Dynamical Systems Approach to Turbulence

湍流研究的动力系统方法

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著

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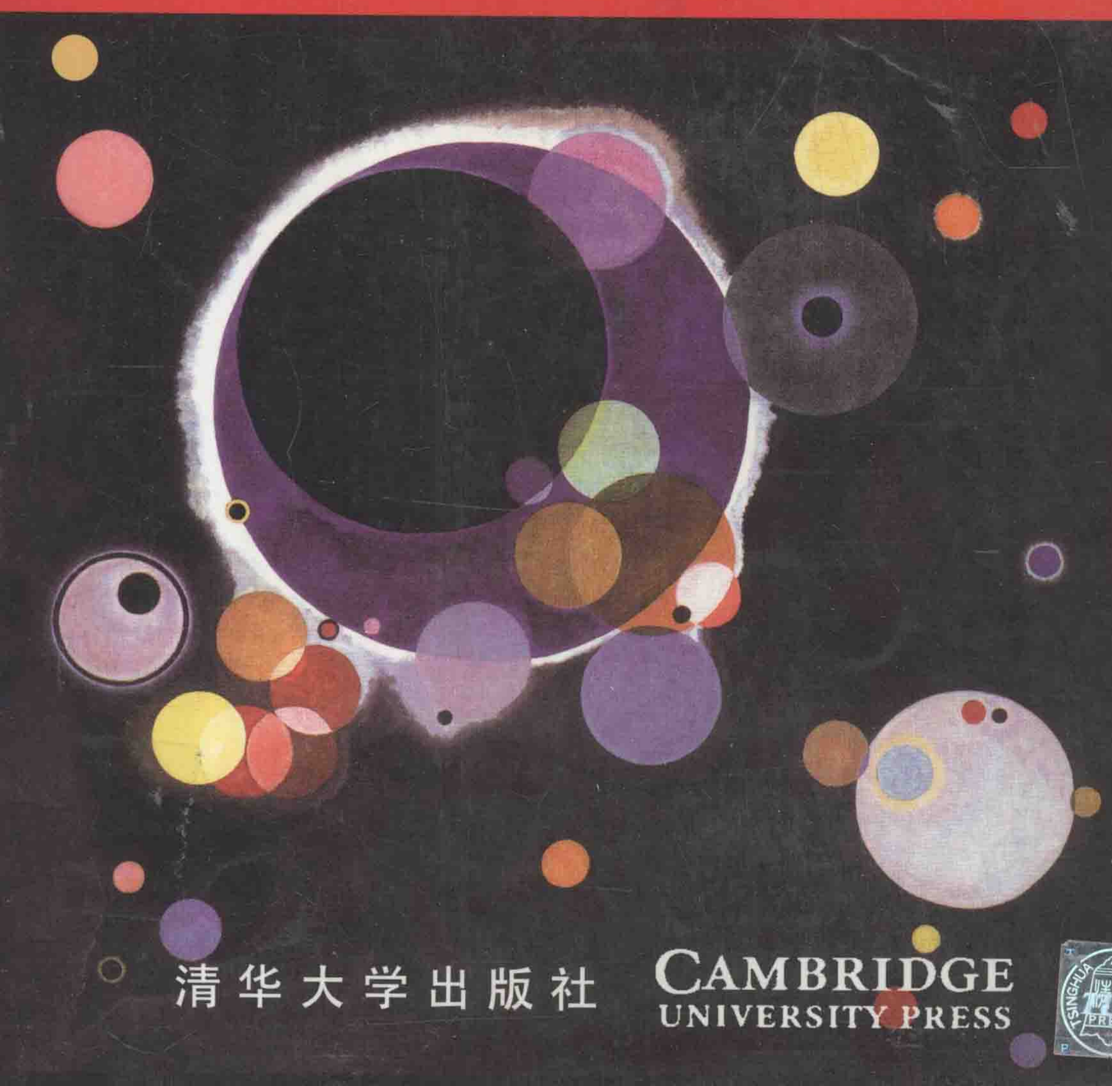

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To the memory of Giovanni

During the first two weeks of June 1996, the four authors of this book were together in Copenhagen to complete the work. Two weeks later, on 29 June, Giovanni Paladin tragically died in a mountain climbing accident on Gran Sasso near L'Aquila in Italy. This was a terrible shock, and we, and many others, feel the incomprehensible loss of a dear colleague and close friend.

