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Recent Advances in Three-Dimensional Fracture Mechanics

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ABSTRACT

Some basic three-dimensional (3D) problems in fracture mechanics are discussed in this paper. Firstly, the interaction between the stress-strain fields and the out-of-plane constraint is analyzed. The weaker singularities of stresses at the crack border in both linear elastic and elastic-plastic materials are shown to be confined to an infinite small zone at the intersection point of the crack front line and the free surface of the cracked body. Therefore, the *K*-based linear elastic fracture mechanics theory and *J*-based nonlinear fracture mechanics theory can be extended to 3D cracked bodies. The influence of the out-of-plane constraint factor *Tz* on the crack tip fields was analyzed and the variations of some important fracture parameters from plane stress to plane strain state are summarized. Then, in consideration the influences of both the in-plane and out-of-plane constraints, a general J-Q_T-Tz or J-A_2-Tz theory is proposed and proven to be more effective. Finally, the 3D effect on fracture of engineering materials is outlined.

1. FUNDAMENTAL EQUATIONS

For a 3D isotropic continuum without body force, the stress tensor $\boldsymbol{\sigma}$ and strain tensor $\boldsymbol{\varepsilon}$ should satisfy the equilibrium and compatibility equations

$$\sigma_{ij,j} = 0, \quad e_{mlk} e_{nij} \varepsilon_{ki,jl} = 0. \tag{1}$$

Where $e_{ijk}=(i-j)(j-k)(k-i)/2$.

In the frame of deformation theory we have

$$\varepsilon_{ij} = \varepsilon_{ij}^{e} + \varepsilon_{ij}^{p} = \frac{1+\upsilon_{ep}}{E_s} S_{ij} + \frac{1-\upsilon_{ep}}{E} \sigma_m \delta_{ij} \tag{2}$$

where E is the Young's module, E_S is secant module, υ_{ep} is the elastic plastic Poisson's ratio

$$\upsilon_{ep} = \frac{1}{2} - \left(\frac{1}{2} - \upsilon\right)\frac{E_s}{E} \tag{3}$$

Consider a sheet element in the normal plane of the crack front line at any point P on the line, the in plane strains can be get from (2) and (3) as

$$\varepsilon_{rr} = \frac{1}{E_s}\left[\left(1-\upsilon_{ep}Tz\right)\sigma_{rr} - \upsilon_{ep}\left(1+Tz\right)\sigma_{\theta\theta}\right],$$
$$\varepsilon_{\theta\theta} = \frac{1}{E_s}\left[\left(1-\upsilon_{ep}Tz\right)\sigma_{\theta\theta} - \upsilon_{ep}\left(1+Tz\right)\sigma_{rr}\right], \tag{4}$$
$$\varepsilon_{\theta\theta} = \frac{1}{E_s}\left(1+\upsilon_{r\theta}\right)\sigma_{r\theta}.$$

And the out-of-plane strain can be written as

$$\varepsilon_{zz} = \frac{1}{E_s}\left(Tz - \upsilon_{ep}\right)\left(\sigma_{rr} + \sigma_{\theta\theta}\right). \tag{5}$$

At the tip of an open mode crack. $\left(\sigma_{rr} + \sigma_{\theta\theta}\right) > 0, \quad \varepsilon_{zz} \le 0 \quad and \quad \sigma_{zz} \ge 0$, so that

$$0 \le \mathbf{Tz} \le \upsilon_{\mathbf{ep}} \le \frac{1}{2}. \tag{6}$$

When the Ramberg-Osgood constitutive relationship is assumed,

$$\upsilon_{ep} = \frac{1}{2} - \left(\frac{1}{2} - \upsilon\right)\frac{1}{1+\alpha\left(\sigma_e / \sigma_0\right)^{n-1}}. \tag{7}$$

