference on grey systems and intelligent services | 2009 | |
| 111 | 王建玲 | 副教授 | 093 | 服务接触视角的品牌延伸研究 | 科技进步与对策 | 2009.26.3 | |

| | | | | | | | |
|-----|-----|-----|-----|--|---|-----------------|--|
| 112 | 胡恩华 | 教授 | 093 | 企业集群创新行为影响因素的实证研究 | 研究与发展管理 | 2009.21.2 | |
| 113 | 陈万明 | 教授 | 093 | 我国高等教育供需关系及发展政策的重新审视 | 中国高教研究 | 2009 年 7 期 | |
| 114 | 李和新 | 工程师 | 090 | 大学生主题教育活动的评价研究 | 经济研究导刊 | 2009.10 | |
| 115 | 李和新 | 工程师 | 090 | 大学生职业生涯规划在高校思想政治教育中的应用探析 | 经济研究导刊 | 2009.12 | |
| 116 | 耿成轩 | 副教授 | 094 | 基于生命周期的家族企业融资行为动态变迁探析 | 管理世界 | 2009.9 | |
| 117 | 耿成轩 | 副教授 | 094 | 家族企业成长与财务治理演进 | 财会研究 | 209.6 | |
| 118 | 耿成轩 | 副教授 | 094 | 不断推进民营企业制度创新 | 人民日报 | 2009.03.27.07 | |
| 119 | 刘思峰 | 教授 | 091 | Emergence and development of grey systems theory | Kybernetes | 2009.38.7 | |
| 120 | 刘思峰 | 教授 | 091 | On the Astray of Complicated Models for Uncertain Systems | 2009 年 IEEE International Conference on Grey Systems and Intelligent Services | 2009 | |
| 121 | 刘思峰 | 教授 | 091 | On Positioned Solution of Linear Programming with Grey Parameters | 2009 IEEE International Conference on Systems, Man, and Cybernetics | 2009 | |
| 122 | 刘思峰 | 教授 | 091 | A Gray Input-Output model based on the Standard Interval Grey Number | IEEE International Conference on Fuzzy Systems | 2009.8.15 | |
| 123 | 刘思峰 | 教授 | 091 | 复杂交通网络中救援点与事故点间的路段重要性评价模型研究 | 中国管理科学 | 2009.1 | |
| 124 | 刘思峰 | 教授 | 091 | 区域国际合作关键技术评价指标的指标体系研究 | 科技进步与对策 | 2009.26.20 | |
| 125 | 林益 | 教授 | 091 | Economic yoyos and some mysteries about child labor | Kybernetes | 2009.38.1.2 | |
| 126 | 江可申 | 教授 | 095 | Integrating Strength of Preference into Status Quo Analysis | An International Meeting on Group Decision and Negotiation | 14-17 June 2009 | |
| 127 | | | | | | | |
| 128 | | | | | | | |
| 129 | | | | | | | |
| 130 | | | | | | | |



A strategic classification support system for brownfield redevelopment

Ye Chen^a, Keith W. Hipel^{b,*}, D. Marc Kilgour^c, Yuming Zhu^d

^a College of Economics and Management, Nanjing University of Aeronautics and Astronautics, Nanjing, Jiangsu 210016, China

^b Department of Systems Design Engineering, University of Waterloo, Waterloo, Ontario N2L 3G1, Canada

^c Department of Mathematics, Wilfrid Laurier University, Waterloo, Ontario N2L 3C5, Canada

^d School of Management, Northwestern Polytechnical University, Xi'an, Shaanxi 710072, China

ARTICLE INFO

Article history:

Received 7 June 2008

Received in revised form

11 October 2008

Accepted 15 October 2008

Available online 6 December 2008

Keywords:

Brownfield redevelopment

Strategic classification

Decision support system

Multiple criteria decision analysis

Rough set theory

ABSTRACT

Brownfield redevelopment (BR) is an ongoing issue for governments, communities, and consultants around the world. It is also an increasingly popular research topic in several academic fields. Strategic decision support that is now available for BR is surveyed and assessed. Then a dominance-based rough-set approach is developed and used to classify cities facing BR issues according to the level of two characteristics, BR effectiveness and BR future needs. The data for the classification are based on the widely available results of a survey of US cities. The unique features of the method are its reduced requirement for preference information, its ability to handle missing information effectively, and the easily understood linguistic decision rules that it generates, based on a training classification provided by experts. The resulting classification should be a valuable aid to cities and governments as they plan their BR projects and budgets.

© 2008 Elsevier Ltd. All rights reserved.

Software availability

Name: 4eMka2

Developer: Michal Lukasik, Maciej Luszczyński, Marcin Szumski and Marcin Zurawski

Contact address: Laboratory of Intelligent Decision Support Systems, Poznań University of Technology, Piotrowo 2, 60-965 Poznań, Poland

First available: 2001

Hardware: Windows 95 or later

Program size: 832KB

Availability: <http://idss.cs.put.poznan.pl/site/4emka.html>

Cost: free for any non-profit activities

Data availability

Name: US National Brownfields Survey

Developer: US Conference of Mayors

Contact address: The US Conference of Mayors, 1620 Eye Street, NW, Washington, DC 20006, USA

First available: 1998

Availability: <http://www.usmayors.org/brownfields>

Cost: free for any non-profit activities

1. Introduction

The world's population increased rapidly over the last century, and was recently estimated at 6.65 billion (US Census Bureau, 2008). In turn, the demand for land also increased rapidly. But land is a scarce good for which production possibilities are minimal, so in most countries the total value of land and related real estate has steadily increased over the last 40 years (Isaac, 2002). It is hardly surprising that effective land use management is now an important issue for communities and governments around the world.

Brownfields are abandoned, idle, or underutilized commercial or industrial properties where an active potential for redevelopment is restrained by known or suspected environmental contamination caused by past actions (USEPA, 2005). It is clear that restoration and redevelopment of brownfields can provide a range of economic, social, and environmental benefits, including restoration of environmental quality and improvement of quality of life for citizens, elimination of health threats, provision of land for housing or commercial purposes, creation of employment opportunities, expansion of the tax base for all levels of government, and reduction in the pressure on urban centers to expand into greenfields (NRTEE, 2003).

Thus, brownfields have recently been the focus of attention for governments, communities, environmental advocates, scientists, and researchers around the world. The US Environmental Protection Agency (USEPA), like agencies of many other governments

* Corresponding author. Tel.: +1 519 888 4567; fax: +1 519 746 4791.
E-mail address: kwhipel@uwaterloo.ca (K.W. Hipel).

around the globe, has supported brownfield redevelopment (BR) with many programs including an assessment and revolving loan fund, cleanup grants, and job training support (USEPA, 2008). Considerable research has addressed BR issues including development of remediation technologies, environmental assessment, risk assessment and management, financial arrangements, and community and public involvement (Brebba, 2006).

Many decision analysis tools have been applied to tackle challenging environmental management problems. For example, a fuzzy multicriteria method presented in Benetto et al. (2008) can help to interpret life cycle assessments by integrating uncertainty evaluations with application in electricity production scenarios. Nonetheless, strategic support for BR decisions at the government and community level is still lacking. One obvious problem is the lack of credible information about a city's situation—current and future—relative to other cities. In this paper, we take a major step toward the provision of this information by developing an approach to classify cities according to two basic BR characteristics. Unfortunately the only available dataset has substantial amounts of missing data, which was a problem for us to overcome.

Our classification approach, based on rough sets, has several features that helped us succeed. First, the method, based on linguistic rules for classification of new alternatives (elements, candidates, or, in this case, cities), retains much of its effectiveness despite missing data. Second, the input includes only minimal criterion preference information (direction only), and makes no assumptions about any pre-defined value (utility) function. The classification rules that are the heart of the system are developed from decisions furnished by DMs (experts) on a set of representative cases. It has been demonstrated that DMs often feel more at ease providing sample decisions than developing rigid model-based preference rules (Greco et al., 2001). Moreover, the fact that preference inputs were minimal made it feasible for us to achieve a consensus expert classification on the test sets.

The remainder of the paper is organized as follows: an overview of research on strategic level-based analysis of BR and the proposed strategic classification system are introduced in Section 2; rough set theory and the dominance-based rough set approach (DRSA) are explained in Section 3; DRSA is applied to produce a strategic classification of US cities according to two BR characteristics in Section 4; and some conclusions are presented in Section 5.

2. Strategic classification for brownfield redevelopment

2.1. Strategic analysis in brownfield redevelopment

Many research initiatives have examined issues in BR at the strategic decision level. These approaches can be roughly summarized into the following categories.

- *Survey-based investigation:* Several studies have relied on extensive surveys to investigate strategic issues in BR. For example, the five BR surveys (1998, 1999, 2000, 2003, 2006) conducted by the US Conference of Mayors examined BR problems faced by local communities throughout the US, focusing on the opportunities lost when properties remain idle and on the benefits of land recycling and return of brownfields to productive uses (US Conference of Mayors, 2008a). Another survey (George Washington University, 2008) estimated that in the US every brownfield acre that is redeveloped saves, on average, at least 4.5 acres of greenfields, which can be left undeveloped or devoted to other uses. Other surveys in BR have assessed project performance (Council for Urban Economic Development, 1999), the meaning of success (Lange and McNeil, 2004), and the characteristics of BR within a specific region (Wernstedt et al., 2008).

- *Qualitative approaches:* Both individual researchers and organizations have provided BR overviews of different regions and countries, or provided qualitative guides to practical BR activities. A comprehensive summary is available (see Brownfields Center, 2008) which includes a “road map” to assist BR stakeholders to identify and select innovative site characterization and cleanup technologies during the redevelopment process (USEPA, 2005); various economic, social, and environmental benefits to stakeholders in BR (NRTEE, 2003); and the brown-field experience overviews of four countries—United Kingdom, Germany, The Netherlands, and Canada—as well as insights into the role of the European Union (IEDC, 2005).
- *Quantitative analysis:* Some quantitative tools to support strategic analysis in BR have been proposed. For example, Chen et al. (2007) designed a two-level decision support procedure to integrate both qualitative and quantitative analysis methods to assist DMs in strategic BR decisions. Real option approaches designed in Erzi (2002) provide models of developers' investment choices incorporating various risk factors. A site ranking model is proposed to select brownfields for redevelopment in Thomas (2002).