Giovanni was born in 1958 in Trieste. He received his education at the University of Rome and wrote his Master's thesis on the subject of 'Dynamical critical phenomena' under the supervision of Luca Peliti (1981). After the thesis he became interested in the theory of dynamical systems and chaotic phenomena, which led to the well-known work on multifractals in collaboration with Roberto Benzi, Giorgio Parisi and Angelo Vulpiani. He continued to develop new ideas within this field now in a very close collaboration with Angelo Vulpiani. From 1982 they co-authored more than 60 papers, among which is a widely cited review on multifractals and a book on products of random matrices. After his Ph.D. at the University of Rome 'La Sapienza' (1987), Giovanni started his 'travelling years'. First he went for a year to the Ecole Normale Superieure, Paris (1987–88) and visited the University of Chicago (1988); then, finally, he spent a year at The Niels Bohr Institute and Nordita in Copenhagen (1989–90). During this time he established links with many groups and individuals, and he kept returning to these places, where he was a treasured guest. While in Copenhagen, Giovanni and Angelo (who remained in close contact) started a collaboration on shell models for turbulence with Mogens Jensen, and this work forms an important part of our book. In 1990 Giovanni became Assistant Professor at the University of L'Aquila, and then in 1992 Associate Professor, again at the University of L'Aquila.

Giovanni was a very gifted and creative scientist. He mastered the techniques of statistical mechanics and dynamical systems to perfection and was able to draw analogies between the various subjects he worked on in a very elegant and productive way. Beyond his technical contributions in research he was also a very good teacher, with a keen interest in the education of young scientists. Six students prepared their Master's thesis under his supervision and two graduate students were working for the Ph.D. In addition Giovanni took great interest in scientific popularization, writing some contributions for encyclopaedias and taking part in many conferences for students and high-school teachers.

He was an extremely sweet and gentle person. Wherever he went, he made close friends immediately and he maintained personal contacts very carefully. One could always speak to Giovanni about anything, and he would listen and answer in his characteristic gentle and original way, always completely honest and deeply absorbed in science, literature, art and music (in particular, Mozart).

A funny aspect of Giovanni was his systematic absent-mindedness. He was basically able to lose anything: keys, books, papers, files, documents, money and so on. On the other hand he was lucky enough to find almost all the lost objects again. Among his friends and collaborators there was a sort of unwritten rule: *it was strictly forbidden to leave the only copy of any important thing with Giovanni.*

Giovanni loved the mountains and went to ski or climb as often as he could – almost every week. There, too, he had a large group of close friends, with whom he shared many adventures. One of his last outings was a long tour climbing and skiing down the volcanoes of northern Patagonia (October 1995). The practice of mountaineering added a new dimension to the purely intellectual side of his life, making it richer and more diverse.

All the many friends, colleagues and students of Giovanni miss him sorely.

We wish to dedicate this book to the memory of Giovanni Paladin.

Angelo, Mogens and Tomas

Preface

During the last few decades the theory of dynamical systems has experienced extremely rapid progress. In the 1980s we became attracted to this field by the enthusiasm and new insights generated by the works of Lorenz, Chirikov, Hénon and Heiles, Ruelle and Takens, Feigenbaum, and Libchaber, to cite just some of the most famous. It was surprising that concepts from low-dimensional dynamical systems, even seemingly abstract mathematical devices such as iterated maps, could be used to describe systems as complicated as unsteady fluids. For fluids under severe constraints, i.e. the experiments on Rayleigh–Bénard convection in small cells, the success was undisputed. For the understanding of turbulence, however, the success has been more limited. Turbulence, which implies spatial as well as temporal disorder, cannot be reduced to a low-dimensional system, and thus a large part of the theory of dynamical systems, in particular regarding bifurcation structures and symbolic dynamics, is basically inapplicable.