When the Maxwell stress functions ϕ_{ii} (i=1,2,3) are introduced, the stress tensor satisfying the equilibrium equation (1) can be expressed as

$$\sigma_{mn} = e_{mkj} e_{nij} \phi_{ij,kl}. \tag{8}$$

2. CORNER SINGULARITY OF 3D ELASTIC CRACKS

It has been shown by previous work that the near tip stresses can be expressed in form of variable separation for 2D cracks and in the interior of a 3D cracked body:

$$\sigma_{ij} = \sum_{k=1}^{\infty} A_k r^{\lambda_k} f_{ij}^{(k)}(\theta), \qquad \lambda_k = \frac{k}{2} - 1.. \tag{9}$$

At corner points where the crack front intersected with the free surfaces, weaker singularity was found in spherical coordinate (ρ, φ, ψ):

$$\sigma_{ij} = A\rho^{-\lambda}\tilde{\sigma}(\varphi, \psi), \qquad \lambda < 1/2. \tag{10}$$

So the dominate term of the Maxwell functions for general 3D cracks can be assumed as

$$\phi_{ii} = r^{f_i(z)}\tilde{\phi}_i(\theta, Tz), \quad Tz = Tz(r, \theta, z) \tag{11}$$

Substituting (11) into the compatibility equation (1) and using (6) it can be gotten finally that:

i) When $\frac{\partial Tz}{\partial z} < \infty$, the stress singularity is the same as in (9), or is $r^{-\frac{1}{2}}$.

ii) When $\frac{\partial Tz}{\partial z} \to \infty$, $r^{-\frac{1}{2}}$ singularity can not be determined. So only in this case a weaker singularity may exist.

For through-cracks in plates with thickness of $2h$ under tension, Tz satisfying the equilibrium and boundary conditions can be expressed as

$$Tz = \upsilon\left[1 - \left|\frac{z}{h}\right|^{g(r)}\right]^2 F(r/h). \tag{12}$$

Then by means of variational method, it can be found that the only stationary value of the complementary energy is $g(r) \to \infty$ at the tip of the crack. So that

$$Tz = \upsilon\left[1 - \left|\frac{z}{h}\right|^{\infty}\right]^2, \qquad r \to 0. \tag{13}$$

or the Tz-z curve tends to a rectangle as $r \to 0$ (Fig.1). The region in which $\frac{\partial Tz}{\partial z} \to \infty$ is infinite small, so is the possible weaker singularity region.

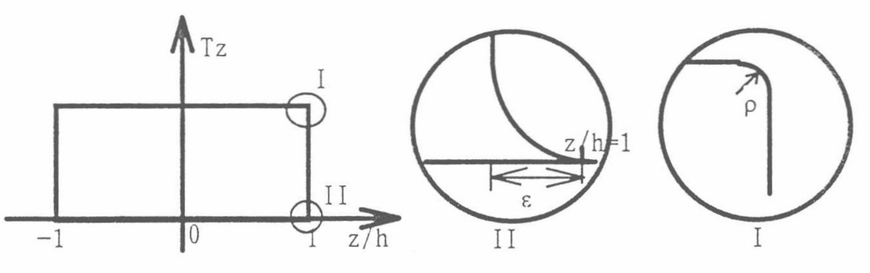

Fig.1 Through thickness variation of Tz $(\varepsilon \to 0 \quad \& \quad \rho \to 0 \quad as \quad r \to 0)$

3. 3D ELASTOPLASTIC CRACK BORDER FIELDS

3.1 Singular Structure of the Fields

Under 3D condition, the dominate term of the crack border stresses can be assumed as

$$\sigma_{ij} = r^{f_{ij}(z)}\tilde{\sigma}_{ij}(\theta, Tz), \qquad Tz = Tz(r, \theta, z) \tag{14}$$