In summary, qualitative BR research, especially survey-based, has been predominant, and there have been relatively few strategic-level quantitative tools to assist in BR decision analysis. Surveys that collect information about BR project attitudes and performance assessments from communities are needed. But so are sophisticated quantitative tools to examine survey data and extract information and knowledge to support decision making. Our proposal accomplishes this goal, as discussed next.

2.2. Strategic BR classification system

Classification constitutes a fundamental technique for assessing and understanding a situation. Much human progress can be attributed to the development of appropriate classification systems, like musical notation or the periodic table in chemistry. Classification not only facilitates understanding and development but also improves decision making. Under the umbrella of multiple criteria decision analysis (MCDA), a strategic classification model is designed for BR and applied to evaluation of the overall performance of BR in different US cities, based on the available survey information (US Conference of Mayors, 2008a). A brief introduction of MCDA and its role in classification analysis is given next.

2.2.1. MCDA and classification in MCDA

MCDA constitutes a set of techniques designed to evaluate and compare alternatives systematically, based on diverse criteria that are typically conflicting. For example, projects can be evaluated according to criteria that reflect environmental, economic, and social objectives. As discussed by (Roy, 1996), there are three fundamentally different models for the assessment of a set of alternatives, **A**:

- *Choice.* Choose the best alternative from **A**.
- *Ranking.* Rank the alternatives of **A** from best to worst.
- *Sorting.* Sort the alternatives of **A** into relatively homogeneous groups, arranged in preference order.

Of the many different approaches that have been proposed to tackle choice and ranking tasks, perhaps the best known are multiattribute utility theory (Keeney and Raiffa, 1976) and the analytic hierarchy process (Saaty, 1980). Recently, the sorting problem or more general classification has received more study (for example, Doumpos and Zopounidis (2002) and Greco et al. (2001)), and has

been generalized to classification by dropping the “preference order” condition (see Chen et al., 2006).

2.2.2. Brownfield redevelopment performance classification in the US

A great deal of data on the BR experience in different US cities has been accumulated in the surveys periodically conducted by the US Conference of Mayors (2008a). The latest information is a 2006 report that summarizes the status of brownfield sites in more than 200 US cities (US Conference of Mayors, 2006). The information includes city population, numbers and areas of brownfield sites, in-progress sites, and redeveloped sites, estimated annual tax revenue gained from BR, estimated jobs created from BR and estimated population capacity. Although the report (US Conference of Mayors, 2006) includes a summary, it is clear that a more quantitative analysis will assist at assessing each city's BR success, and its correlation to other information. Since more than 200 cities are included in the survey, it is more efficient to group them into manageable and meaningful categories first, in order to design or identify policies appropriate to each group. We demonstrate below that such a classification is feasible and useful.

The general plan for our strategic classification system is shown in Fig. 1. Let $A = \text{Brownfields Center}\{a^1, \dots, a^{|A|}\}$ denote the set of cities under review. We focus on two key BR characteristics of cities: *effectiveness* and *future needs*. The evaluation of each characteristic is based on a specific set of criteria; we use $C = \{c_1, \dots, c_j, \dots, c_{|C|}\}$ to denote the criteria for assessment of BR effectiveness and $C' = \{c'_1, \dots, c'_j, \dots, c'_{|C'|}\}$ to denote the criteria for evaluation of future needs. Specifically, the comprehensive survey data (US Conference of Mayors, 2006) appropriate to these characteristics is as follows:

For C (BR effectiveness)

1. c_1 : number of redeveloped brownfield sites;
2. c_2 : area of redeveloped brownfield sites (acres);
3. c_3 : number of in-progress brownfield sites;
4. c_4 : area of in-progress brownfield sites (acres);
5. c_5 : conservative estimate tax revenue tax gained (\$);
6. c_6 : optimistic estimate tax revenue tax gained (\$);
7. c_7 : actual tax revenue tax gained (\$);
8. c_8 : jobs created during redevelopment;

9. c_9 : permanent jobs created.

For C' (BR future needs)

1. c'_1 : population;
2. c'_2 : estimated number of brownfield sites;
3. c'_3 : estimated area of brownfield sites (acres);
4. c'_4 : number of redeveloped brownfield sites;
5. c'_5 : area of redeveloped brownfield sites (acres);
6. c'_6 : number of in-progress brownfield sites;
7. c'_7 : area of in-progress brownfield sites (acres);
8. c'_8 : estimated capacity to absorb population (with no increase in infrastructure);
9. c'_9 : estimated number of “moth-balled” sites (A “moth-balled” site is a site that is probably a brownfield but cannot be assessed due to lack of cooperation by the owner.).

Note that other criteria that may be relevant to the evaluation of BR performance could not be included in C and C' , because data was not available. For example, criteria referring to redevelopment cost, type of contamination and location of the BR within a city could contribute to the assessment of the potential redevelopment ability of a city. Furthermore, the extent of ongoing reclamation work negatively influences the ability of a city to undertake new initiatives. Criteria including capacity employed, budget spent on past projects and short-term area demand may better express the city's priority for new reclamation projects. It would be easy to incorporate additional criteria in the proposed method should suitable data become available.

A criterion is called positive if a higher measure on the criterion is associated with more of the characteristic, and negative if a higher measure on the criterion is associated with less of the characteristic. For example, all criteria in C are positive in that a higher measure on any one of the criteria is *prima facie* associated with greater BR effectiveness, whereas in C' criteria c'_1, c'_2, c'_3 and c'_9 are positive while c'_4, c'_5, c'_6, c'_7 and c'_8 are negative, i.e. higher measures on any criterion in the latter subgroup is associated with reduced values of BR future needs.

As depicted in Fig. 1, all cities are evaluated according to all criteria in each of the two sets and then classified into three groups, I, II and III, representing high, medium, and low values of the two

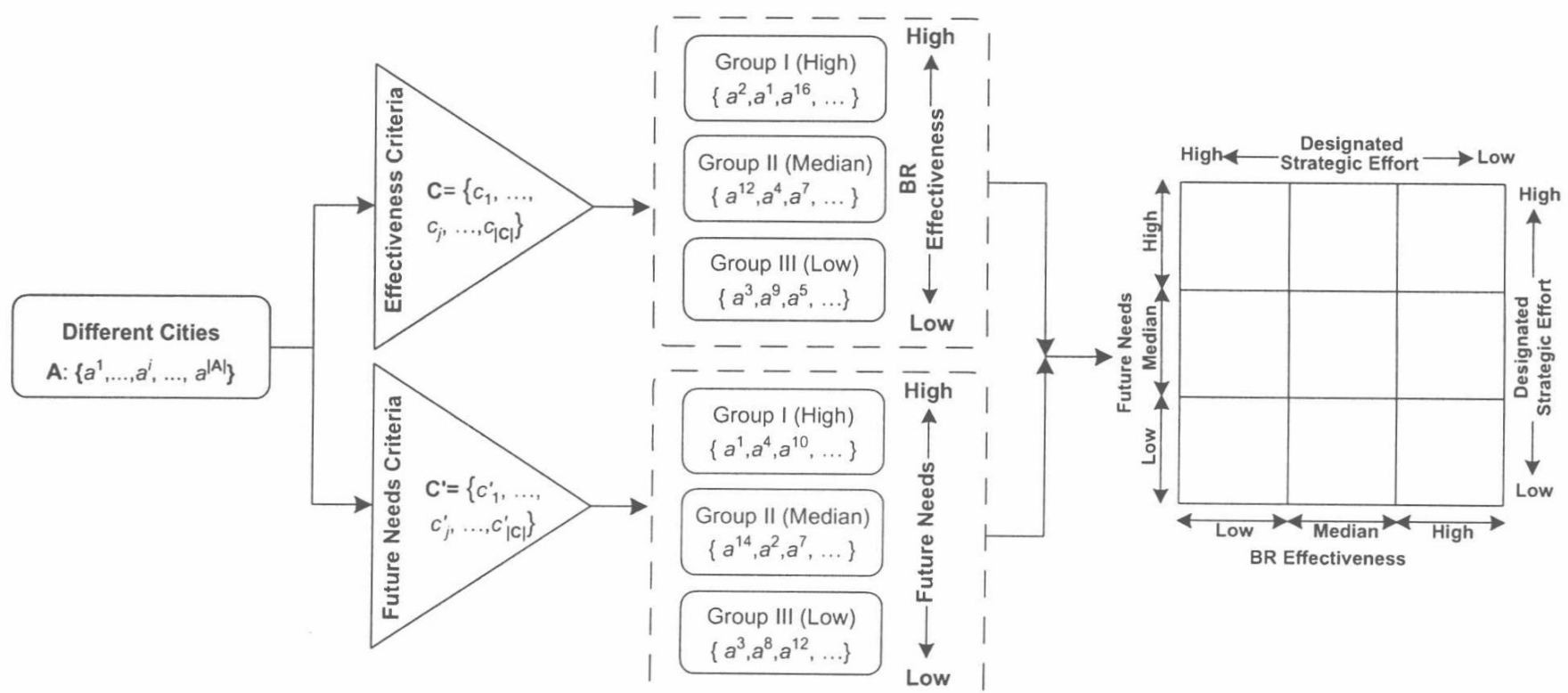


Fig. 1. Strategic BR classification framework.

characteristics. Of course the number of groups can be adjusted for convenience, and taking into account the amount of data available. Clearly strategic planning should take into account the characteristics of a city. For example, a city with a low level of BR effectiveness needs extra effort from governments and community groups, while a city with a low level of BR future needs can begin to reduce the priority of its BR programs.