The aim of this book is to show that there are dynamical systems that are much simpler than the Navier–Stokes equations but that can still have turbulent states and for which many concepts developed in the theory of dynamical systems can be successfully applied. In this connection we advocate a broader use of the word 'turbulence', to be made precise in the first chapter, which emphasizes the common properties of a wide range of natural phenomena. Even for the case of fully developed hydrodynamical turbulence, which contains an extreme range of relevant length scales, it is possible, by using a limited number of ordinary differential equations (the so-called shell models), to reproduce a surprising variety of relevant features.

This book reflects to a large extent our own scientific interests during the last decade or so. These interests were strongly influenced by many outstanding colleagues. In particular we would like to express our thanks for guidance, inspiration and encouragement to P. Bak, G. Grinstein, L. P. Kadanoff, A. Libchaber, G. Parisi, I. Procaccia and D. Rand.

We are also deeply grateful to P. Alstrøm, E. Aurell, R. Benzi, L. Biferale, G. Boffetta, E. Bosch, O. B. Christensen, C. Conrado, A. Crisanti, P. Cvitanović, M. Falcioni, Y. He, J. M. Houlrik, G. Huber, C. Jayaprakash, J. Krug, R. Lima, R. Livi, J. Lundbek Hansen, V. L'Vov, D. Mukamel, E. Ott, A. W. Pedersen, A. Pikovsky, A. Provenzale, S. Ruffo, K. Sneppen, S. Vaienti, M. Vergassola, W. van de Water, I. Webman, R. Zeitak and Y.-C. Zhang, who participated in obtaining many of the results discussed in this book.

Finally we thank E. Aurell, M. Falcioni, J. Krug, Y. Pomeau, and N. Schörghofer for valuable comments on the first draft of the manuscript.

Introduction

The traditional description of turbulence (as summarized in the monograph by Monin and Yaglom [1971, 1975]) employs statistical methods, truncation schemes in the form of approximate closure theories and phenomenological models (e.g. Kolmogorov's theories of 1941 and 1962). The complementary point of view guiding our description of turbulence is to regard the Navier–Stokes equations, or other partial differential equations describing turbulent systems, as a deterministic dynamical system and to regard the turbulence as a manifestation of deterministic chaos.

In the case of fully developed turbulence the direct simulation of the Navier–Stokes equations is prohibitively difficult owing to the large range of relevant length scales. It is thus important to study simplified models, and a large part of this book is devoted to the introduction and investigation of such models. We shall give an introduction to the dynamical systems approach to turbulence and show the applicability of methods borrowed from dynamical systems to a wide class of dynamical states in spatially extended systems, for which we shall use the general term turbulence.

It is important to note that the dynamical models employed to describe turbulent states are not low-dimensional. In flows with high Reynolds numbers or in chaotic systems of large spatial extent, the number of relevant degrees of freedom is very large, and our primary interest is to explore properties that are well defined in the 'thermodynamic limit', where the system size (or Reynolds number) becomes very large.

Some of the main concepts and characteristics of this approach to turbulence are

1. The use of quantifiers of chaotic dynamics, such as Lyapunov exponents, entropies and dimensions. These concepts provide a very precise determination of e.g. the onset of turbulence and an understanding of the interplay between temporal chaos and spatial scales. In addition they provide information about the propagation of information and disturbances.
2. The use of the geometrical description of (multi)fractal objects, i.e. concepts borrowed from the thermodynamical formalism for dynamical systems. These concepts provide a framework for understanding subtle statistical properties, such as intermittency, and allow detailed comparison with experimental data.
3. Direct simulation of simplified dynamical models of turbulence, such as shell models, coupled maps or amplitude equations. Here it is important that we view these systems as non-linear dynamical systems in their full complexity, for which the more traditional study of stationary and periodic states and their stability is only the first step.
4. The fact that we deal with systems with a large number of degrees of freedom means that many concepts from statistical mechanics, and in particular from the theory of critical behaviour, become relevant. In fact, our goal is to understand the 'thermodynamic limit' of deterministically chaotic systems – as opposed to noisy systems (i.e. Langevin equations).

The layout of the book is as follows:

In **Chapter 1** we introduce various concepts and models to be described in detail later. After reading that chapter, the reader will know what we mean by turbulence and what the rest of the book is about. This should allow the reader to pick out which parts to read next.