A dominance-based rough set approach (DRSA) is a good choice as the primary analysis tool to classify cities according to the two characteristics because it requires minimal direct preference information, and incorporates easily understood linguistic rule-based judgments. A brief introduction of rough set theory and the DRSA is given next.

3. The dominance-based rough set approach

3.1. Rough set theory

Rough set theory was introduced by Pawlak (1982) in the early 1980s as a tool to describe dependencies among attributes, to evaluate the significance of individual attributes, and hence, to provide “automated transformation of data into knowledge” (Pawlak, 1982). Its unique approach to uncertainty or vagueness in data makes rough set theory complementary to probability theory, evidence theory, fuzzy set theory, and other methods. Recent advances in rough set theory have made it a powerful tool for data mining, pattern recognition, and information representation. For example, a rule-based procedure based on the modified rough set induction method has been found to be an efficient tool for the environmental life cycle assessment of alternative energy sources (Tan, 2005). A comprehensive literature review of rough set theory, including new research directions and applications, appears in Pawlak and Skowron (2007).

An important principle of rough sets of alternatives is that all relevant information, including both conditions and decision attributes, are expressed in the data (Pawlak, 1982). Condition attributes refer to the characteristics of the alternatives; for example, condition attributes describing a firm may include size, profitability, liquidity ratios and market position. Decision attributes define a partition of the alternatives into groups reflecting the overall situation of condition attributes. In terms of MCDA, condition and decision attributes can be interpreted as, respectively, criteria and final decision assessments (such as sorting results).

As pointed out in Greco et al. (2001), the original rough set approach cannot efficiently extract knowledge (such as a DM's preferences) from the analysis of a case set involving a human's decision judgments. The Dominance-based Rough Set Approach (DRSA) is an extension of rough set theory that is highly effective for MCDA because it replaces the *indiscernibility* relation with a *dominance* relation, providing a means to deal with inconsistent comparisons of alternatives according to a criterion, and in preference-ordered classes (groups). The main idea of DRSA is summarized below.

3.2. The dominance-based rough set approach

3.2.1. The basic structure

In an MCDA sorting problem, a set of alternatives $A = \{a^1, \dots, a^i, \dots, a^{|A|}\}$ is to be assigned to an ordered partition, $CI = \{cl_1, \dots, cl_t, \dots, cl_{|CI|}\}$. The subsets in the partition are called classes, and the assignment is to reflect a set of criteria $C = \{c_1, \dots, c_j, \dots, c_{|C|}\}$. The intent is that, if $1 \leq g < h \leq |CI|$, then the DM prefers every alternative in cl_g to any alternative in cl_h . Hence, CI is often written $cl_1 < cl_2 < \dots < cl_{|CI|}$, where $<$ denotes “is preferred to”.

The DRSA requires a representative case set for data training and elicitation of linguistic decision rules. The inferred rules are then

applied globally to evaluate all alternatives. The DM is asked to partition $T = \{t^1, \dots, t^i, \dots, t^{|T|}\}$ into CI in such a way that $cl_t \neq \emptyset$ and $\bigcup_{t=1}^{|T|} cl_t = T$ for all $t = 1, \dots, |T|$ and, of course, so that the subsets of the partition are arranged according to preference, as described above.

The procedure to apply DRSA to sort A is illustrated in Fig. 2. Note that m_j^i is the performance (consequence) measurement of case t^i over criterion j . It is assumed that the DM's preference over performance on each criterion is monotonic, i.e., that each criterion c_j is either positive or negative. Once this preference direction is specified for each criterion, the DRSA can be utilized to extract a set of linguistic rules, R , that captures the preferential information inherent in the sorting of T supplied by the DM, and apply R to sort A into $|CI|$ classes, $cl_1 < cl_2 < \dots < cl_{|CI|}$.

Define the *upward union* (denoted by the superscript “ \geq ”) and the *downward union* (denoted by the superscript “ \leq ”) by $cl_r^{\geq} = \bigcup_{s \leq r} cl_s$ and $cl_r^{\leq} = \bigcup_{s \geq r} cl_s$, respectively, where $r = 1, \dots, |CI|$. It is easy to demonstrate that $cl_{|CI|}^{\geq} = cl_1^{\leq} = CI$, $cl_1^{\geq} = cl_1$ and $cl_{|CI|}^{\leq} = cl_{|CI|}$.

3.2.2. Approximation of partitions

Fix a subset of the criteria $P \subseteq C$. Given the sorting of the case set T supplied by the DM, we can define D_P , a binary preference relation on T where, for any $t^i, t^j \in T$, $t^i D_P t^j$ means that “ t^i is at least as preferred as t^j with respect to P ” in the sorting of T . Assume that D_P is a complete preorder, i.e. D_P is a relation that is reflexive, transitive, and complete. Given $t^i \in T$, define the P -dominating set $t^i D_P^+ = \{t^j \in T : t^i D_P t^j\}$, and the P -dominated set for t^i , $D_P^-(t^i) = \{t^j \in T : t^j D_P t^i\}$, respectively.

Next, define P -lower and P -upper approximations of cl_t^{\geq} to be $\underline{P}(cl_t^{\geq}) = \{t^i \in T : D_P^+(t^i) \subseteq cl_t^{\geq}\}$ and $\overline{P}(cl_t^{\geq}) = \{t^i \in T : D_P^-(t^i) \cap cl_t^{\geq} \neq \emptyset\}$, respectively. Similarly, the P -lower and P -upper approximations of cl_t^{\leq} are $\underline{P}(cl_t^{\leq}) = \{t^i \in T : D_P^-(t^i) \subseteq cl_t^{\leq}\}$ and $\overline{P}(cl_t^{\leq}) = \{t^i \in T : D_P^+(t^i) \cap cl_t^{\leq} \neq \emptyset\}$, respectively.

It is easy to verify that the P -lower and P -upper approximations defined above satisfy the following conditions for each $r = 1, 2, \dots, |CI|$: $\underline{P}(cl_r^{\geq}) \subseteq cl_r^{\geq} \subseteq \overline{P}(cl_r^{\geq})$ and $\underline{P}(cl_r^{\leq}) \subseteq cl_r^{\leq} \subseteq \overline{P}(cl_r^{\leq})$. Now define the P -boundaries of cl_r^{\geq} and cl_r^{\leq} by $Bn_P(cl_r^{\geq}) = \overline{P}(cl_r^{\geq}) - \underline{P}(cl_r^{\geq})$ and $Bn_P(cl_r^{\leq}) = \overline{P}(cl_r^{\leq}) - \underline{P}(cl_r^{\leq})$, respectively. Then a reasonable measure of the quality of the approximation to CI using only the criteria P on the test set T is

$$\gamma_P(CI) = \frac{|T - [(\bigcup_{cl_t \in CI} Bn_P(cl_t^{\geq})) \cup (\bigcup_{cl_t \in CI} Bn_P(cl_t^{\leq}))]|}{|T|}$$

3.2.3. Decision rules elicitation

The approximations obtained through dominance analysis can be used to construct decision rules that capture the preference information underlying the sorting of the case set. Assume that all

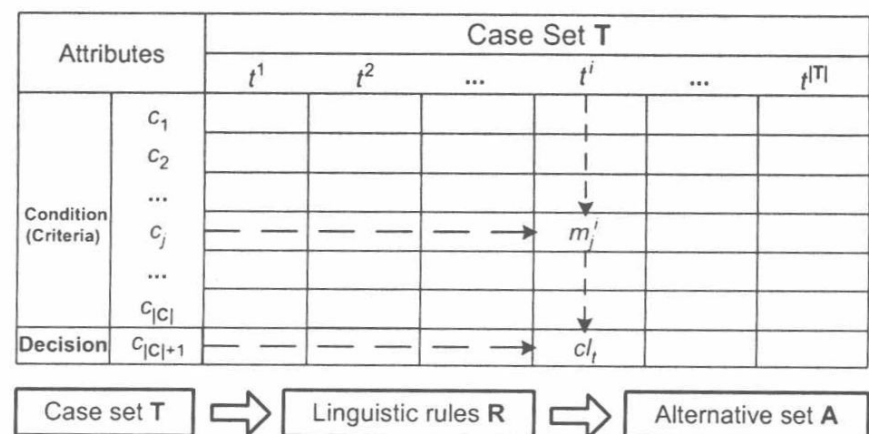


Fig. 2. Procedure to apply DRSA.

Table 1
Representative case set for BR effectiveness, **T**

| City | c_1 | c_2 | c_3 | c_4 | c_5 | c_6 | c_7 | c_8 | c_9 | Group |
|-----------------------------|-------|-------|-------|-------|-----------|-----------|-----------|-------|-------|-------|
| Akron (t^1) | 9 | 40 | 7 | 27 | 1900000 | 2500000 | 1650000 | 1100 | 1400 | H |
| Albuquerque (t^2) | 5 | 15 | 5 | 20 | 5000000 | 1300000 | 7920000 | 150 | 450 | H |
| Athens (t^3) | 1 | 2 | 1 | 100 | 500000 | 1000000 | 100000 | 30 | 180 | L |
| Baton Rouge (t^4) | 28 | 41 | 12 | 21 | 250000 | 500000 | 175000 | 75 | 175 | M |
| Buffalo (t^5) | 18 | 300 | 5 | 150 | 5000000 | 15000000 | 350000 | 725 | 825 | H |
| Charleston (t^6) | 6 | 30 | 7 | 35 | 1000000 | 2500000 | 57000 | 35 | 75 | L |
| Clearwater (t^7) | 70 | 100 | 20 | 45 | 2000000 | 3500000 | 1000000 | 729 | 773 | H |
| Columbus (t^8) | 12 | 50 | 2 | 75 | 2000000 | 200000 | 500000 | 1500 | 2000 | M |
| Elizabeth (t^9) | 12 | 195 | 9 | 30 | 30000000 | 45000000 | 6600000 | 5250 | 7250 | H |
| Emeryville (t^{10}) | 30 | 150 | 50 | 50 | 3000000 | 6000000 | 2000000 | 8000 | 8500 | H |
| Fitchburg (t^{11}) | 2 | 12 | 2 | 12 | 100000 | 250000 | 200000 | 20 | 50 | L |
| Frisco (t^{12}) | 18 | 82 | 5 | 8 | 140000001 | 180000001 | 103885604 | 15 | 50 | H |
| Indianapolis (t^{13}) | 40 | 8 | 15 | 10 | 5000000 | 10000000 | 1000000 | 225 | 725 | H |
| La Crosse (t^{14}) | 5 | 10 | 3 | 5 | 1000000 | 4000000 | 600000 | 900 | 925 | M |
| Lafayette (t^{15}) | 2 | 5 | 1 | 3 | 1000000 | 10000000 | 15000000 | 150 | 350 | M |
| Las Vegas (t^{16}) | 6 | 10 | 5 | 8 | 150000 | 750000 | 35356 | 136 | 290 | L |
| Lynn (t^{17}) | 8 | 5 | 1 | 19 | 8000000 | 20000000 | 400000 | 100 | 110 | L |
| Montgomery (t^{18}) | 2 | 11 | 1 | 2 | 1000000 | 6000000 | 1000000 | 300 | 550 | M |
| New Orleans (t^{19}) | 28 | 60 | 11 | 35 | 5000000 | 20000000 | 2000000 | 300 | 785 | M |
| Ocala (t^{20}) | 12 | 20 | 5 | 18 | 25000 | 50000 | 60000 | 300 | 320 | L |
| Pittsburgh (t^{21}) | 25 | 700 | 10 | 200 | 5000000 | 50000000 | 10000000 | 10000 | 15000 | H |
| Richmond (t^{22}) | 5 | 20 | 2 | 10 | 300000 | 1000000 | 1000000 | 60 | 210 | M |
| Rochester (t^{23}) | 20 | 285 | 13 | 75 | 50000000 | 150000000 | 4000000 | 193 | 233 | H |
| Springfield (t^{24}) | 1 | 3 | 3 | 13 | 15000000 | 36000000 | 1300000 | 20 | 35 | M |
| Trenton (t^{25}) | 50 | 100 | 20 | 50 | 1500000 | 2500000 | 1000000 | 500 | 1000 | H |
| Waco (t^{26}) | 8 | 38 | 3 | 13 | 637500 | 900000 | 176000 | 300 | 750 | M |
| West Hollywood (t^{27}) | 1 | 4 | 1 | 3 | 4000000 | 10000000 | 1500000 | 800 | 925 | H |
| Winston-Salem (t^{28}) | 5 | 50 | 2 | 40 | 200000 | 1000000 | 75000 | 30 | 35 | L |

criteria are positive, i.e. that $m_j(t^i) \geq m_j(t^l)$ implies $t^i D_{c_j} t^l$ for all $c_j \in \mathbf{C}$ and all $t^i, t^l \in \mathbf{T}$. Then, three types of decision rules, to be used to sort **A** into **Cl**, can be generated from the non-empty set of criteria $\mathbf{P} \subseteq \mathbf{C}$.

- D_{\geq} -decision rule: If $m_j(t^i) \geq r_j$, for all $c_j \in \mathbf{P}$, then $t^i \in cl_{\geq}^{\mathbf{P}}$, where $r_j \in \mathbb{R}$ is a performance threshold for criterion c_j .
- D_{\leq} -decision rule: If $m_k(t^i) \leq r_k$, for all $c_k \in \mathbf{P}$, then $t^i \in cl_{\leq}^{\mathbf{P}}$, where $r_k \in \mathbb{R}$ is a performance threshold for criterion c_k .

- $D_{\geq \leq}$ -decision rule: If $m_j(t^i) \geq r_j$, for all $c_j \in \mathbf{O}$ and $m_k(t^i) \leq r_k$, for all $c_k \in \{\mathbf{P} - \mathbf{O}\}$, then $t^i \in cl_t \cup cl_{t+1} \cup \dots \cup cl_s$, where $r_j, r_k \in \mathbb{R}$ are performance thresholds for criteria c_j and c_k , respectively.

A set of decision rules is *complete* iff every case in the case set **T** can be classified into one or more groups according to the rules, i.e. no alternative remains unclassified. A set of decision rules is *minimal* if it is complete and non-redundant, i.e. exclusion of any rule makes

Table 2
Representative case set for BR future needs, **T'**

| City | c'_1 | c'_2 | c'_3 | c'_4 | c'_5 | c'_6 | c'_7 | c'_8 | c'_9 | Groups |
|---------------------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| Akron (t^1) | 217074 | 44 | 200 | 9 | 40 | 7 | 27 | 6000 | 12 | M |
| Bridgeport (t^2) | 139529 | 250 | 300 | 7 | 15 | 5 | 15 | 7000 | 20 | M |
| Buffalo (t^3) | 292648 | 300 | 2000 | 18 | 300 | 5 | 150 | 300000 | 20 | L |
| Calumet City (t^4) | 39071 | 11 | 46 | 4 | 6 | 2 | 3 | 0 | 7 | H |
| Charleston (t^5) | 96650 | 100 | 600 | 6 | 30 | 7 | 35 | 0 | 6 | H |
| Council Bluffs (t^6) | 58268 | 5 | 30 | 1 | 3 | 1 | 2 | 10000 | 3 | M |
| Detroit (t^7) | 951270 | 1000 | 10000 | 150 | 3000 | 50 | 1500 | 300000 | 600 | H |
| Easthampton (t^8) | 15994 | 12 | 60 | 3 | 6 | 3 | 8 | 3000 | 2 | M |
| Emeryville (t^9) | 6882 | 150 | 200 | 30 | 150 | 50 | 50 | 5000 | 50 | M |
| Gainesville (t^{10}) | 95447 | 75 | 200 | 3 | 15 | 6 | 40 | 5000 | 40 | M |
| Harrisburg (t^{11}) | 48950 | 18 | 90 | 7 | 9 | 5 | 20 | 25000 | 9 | L |
| Indianapolis (t^{12}) | 791926 | 400 | 500 | 40 | 8 | 15 | 10 | 50000 | 25 | L |
| Las Vegas (t^{13}) | 478434 | 20 | 40 | 6 | 10 | 5 | 8 | 25000 | 5 | M |
| Lowell (t^{14}) | 105167 | 365 | 1000 | 30 | 100 | 20 | 50 | 10 | 5 | H |
| Marlborough (t^{15}) | 36255 | 3 | 6 | 1 | 3 | 2 | 3 | 12000 | 1 | L |
| Ocala (t^{16}) | 45943 | 140 | 300 | 12 | 20 | 5 | 18 | 5000 | 3 | M |
| Owensboro (t^{17}) | 54067 | 100 | 180 | 1 | 2 | 1 | 2.5 | 10000 | 4 | H |
| Pittsburg (t^{18}) | 56769 | 15 | 250 | 3 | 20 | 4 | 50 | 2500 | 5 | M |
| Port Arthur (t^{19}) | 57755 | 30 | 226 | 1 | 1 | 15 | 31 | 4000 | 14 | M |
| Providence (t^{20}) | 173618 | 250 | 2500 | 1 | 10 | 10 | 100 | 15000 | 25 | H |
| Rochester (t^{21}) | 219773 | 1000 | 975 | 20 | 285 | 13 | 75 | 75000 | 200 | H |
| Rock Island (t^{22}) | 39684 | 50 | 50 | 5 | 27 | 5 | 15 | 10000 | 1 | M |
| South Bend (t^{23}) | 107789 | 243 | 350 | 11 | 137 | 4 | 180 | 30000 | 200 | H |
| Terre Haute (t^{24}) | 59614 | 10 | 80 | 2 | 10 | 2 | 65 | 10000 | 6 | L |
| Toledo (t^{25}) | 313619 | 100 | 1200 | 7 | 345 | 6 | 360 | 90 | 10 | H |
| Westland (t^{26}) | 86602 | 9 | 111 | 1 | 35 | 1 | 35 | 20000 | 1 | L |
| Wilmington (t^{27}) | 72664 | 275 | 1500 | 100 | 100 | 10 | 20 | 50000 | 5 | L |

4eMka2 - BR effectiveness.isf - [Browse Generated Rules (Minimal Cover Algorithm)]

File Show Calculate Classify Report Window Help

Filter Options:
Relative Strength: - Support: -

Generated Rules: 13 Displayed Rules: 13

| Number | Condition | Decision | Support | Relative Strength [%] |
|--------|---|------------------|---------|-----------------------|
| 1. | (c7 <= 100000) | Class at most L | 5 | 71.43 |
| 2. | (c5 <= 100000) | Class at most L | 2 | 28.57 |
| 3. | (c3 <= 1) & (c9 <= 110) | Class at most L | 1 | 14.29 |
| 4. | (c5 <= 1000000) | Class at most M | 12 | 75.00 |
| 5. | (c6 <= 200000) | Class at most M | 2 | 12.50 |
| 6. | (c8 <= 300) & (c7 <= 2000000) & (c3 <= 11) | Class at most M | 12 | 75.00 |
| 7. | (c2 >= 82) | Class at least H | 8 | 66.67 |
| 8. | (c1 >= 40) | Class at least H | 3 | 25.00 |
| 9. | (c7 >= 1500000) & (c9 >= 450) & (c7 >= 7920000) | Class at least H | 2 | 16.67 |
| 10. | (c8 >= 800) & (c7 >= 1500000) | Class at least H | 5 | 41.67 |
| 11. | (c7 >= 500000) | Class at least M | 18 | 85.71 |
| 12. | (c3 >= 12) | Class at least M | 6 | 28.57 |
| 13. | (c9 >= 750) | Class at least M | 12 | 57.14 |

Fig. 3. Decision rules generated for BR effectiveness.

the set incomplete (Greco et al., 2001). Let $R = \{r_1, \dots, r_i, \dots, r_{|R|}\}$ be a set of decision rules generated from the classification of T . The relative strength of decision rule r_i is $\beta(r_i)$, the ratio of the number of cases supporting r_i to the cardinality of the lower approximation of the classes with which the decision rule r_i is associated. The most popular rule induction for DRSA is DOMLEM, which has been implemented in the software 4eMka2 (ICS, 2008).