In **Chapter 2** we go through the phenomenology of fully developed hydrodynamic turbulence, centred around the ideas of Kolmogorov. We introduce e.g. structure functions, scale invariance and the multifractal description of turbulence, and this should give a good background for the more detailed expositions in chapters 3, 6, 8 and 9.

Chapter 3 is mainly concerned with shell models for fully developed turbulence, which are introduced and treated in detail. The idea is to capture basic ingredients, such as conservation laws and the energy cascade in 3D turbulence, by a chaotic dynamical system with a reasonable number of equations. By studying a system of around 20 coupled differential equations one can obtain results on issues such as the scaling exponents of the structure functions, intermittency corrections and the probability distribution of velocity gradients, in agreement with experiments.

In **Chapter 4** coupled map lattices are introduced and selected topics are discussed in more detail. One main aim is to show that these models are helpful in understanding the interplay between chaos and turbulence, i.e. what happens when a chaotic system becomes larger, and to understand the influence of conservation laws and symmetry breaking. Spatio-temporal intermittency is treated in detail as an example of a deterministic 'phase transition'. and examples are shown of how to model aspects of turbulent Rayleigh–Bénard convection.

Chapter 5 is concerned with turbulence in amplitude equations, i.e. equations derived by expansions around an instability. It is centred around the complex Ginzburg–Landau equation, which models a rich variety of physical, chemical and biological systems in which a coherent periodic state exists. The main issues are the interplay between periodic states, spirals, turbulent states and so-called vortex–glass states, disordered states with many spirals. The chapter ends with a discussion of the Kuramoto–Sivashinsky equation and generalizations thereof.

In **Chapter 6** we discuss predictability in systems with many degrees of freedom. In chaotic systems the distance between two initially close trajectories diverges exponentially, which implies that prediction is feasible only up to a predictability time inversely proportional to the maximum Lyapunov exponent. This simple scenario fails in realistic situations, where many characteristic times are involved, especially if non-infinitesimal perturbations are applied. A relevant example is weather forecasting, where we focus on the prediction of the large-scale motion.

Chapter 7 deals with the dynamics of interfaces and surfaces. Here we are mostly dealing with noise-driven dynamical systems, which generate rough, scale-invariant fronts modelling e.g. the motion of a viscous fluid in a porous medium. Other fronts (e.g. flame fronts) can be modelled by deterministic equations, notably the Kuramoto–Sivashinsky equation, and one important issue is in what sense the deterministic and the noise-driven systems are alike. A large part of the chapter is devoted to 'extremal' models with non-local interactions, relevant for strongly pinned systems. The relation to self-organized criticality and to directed percolation is discussed in detail.

Lagrangian chaos is the subject of **Chapter 8**. The motion of a fluid particle is described by a low-dimensional dynamical system and can be chaotic even in the absence of (Eulerian) chaos in the velocity field. This fact is of great importance to mixing, transport and diffusion in fluids. The use of techniques of dynamical systems allows one to determine e.g. the scaling range of the Batchelor law for passive scalar fluctuations at small scales or the connection between variations of the effective Lyapunov exponent and the strong spatial fluctuations of the magnetic field in the dynamo problem.

This leads naturally to the problem of chaotic diffusion which is treated in **Chapter 9**. The main issue is: what does the structure of the velocity field tell us about the diffusion of fluid particles, e.g. whether it is anomalous and, if it is normal, how to compute the diffusion coefficient. In the last part we show that velocity fields generated by models from chapters 5 and 7 can give rise to anomalous diffusion.

Concepts from the theory of dynamical systems are used throughout the book, and we assume that the reader has some familiarity with these notions. For completeness we do, however, review some of the basic features, such as the calculation of Lyapunov exponents and the theory of the Hopf bifurcation in the **Appendices**. These also contain introductions to the theory of convective instabilities, linear front propagation, multifractality and directed percolation.

The aim of this book is to show that concepts and techniques developed in the context of chaotic dynamical systems play a key role in the understanding of turbulent states in spatially extended systems. We hope that we have managed to convey the richness and ubiquity of such turbulent states as well as the basic features which bind them together.

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