4. Data analysis and results

Our objective was to classify a data set consisting of 201 US cities with brownfield redevelopment (BR) projects (see US Conference of Mayors, 2006) according to BR effectiveness and BR future needs. We followed the DRSA analysis procedure described in Section 3. Our plan was to use a relatively small test set in order to present

4eMka2 - Demanding for future attention.isf - [Browse Generated Rules (Minimal Cover Algorithm)]

File Show Calculate Classify Report Window Help

Filter Options:
Relative Strength: - Support: -

Generated Rules: 16 Displayed Rules: 16

| Number | Condition | Decision | Support | Relative Strength [%] |
|--------|--|------------------|---------|-----------------------|
| 1. | (c'3 <= 6) | Class at most L | 1 | 14.29 |
| 2. | (c'4 >= 40) & (c'3 <= 1500) | Class at most L | 2 | 28.57 |
| 3. | (c'2 <= 10) & (c'7 >= 35) | Class at most L | 2 | 28.57 |
| 4. | (c'8 >= 300000) & (c'9 <= 20) | Class at most L | 1 | 14.29 |
| 5. | (c'1 <= 48950) & (c'8 >= 25000) | Class at most L | 1 | 14.29 |
| 6. | (c'9 <= 3) | Class at most M | 6 | 33.33 |
| 7. | (c'2 <= 44) & (c'6 >= 4) | Class at most M | 5 | 27.78 |
| 8. | (c'5 >= 15) & (c'3 <= 300) | Class at most M | 8 | 44.44 |
| 9. | (c'8 <= 90) | Class at least H | 4 | 44.44 |
| 10. | (c'9 >= 200) | Class at least H | 3 | 33.33 |
| 11. | (c'3 >= 2500) | Class at least H | 2 | 22.22 |
| 12. | (c'7 <= 2.5) & (c'9 >= 4) | Class at least H | 1 | 11.11 |
| 13. | (c'8 <= 7000) | Class at least M | 12 | 60.00 |
| 14. | (c'7 <= 2.5) | Class at least M | 2 | 10.00 |
| 15. | (c'1 >= 478434) & (c'8 <= 25000) | Class at least M | 1 | 5.00 |
| 16. | (c'7 <= 15) & (c'2 >= 50) & (c'4 <= 5) | Class at least M | 2 | 10.00 |

Fig. 4. Decision rules generated for BR future needs.

Table 3
Classification results

| | | BR effectiveness | | | | | | Summary |
|-----------------|------------------|------------------|----------|----------|--------------|--------------|------------------|---------|
| | | <i>H</i> | <i>M</i> | <i>L</i> | <i>H ∪ M</i> | <i>M ∪ L</i> | <i>H ∪ M ∪ L</i> | |
| BR future needs | <i>H</i> | 5 | 11 | 5 | 1 | 2 | 14 | 38 |
| | <i>M</i> | 6 | 8 | 2 | 3 | 5 | 7 | 31 |
| | <i>L</i> | 4 | 2 | 3 | 4 | 4 | 7 | 24 |
| | <i>H ∪ M</i> | 2 | 3 | 3 | 4 | 3 | 15 | 30 |
| | <i>M ∪ L</i> | 0 | 2 | 4 | 5 | 3 | 7 | 21 |
| | <i>H ∪ M ∪ L</i> | 8 | 8 | 5 | 3 | 10 | 23 | 57 |
| | Summary | 25 | 34 | 22 | 20 | 27 | 73 | 201 |

a problem of manageable size to our experts (and not to demand too much time).

4.1. Expert and representative case set identification

The input for DRSA depends on a case set of representative cases for which experts' holistic assessments is required for each BR characteristic. Formally, the 201 cities listed in the survey report constitute the alternative set, *A*. However, only 28 cities had complete information for all nine criteria for BR effectiveness (*C*), and only 27 cities had complete information for all nine criteria for BR future needs, *C'*. Therefore the case set on each characteristic consisted of the 27 or 28 cities for which complete information was available. In principle, it is not a problem to generalize this data to all 201 cities, since the DRSA can handle missing data. The two case sets were denoted *T* (for BR effectiveness) and *T'* (for BR future needs), respectively.

We approached several groups of experts, including members of our research group and their colleagues at four universities, officials in charge of BR activities in the City of Kitchener, Ontario, and project managers for professional environmental consultants. Depending on the time they had available, we presented experts with the full case sets, *T* and *T'*, or randomly selected subsets of appropriate size. The experts were asked to assign cities into three pre-defined groups, *H*, *M*, and *L*, representing high, medium, and low groups of BR effectiveness and BR future needs. There were some variations of judgment initially. We shared the data, and after a few iterations, a unanimous judgment was reached. The complete data sets and classifications of *T* and *T'* are shown in Tables 1 and 2, respectively.

4.2. Decision rule generation

The software, 4eMka2 (ICS, 2008), was employed for data training and decision rule generation. The main steps are *data input*, *dominance relationship identification*, and *linguistic decision rule elicitation*. Using the minimal cover algorithm (Greco et al., 2001), 13 rules were identified for classification of BR effectiveness, as shown in Fig. 3, and 16 rules were generated for classification of BR future needs, as displayed in Fig. 4.

4.3. Results and insights

The linguistic decision rules were then applied to classify all 201 cities for the two BR characteristics. The detailed calculations are omitted here. The overall classification results can be conveniently displayed in the joint classification matrix shown Table 3. Ambiguities in the classification are indicated using "U" to represent "or". For example, a city labelled "*H ∪ M*" belongs to either the *H* or *M* group, and a city labelled "*H ∪ M ∪ L*" could be placed in any group, i.e., it cannot be distinguished by the decision rules.

Some observations directly from Table 3 are summarized next.

- Information for roughly 30% of the cities (73 for BR effectiveness and 57 for future need) is insufficient to classify them in

any way at all, i.e. they are placed in *H ∪ M ∪ L*. The inability to classify these cities is primarily due to the large amount of missing data.

- In fact, an enormous amount of data is missing from the survey results; less than 15% of the cities had complete data sets. DRSA, because of its unique reliance on linguistic rule-based decision judgments, can often overcome problems of missing information. For example, if a city has 50 redeveloped brownfield sites, then according to decision rule 8 of Fig. 3 (if $c_1 \geq 40$, set in class *H*) then the city is classified as high on the BR effectiveness characteristic. In other words, such a city can be definitively put into class *H* on BR effectiveness, even though all other performance measures may be missing. Thus, we conclude that DRSA did a good job despite all of the missing information in this case. The proportion of ambiguous or inconclusive classifications (group *H ∪ M ∪ L*) to cases with missing data was 73:173 for BR effectiveness and 57:174 for future needs, roughly 40% for each characteristic.
- Overall, more cities were assessed as high on future needs (there were 38 cities in class *H*) than were evaluated as low on BR effectiveness (22 in class *L*). This suggests that the experts were more concerned about future demand for brownfield redevelopment than about the success of previous redevelopment efforts.

4.4. Implications for practice

Based on the analytic results displayed above, we suggest some implications for practice.

- Improvements in the quality of BR surveys of US cities (US Conference of Mayors, 2006) would enable quantitative analysis tools to produce more conclusive and precise results. The online survey submission system that has recently been established may make information collection more convenient and expeditious (US Conference of Mayors, 2008b). Better information will give all cities a better picture of their position with respect to brownfield redevelopment.
- In procedures relying on holistic group judgments, the determination of representative cases plays a role in the generation of linguistic classification rules that is difficult to measure, but potentially important. For this reason, it would be interesting to cross-validate the study using the judgments of different sets of DMs, perhaps on subsets of the cities. Also, more experts should be approached to obtain a broader sampling of test-case classifications. An alternative might be to obtain more comprehensive evaluations, perhaps facilitated by a website to encourage not only a broader range of experts but also a survey of a larger number of cases. But expanding the test set in this way will probably make it impractical to reach a unanimous judgment, as was done above. Then the first step to finding a consensus view would be to employ statistical tests such as the Friedman test (Friedman, 1937) to detect whether there is

any significant difference among ordinal rankings across evaluations. Aggregation procedures can then be designed to generate an overall result, which can be fed into the DRSA for elicitation of decision rules.

- Additionally, it is useful to compare cities' efforts to reclaim land and to assess the potential that can be tapped. This assessment may be important, e.g. to allocate funds from higher levels of government as effectively as possible. The proposed method can be adapted to prioritize redevelopment projects. In future research, a variety of classification tools including rough set theory will be compared in terms of their classification ability for brownfield redevelopment. One potentially useful data source is a report prepared by the US Council for Urban Economic Development (1999) summarizing hundreds of US brownfield redevelopment projects, identifying important evaluation criteria and proposing benchmarks on each criterion that "can be evaluated and [applied] to a wide variety of projects".

5. Conclusions

Brownfields are the legacy of a century of industrialization (NRTEE, 2003). But they are much more than just contaminated or blighted land; they represent opportunities for large-scale urban improvements, and for reduction of expansion pressure into surrounding greenfields. In this paper, brownfield redevelopment (BR) and strategic decision support for BR are broadly reviewed, and a rough set-based approach is designed for strategic classification of cities, based on a published BR survey of US cities with BR records. The proposed method classifies cities according to their levels of BR effectiveness and BR future needs. Ready availability of a credible classification of cities will be useful to all of them as they make their strategic plans with respect to BR projects and budgets.

Acknowledgements

This work was supported in part by the Natural Sciences and Engineering Research Council of Canada through the project "Systems Engineering Approaches for Brownfield Redevelopment" (STPGP 336683-06), by the National Social Sciences Foundation of China through the project entitled "Research on Ecological Environment and Society Harmony Oriented Project Integrated Evaluation for Typical Western Brownfield Redevelopment" (07BJY031) and by the Provincial Social Sciences Foundation in Shaanxi, China through the project "Market Mechanism and Policy System for Brownfield Redevelopment in Shaanxi Province (08E023). The authors wish to express their sincere appreciation to two anonymous referees whose comments helped us to improve the paper significantly.

References

- Benetto, E., Dujet, C., Rousseaux, P., 2008. Integrating fuzzy multicriteria analysis and uncertainty evaluation in life cycle assessment. *Environmental Modelling & Software* 23 (12), 1461–1467.
- Brebbia, C.A. (Ed.), 2006. *Brownfields III: Prevention, Assessment, Rehabilitation and Development of Brownfield Sites* (Brownfields 2006). WIT Press, Southampton, UK.
- Brownfields Center, 2008. *Brownfields Bibliography*. <http://www.brownfieldscenter.org/big/bibliography.shtml> (accessed March 12, 2008).
- Chen, Y., Kilgour, D.M., Hipel, K.W., 2006. Multiple criteria classification with an application in water resources planning. *Computers and Operations Research* 33 (11), 3301–3323.
- Chen, Y., Witmer, J.A., Hipel, K.W., Kilgour, D.M., 2007. Strategic decision support for brownfield redevelopment, in: *Proceedings of the 2007 IEEE International Conference on Systems, Man and Cybernetics*, Montreal, Quebec, Canada, pp. 1860–1865.
- Council for Urban Economic Development, 1999. *Brownfields Redevelopment: Performance Evaluation*. Council for Urban Economic Development, Washington, DC.
- Doumpos, M., Zopounidis, C., 2002. *Multicriteria Decision Aid Classification Methods*. Kluwer, Dordrecht.
- Erzi, I., 2002. An analysis of uncertainty in brownfield redevelopment using real options, PhD thesis, Carnegie Mellon University, Pittsburgh, PA.
- Friedman, M., 1937. The use of ranks to avoid the assumption of normality implicit in the analysis of variance. *Journal of the American Statistical Association* 32 (200), 675–701.
- George Washington University, 2008. *Public Policies and Private Decisions Affecting the Redevelopment of Brownfields: An Analysis of Critical Factors, Relative Weights and Areal Differentials*. <http://www.gwu.edu/~eem/Brownfields/> (accessed March 12, 2008).
- Greco, S., Matarazzo, B., Slowinski, R., 2001. Rough set theory for multicriteria decision analysis. *European Journal of Operational Research* 129, 1–47.
- ICS, 2008. *4eMka2 software*. Institute of Computing Science (ICS), Poznan University of Technology, Poland. <http://idss.cs.put.poznan.pl/site/4emka.html> (accessed February 18, 2008).
- IEDC, 2005. *International Brownfield Redevelopment*. International Economic Development Council, Washington, DC.
- Isaac, D., 2002. *Property Valuation Principles*. Palgrave, London.
- Keeney, R.L., Raiffa, H., 1976. *Decision With Multiple Objectives: Preferences and Value Tradeoffs*. Wiley, New York.
- Lange, D., McNeil, S., 2004. Clean it and they will come? Defining successful brownfield development. *Journal of Urban Planning and Development* 130 (2), 101–108.
- NRTEE, 2003. *Cleaning up the Past, Building the Future: A National Brownfield Redevelopment Strategy for Canada*. National Round Table on the Environment and the Economy, Ottawa, Canada.
- Pawlak, Z., 1982. Rough sets. *International Journal of Computer and Information Sciences* 11, 341–356.
- Pawlak, Z., Skowron, A., 2007. Rudiments of rough sets. *Information Sciences* 177, 3–27.
- Roy, B., 1996. *Multicriteria Methodology for Decision Aiding*. Kluwer, Dordrecht.
- Saaty, T.L., 1980. *Analytic Hierarchy Process*. McGraw-Hill, New York.
- Tan, R.R., 2005. Rule-based life cycle impact assessment using modified rough set induction methodology. *Environmental Modelling & Software* 20 (5), 509–513.
- Thomas, M.R., 2002. A weighted, multi-attribute, site prioritization and selection process for brownfield redevelopment. *Environmental Practice* 4, 95–106.
- US Census Bureau, 2008. *World POPClock Projection*. <http://www.census.gov/ipc/www/popclockworld.html> (accessed March 6, 2008).
- US Conference of Mayors, 2006. *Recycling, Americas Land: A National Report on Brownfields Redevelopment*, Vol. VI.
- US Conference of Mayors, 2008a. *Brownfields*. <http://www.usmayors.org/brownfields> (accessed March 12, 2008).
- US Conference of Mayors, 2008b. *Brownfield survey online submission*. <http://formdesk.com/usmayors/brownfields2006/> (accessed March 6, 2008).
- USEPA, 2005. *Road Map to Understanding Innovative Technology Options for Brownfields Investigation and Cleanup*, fourth ed. US Environmental Protection Agency, EPA-542-B-05-001.
- USEPA, 2008. *Brownfields Funding Information*. US Environmental Protection Agency. <http://www.epa.gov/brownfields/applicat.htm> (accessed March 6, 2008).
- Wernstedt, K., Crooks, L., Hersh, R., 2008. *Brownfields Redevelopment in Wisconsin: A Survey of the Field, Resources for the Future*. <http://www.rff.org> (accessed March 12, 2008).

Using a Benchmark in Case-Based Multiple-Criteria Ranking

Ye Chen, D. Marc Kilgour, and Keith W. Hipel, *Fellow, IEEE*

Abstract—A benchmark-based method is proposed for multiple-criteria ranking, and a case study is presented to demonstrate that the procedure can be efficient and effective in practice. Multiple-criteria ranking aims to help a decision maker (DM) assess a finite set of alternatives according to several criteria, usually conflicting, in order to rank the full set. The relation of benchmarks to multiple-criteria decision analysis is investigated systematically, and then, an approach based on distance from a benchmark is designed to incorporate information about a DM's judgements so as to produce a full ranking. The procedure is applied to rank 81 U.S. brownfield redevelopment projects based on available data and an accepted benchmark.

Index Terms—Case-based distance approach (CBDA), multiple-criteria decision analysis (MCDA), multiple-criteria ranking, preference disaggregation, project evaluation.

I. MOTIVATION

MULTIPLE-CRITERIA decision making (MCDM) is often supported by a set of techniques that help decision makers (DMs) identify, compare, and evaluate alternatives according to diverse, usually conflicting, criteria. For example, it may be desirable to assess development plans in light of environmental, economic, legal, social, and other criteria. Generally speaking, MCDM can be divided into two main branches: multiple-objective optimization (MOO) and multiple-criteria decision analysis (MCDA).

MOO focuses on applying mathematical algorithms to identify alternatives that are optimal or efficient, under certain constraints, with respect to a few objectives that are expressed mathematically using decision variables [23]. The decision variables are usually continuous; therefore, most MOO problems have infinitely many alternatives which are defined by distinct combinations of values for decision variables.

Manuscript received February 4, 2008. First published December 22, 2008; current version published February 19, 2009. This work was supported by the Natural Sciences and Engineering Research Council of Canada under the Project "Systems Engineering Approaches for Brownfield Redevelopment" (STPGP 336683-06).

Y. Chen is with the College of Economics and Management, Nanjing University of Aeronautics and Astronautics, Nanjing, Jiangsu 210016, China (e-mail: chenye@nuaa.edu.cn).

D. M. Kilgour is with the Department of Mathematics and the Laurier Centre for Military Strategic and Disarmament Studies, Wilfrid Laurier University, Waterloo, ON N2L 3C5, Canada, and also with the University of Waterloo, Waterloo, ON N2L 3G1, Canada (e-mail: mkilgour@wlu.ca).

K. W. Hipel is with the Department of Systems Design Engineering, University of Waterloo, Waterloo, ON N2L 3G1, Canada (e-mail: kwhipel@engmail.uwaterloo.ca).

Color versions of one or more of the figures in this paper are available online at <http://ieeexplore.ieee.org>.

Digital Object Identifier 10.1109/TSMCA.2008.2010135

MCDA, on the other hand, aims to help DMs assess and compare alternatives from a finite set and includes techniques to assist in eliciting their preferences. The criteria are predefined and usually conflicting. As discussed by Roy [21], three *problématiques* (fundamental problems) can be applied to the assessment of a set of alternatives A .

- 1) *Choice*. Choose the best alternative from A .
- 2) *Sorting*. Sort the alternatives of A into relatively homogeneous groups, which can then be arranged in preference order.
- 3) *Ranking*. Rank the alternatives of A from best to worst.

One primary point of comparison of MOO and MCDA is that MOO focuses on *choice* problems with infinitely many alternatives, whereas MCDA considers all three problems, although within a finite set of alternatives. Over the last 40 years, many MCDA methods have been proposed for *choice* and *ranking* problems, including the multiattribute utility theory [19], the analytic hierarchy process [22], and outranking methods [20]. Recently, MCDA research on *sorting* has come to the fore. Doumpos and Zopounidis [7] wrote the first book on this subject and presented a comprehensive review of the MCDA literature on sorting [28]. A related branch of MCDA sorting research is the extension of traditional sorting to nominal classification, which has been carried out by Chen *et al.* [4] and Malakooti and Yang [16].

In general, a *benchmark* is a standard by which something is evaluated or measured. Originally, a "benchmark" was a chiseled horizontal mark, made by a surveyor, into which an angle iron could be placed to bracket (or "bench") a leveling rod, ensuring that the rod could be positioned in exactly the same place in the future [26]. The concept of benchmarks is now applied in many distinct contexts. The prices of West Texas Intermediate and Brent Blend are crude oil benchmarks and are widely accepted as indicators of prices in the petroleum sector. In computing, times and other execution characteristics of programs called benchmarks are used as measures of hardware or software performance. In strategic management, it is common to benchmark an organization, which means to compare its performance to best practices in its industry or sector.

In this paper, the possibility of applying benchmarks in MCDA is investigated, and an approach to multiple-criteria ranking problems based on distance to a benchmark is proposed. To begin, Section II summarizes MCDA and discusses how benchmarks could be valuable in MCDA. Then, Section III details a distance approach that uses benchmarks to solve ranking problems. In Section IV, a case study is presented to show how the method could be applied to evaluate data

| | | Alternatives | | | |
|----------|----------|--------------|-------|--------------|-------|
| | | a^1 | a^2 | \dots | a^n |
| Criteria | c_1 | | | \vdots | |
| | c_2 | | | \downarrow | |
| | \vdots | | | | |
| | c_q | | | m_j^i | |

Fig. 1. Performance matrix in MCDA.

on brownfield redevelopment projects. Some conclusions are offered in Section V.

II. MCDA AND BENCHMARKS

A. MCDA Analysis Procedure and Basic Structure

The analysis of an MCDA problem involves three key steps: 1) problem construction, the process of defining objectives, translating them into criteria, identifying all possible alternatives, and measuring the *consequence* (performance) of each alternative on each criterion; 2) preference elicitation and aggregation, the process of modeling the DM's preferences for performance on each criterion and the DM's relative weights for the different criteria, thereby obtaining an overall evaluation of each alternative; and 3) implementation, the process of assessing the evaluations of the alternatives in order to choose from, sort, or rank \mathbf{A} as aid to decision making.

The fundamental structure of an MCDA problem, the performance matrix (or information matrix), is established in step 1), as shown in Fig. 1. Here, $\mathbf{A} = \{a^1, a^2, \dots, a^n\}$ is the set of alternatives, and $\mathbf{C} = \{c_1, c_2, \dots, c_q\}$ is the set of criteria. The consequence of alternative a^i measured on criterion c_j , denoted as m_j^i , appears in the j th row and i th column of the table. Note that a consequence is a direct measurement of the success of an alternative according to a criterion, such as cost in dollars or capacity in millions of liters per day. Insofar as possible, it is an objective physical measurement and does not include preferential information.

The DM's preferences are a fundamental input in any MCDA problem; therefore, their elicitation and representation are obviously important. It is natural to distinguish two kinds of preferences, preferences on consequences or *values*, and preferences on criteria or *weights*.

Values are refined data reflecting the assessment of a consequence according to the DM's needs and objectives. The relationship between consequence and value can be expressed as $v_j^i = f_j(m_j^i)$, where $v_j^i = v_j(a^i)$ and m_j^i are a value and a consequence, respectively; $f_j(\cdot)$ maps a consequence datum to its value, usually a real number, representing its worth to the DM. The DM's values over all criteria for alternative a^i constitute the *value vector* $\mathbf{v}(a^i) = (v_1(a^i), v_2(a^i), \dots, v_q(a^i))$.

Weights, or preferences on criteria, reflect the relative importance of each criterion relative to the others. The weight for criterion $c_j \in \mathbf{C}$ is $w_j \in \mathbb{R}^+$. Note that no criterion is negligible; therefore, $w_j > 0$ for all $c_j \in \mathbf{C}$. Since preferences on criteria are relative, weights are normalized by requiring $\sum_{j=1}^q w_j = 1$. The *weight vector* is $\mathbf{w} = (w_1, w_2, \dots, w_q)$.

After the preferences have been acquired, a global model is used to aggregate preferences for alternatives and use them to solve the required *problématique*. For $a^i \in \mathbf{A}$, $V(a^i) = F(\mathbf{v}(a^i), \mathbf{w})$, where $V(a^i) \in \mathbb{R}$ is the evaluation of alternative a^i and $F(\cdot)$ is the real-valued mapping from value vectors $\mathbf{v}(a^i)$ and weight vector \mathbf{w} that produces the evaluation of a^i . A typical example is the *linear additive value function* $V(a^i) = \sum_{j=1}^q w_j \cdot v_j(a^i)$.

B. Application of Benchmarks in MCDA

MOO and MCDA procedures that use a specific point, analogous to a benchmark, include goal programming [3], compromise programming [27], reference point approaches [25], Technique for Order Preference by Similarity to Ideal Solution [9], the aspiration level interactive method [14], and the elimination method [15]. In these procedures, the predefined point may be related to the data but is typically an ideal or anti-ideal point and is unrealizable. Benchmarks, on the other hand, are realizable points of reference. For example, feasible benchmarks for performance, often called best practices, have been established and are widely used in performance evaluations of businesses and other organizations [2]. In this paper, we consider measurements relative to a realizable point of reference, such as a median. In other words, a benchmark is, or could be, achieved by an actual alternative.

As far as we know, there has been no analysis of how to use such a point of reference in solving MCDA problems. In this paper, we explore systematically the utilization of a benchmark in MCDA and develop an approach based on distance from a benchmark to solve ranking problems.

Except for preference information, the data of an MCDA problem—the set of alternatives \mathbf{A} , the set of criteria \mathbf{C} , and the consequence of each alternative on each criterion—are shown in Fig. 1. The solution of the MCDA problem must apply the DM's preferences, over consequences and criteria, to these data. We henceforth assume that all consequences (entries of the performance matrix) are real numbers and that the DM's preferences over consequences are *monotonic*, i.e., that $\mathbf{C} = \mathbf{C}^+ \cap \mathbf{C}^-$, where \mathbf{C}^+ is the set of *positive criteria* (the greater the consequence, the more preferred the alternative, *ceteris paribus*, e.g., capacity) and \mathbf{C}^- is the set of *negative criteria* (the greater the consequence, the less preferred the alternative, e.g., cost).

Alternative a^l *dominates* alternative a^m iff $m_j(a^l) \geq m_j(a^m)$ for all $c_j \in \mathbf{C}^+$ and $m_j(a^l) \leq m_j(a^m)$ for all $c_j \in \mathbf{C}^-$, provided at least one of these inequalities is strict. If so, we write $a^l \succ a^m$. Alternatives a^l and a^m are *nondominating* iff neither $a^l \succ a^m$ nor $a^m \succ a^l$. We denote the nondominance of a^l and a^m by $a^l \sim a^m$.

Suppose that $b = (b_1, b_2, \dots, b_q) \in \mathbb{R}^q$ is fixed and satisfies $\min_{a^i \in \mathbf{A}} m_j(a^i) < b_j < \max_{a^i \in \mathbf{A}} m_j(a^i)$ for all $c_j \in \mathbf{C}$. Then, we can interpret b as a *benchmark* and compare all alternatives in \mathbf{A} to it. Clearly, \mathbf{A} is partitioned into three subsets by b . The *superior group* of alternatives is $\mathbf{A}^+(b) = \mathbf{A}^+ = \{a^i \in \mathbf{A} : a^i \succ b\}$, the *inferior group* of alternatives is $\mathbf{A}^-(b) = \mathbf{A}^- = \{a^i \in \mathbf{A} : b \succ a^i\}$, and the *nondominant group* of alternatives is $\mathbf{A}^\sim(b) = \mathbf{A}^\sim = \mathbf{A} - \mathbf{A}^+(b) - \mathbf{A}^-(b)$.

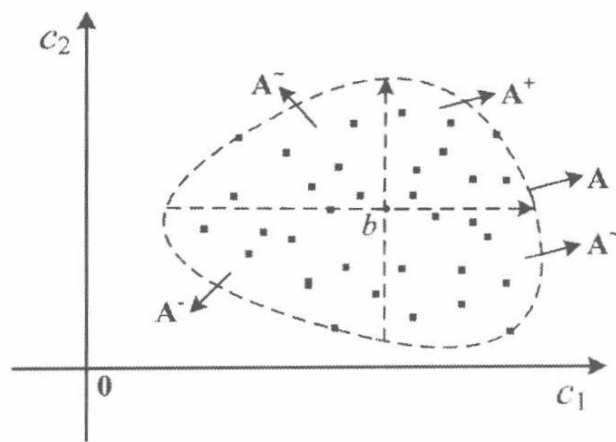


Fig. 2. Relationship among A^+ , A^- , and A^\sim .

Fig. 2 shows these three subsets in the context of two positive criteria. Note that $A^\sim(b)$ has two components unless the benchmark b is itself an alternative, i.e., unless $b \in A$.

III. BENCHMARKS, DISTANCES, AND RANKING

In principle, all of the DM's preference information is required to generate a full ranking of A . In MCDA, this information is normally disaggregated into preferences on consequences (values, given by real functions of one variable) and preferences on criteria (weights, given as a weight vector). The efficient acquisition of this preference information is the key challenge of MCDA. There are two broad classes of techniques for preference elicitation, direct judgment and case based. Both employ a model of preference which, when calibrated, represents the DM's preferences explicitly. In direct-judgment methods, the DM is asked to specify the model by providing explicit values for all parameters. In case-based approaches, the DM is asked to make global preference judgements on selected cases (alternatives or potential alternative), which are then input into optimization programs that find the parameters of the preference model that is most consistent with the input. Here, we adapt previous work [5] to show how benchmarks can be integrated into case-based methods based on distance models of preference.

The case-based distance approach (CBDA) originates from preference disaggregation methodologies [11], [12]. The CBDA begins with a predefined ideal (or anti-ideal) point and supposes that the DM's preference can be identified with distance from this reference point. Specifically, decreasing preference corresponds to increasing distance from the ideal point (or decreasing distance to the anti-ideal point), but only if the "right" distance metric is employed. For example, under the weighted Euclidean distance assumption, optimization programs are designed to identify the most suitable information of criterion weights and group thresholds for multiple-criteria sorting problems in [5]. We now extend this approach to reference points that are not extreme, like ideal or anti-ideal points, but instead, are achievable benchmarks.

A. Procedural Inputs

Assume that a fixed benchmark $b \in \mathbb{R}^q$ is given. The DM is asked to provide three nonempty case sets T_+ , T_\sim , and T_- that are representative of A^+ , A^\sim , and A^- , respectively.

Note that \succ and \prec indicate dominance relationships, and \sim indicates that there is no dominance relationship. Consistent with these usages, \succ_P and \prec_P indicate preference and \sim_I lack of preference or indifference. In particular, $t \succ_P b$ if $t \in T_+$, $t \sim_I b$ if $t \in T_\sim$, and $t \prec_P b$ if $t \in T_-$. The DM may construct the case sets by selecting alternatives from A , by modifying historical records or simply based on experience.

Suppose that the DM provides $T_+ = \{t_+^1, \dots, t_+^r, \dots, t_+^{m_+}\}$, $T_\sim = \{t_\sim^1, \dots, t_\sim^r, \dots, t_\sim^{m_\sim}\}$, and $T_- = \{t_-^1, \dots, t_-^r, \dots, t_-^{m_-}\}$. Note that $|T_+| = m_+$, $|T_\sim| = m_\sim$, and $|T_-| = m_-$. Define $T = T_+ \cup T_\sim \cup T_-$.

The DM is now asked to rank T in decreasing order of preference. For example, if $1 \leq g < h \leq m_+$, then either $t_+^g \succ_P t_+^h$ or $t_+^g \sim_I t_+^h$ and similarly within T_\sim and T_- . The monotonicity of preference implies that any element of T_+ is strictly preferred to any element of T_- but implies nothing about the relative preference of an element of T_+ and an element of T_\sim , or about the relative preference of an element of T_\sim and an element of T_- . In most optimization software, including Matlab and Lingo, there is no distinction between calculations based on constraints such as "equal or greater than" (\geq) versus "greater than" ($>$); for this reason, we do not emphasize weak preferences (\succeq_P) here and usually assume that all preferences are strict.

B. Distance Assumptions

Define $A' = A \cup T$. For each $c_j \in C$, set

$$d_j^{\max} = \max_{a \in A'} |m_j(a) - m_j(b)| \quad (1)$$

where $|x|$ is the absolute value of $x \in \mathbb{R}$. Then, d_j^{\max} is the *normalization factor* for criterion c_j , and the signed normalized distance between $a \in A'$ and b on criterion c_j is

$$d_j(a, b) = d_j(a) = \frac{m_j(a) - m_j(b)}{d_j^{\max}}. \quad (2)$$

Note that $0 \leq d_j(a) \leq 1$ if $a \in A^+ \cup T_+$, $-1 \leq d_j(a) \leq 0$ if $a \in A^- \cup T_-$, and $-1 \leq d_j(a) \leq 1$ if $a \in A^\sim \cup T_\sim$.

To aggregate the signed normalized distances between $a \in A'$ and b over all q criteria, we use a distance related to the so-called p -norm, where $p \in \mathbb{N}^+$. The most commonly used norms are $p = 1$ and $p = 2$. Given a weight vector $w = (w_1, w_2, \dots, w_q)$, the weighted signed p -power distance is

$$DW(a) = \sum_{j=1}^q w_j \cdot |d_j(a)|^p \cdot \frac{d_j(a)}{|d_j(a)|}.$$

Note that $DW(a)$ may be positive or negative, depending on whether the positive or negative values of $d_j(a)$ predominate. The signed distance from alternative $a \in A'$ to b is then

$$D(a, b) = D(a) = \sqrt[p]{|DW(a)|} \cdot \frac{DW(a)}{|DW(a)|}. \quad (3)$$

When $p = 1$, D is the signed 1-norm or signed Manhattan distance from the benchmark, and it is easy to verify that

$D(a) = \sum_{j=1}^q w_j \cdot d_j(a^i)$. When $p = 2$, D is the signed 2-norm or signed Euclidean distance from b , and

$$D(a) = \begin{cases} \sqrt{DW(a)}, & \text{if } DW(a) \geq 0 \\ -\sqrt{|DW(a)|}, & \text{if } DW(a) < 0. \end{cases}$$

Both of these signed distance norms have clear geometric interpretations and are usually easy for the DM to understand.

It is obvious that $D(a) \in [-1, 1]$ for all $a^i \in \mathbf{A}'$; greater values of $D(a)$ represent more preferred outcomes relative to the benchmark b . Also, if $D(a) = 0$, then $a \sim_I b$.

C. Graphical Demonstration

Fig. 3 shows the idea in the context of two positive criteria with the signed $p = 1$ and $p = 2$ norms. The case sets \mathbf{T}^+ , \mathbf{T}^- , and \mathbf{T}^\sim are displayed in the first panel in the original consequence space. Note that there are five, nine, and six alternatives in \mathbf{T}^+ , \mathbf{T}^- , and \mathbf{T}^\sim , respectively.

In the second panel, (1) and (2) have been used to transfer the case sets to normalized consequence space. Here, c'_1 and c'_2 are the normalized versions of criteria c_1 and c_2 , respectively; notice that the origin of the coordinate system is now replaced by the benchmark b .

In the third panel, (3) has been used to measure the relative preference of each alternative in \mathbf{T} according to its signed weighted distance from b . Under the assumptions that $p = 1$ and that c'_1 and c'_2 are weighted equally, the parallel straight lines from lower right to upper left contain points that are equally preferred because they are at equal aggregated distances to the benchmark b . For instance, the line passing through b includes all alternatives equally preferred to b (i.e., at distance zero from b); any line below and to the left represents equally preferred alternatives that happen to be less preferred than b (at negative distance from b); and any line above and to the right represents equally preferred alternatives that are preferred to b (at positive distance from b).

Similarly, if $p = 2$ and c'_1 and c'_2 remain equally weighted, the alternatives preferred to b (at a fixed positive distance from b) lie on an arc of a circle in the first quadrant (upper right) and then become rectangular hyperbolae on crossing the horizontal or vertical axis. All of these hyperbolae are asymptotic to the 0-distance line, which is the straight line passing through the benchmark b . Thus, all equally preferred lines (other than the 0-distance line) start off close to the 0-distance line at the bottom right, then slowly separate from it, then bend around, and finally approach the 0-distance line again in the upper left. In the case when c'_1 and c'_2 are not equally preferred, the circle and rectangular hyperbola in the aforementioned description are replaced by an ellipse and a nonrectangular hyperbola.

In the fourth panel, the signed distances from b of the alternatives in the different case sets are shown. Note that the distances of all alternatives in \mathbf{T}^+ are strictly positive and the distances of all alternatives in \mathbf{T}^- are strictly negative. In \mathbf{T}^\sim , three alternatives are preferred to b (at positive distance from b), two alternatives happen to be indifferent to b (at distance zero), and four alternatives are less preferred than b (at negative distance from b).

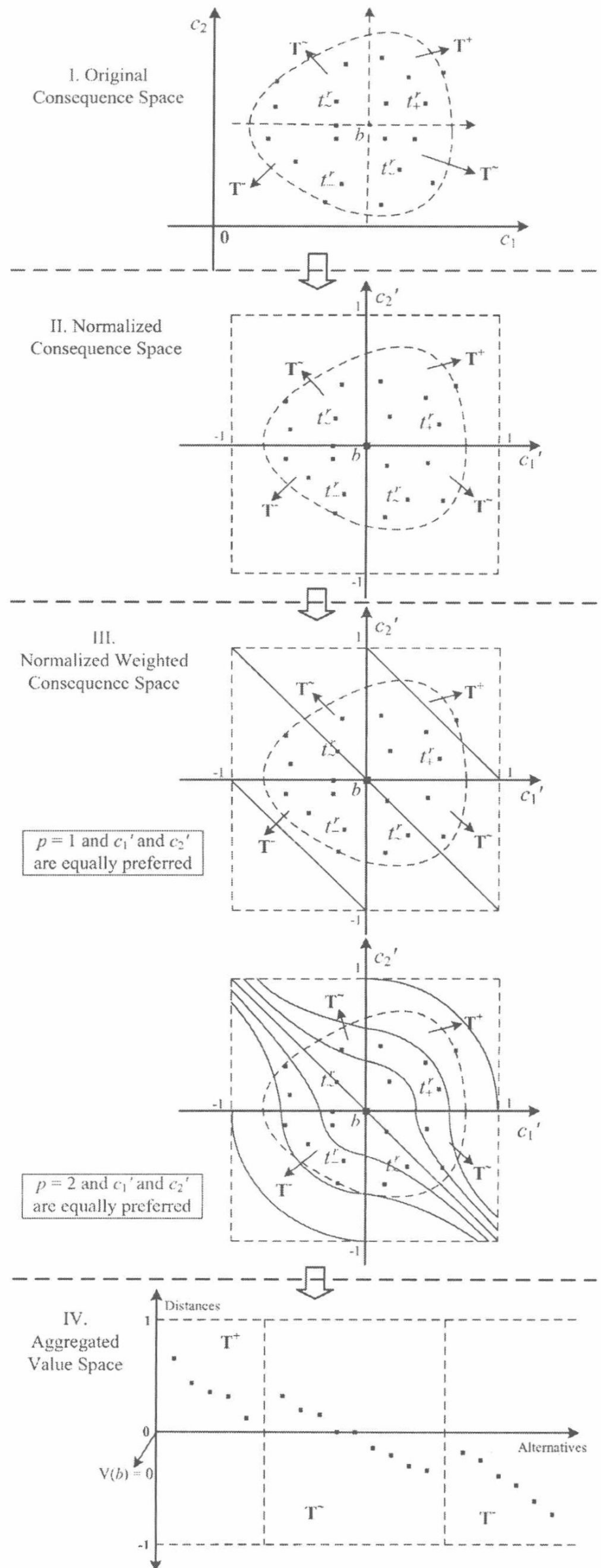


Fig. 3. Benchmark-based distance with two positive criteria under $p = 1, 2$.