By using the basic equations of the problem as well as the properties of Tz, it can be found that:

i) Providing $\frac{\partial Tz}{\partial z} < \infty$, the singularity and angular distribution of stresses and strains are functions of the triaxial stress constraint Tz, that is

$$\sigma_{ij} = Kr^{f-2}\tilde{\sigma}_{ij}(\theta, Tz), \qquad \varepsilon_{ij} = \frac{3}{2}\alpha K^n r^{f-2}\tilde{\varepsilon}_{ij}(\theta, Tz), \qquad (i, j = 1,2) \tag{15}$$

and

$$\sigma_{zz} = TzKr^{f-2}\left(\tilde{\sigma}_{xx} + \tilde{\sigma}_{yy}\right), \quad \varepsilon_{zz} = \left(Tz - \frac{1}{2}\right)K^n r^{n(f-2)}\tilde{\varepsilon}_{zz}(\theta, Tz) \tag{16}$$

The traverse shear stresses and strains are one order lower as $r \to 0$.In this case the problem can be simplified to a quasi-planar problem with Tz being considered. This makes it possible to solve the problem analytically.

ii) In the case of $\frac{\partial Tz}{\partial z} \to \infty$, singular structure of the fields can not be settled. It is similar to the corner problem in 3D elastic cracks. Again, this region will be infinite small and not important in application.

3.2 Asymptotic Solution Under Triaxial Stress Constraint

In the case of $\frac{\partial Tz}{\partial z} < \infty$, all of the singular stresses in a 3D cracked body bear relation to only one stress function ϕ_{33}, and it can be get from (14) that

$$\phi_{33} = Kr^{f(z)}\tilde{\phi}(\theta, Tz), \tag{17}$$

where $f(z)$ is a function of Tz.

Substituting (17) into (8),(2) and (1), the governing equation in a strain hardening material can be obtained under given Tz

$$\frac{1+Tz}{3}\left\{a_1\left[\tilde{\sigma}_e^{\,n-1}\left(f^2\tilde{\phi}+\tilde{\phi}\tilde{\phi}''\right)\right]-\left[\tilde{\sigma}_e^{\,n-1}\left(f^2\tilde{\phi}+\tilde{\phi}''\right)\right]''\right\}+\left[\tilde{\sigma}_e^{\,n-1}\left(f\ \tilde{\phi}+\tilde{\phi}''\right)\right]''$$
$$+a_2\left(\tilde{\sigma}_e^{\,n-1}\tilde{\phi}'\right)+n(f-2)\left[\tilde{\sigma}_e^{\,n-1}\left(a_3\tilde{\phi}+\tilde{\phi}''\right)\right]=0 \tag{18}$$

where $(\ )'=\frac{\partial}{\partial\theta}$, a_i are functions of f and n.

For a stress free mode I crack in homogeneous continuum, the problem can be summarized as a two point boundary problem of (18) with

$$\tilde{\phi}(\pi)=\tilde{\phi}'(\pi)=\tilde{\phi}'''(0)=\tilde{\phi}'(0)=0. \tag{19}$$

Solution of (18) and (19) shows that both the exponent of singularity f and angular distributions of the fields change with *Tz*. (*f*-2) is highest at *Tz*=0 and 0.5 and agrees with 2D HRR solution, but when 0<Tz<0.5, the singularity becomes weaker and Rice's line energy integral *J* will no longer be path independent. The amplitude coefficient *K* is related to *J* by

$$J=\alpha\varepsilon_0\sigma_0K^{n+1}r^{(n+1)(f-2)}I(n,Tz). \tag{20}$$

Substituting (20) into (15) leads to

$$\sigma_{ij}=\left[\frac{J}{\alpha\varepsilon_0\sigma_0I(n,Tz)r}\right]^{1/(n+1)}\tilde{\sigma}_{ij}(\theta,Tz),$$
$$\varepsilon_{ij}=\frac{3}{2}\alpha\left[\frac{J}{\alpha\varepsilon_0\sigma_0I(n,Tz)r}\right]^{n/(n+1)}\tilde{\varepsilon}_{ij}(\theta,Tz). \tag{21}$$

3.3 Distribution of *Tz* in Front of 3D Cracks

In front of a mode I through-crack in finite thick plate, Tz can be predicted very well by the following expression (where $\xi=r/2h$)

$$Tz=\upsilon_{ep}\left(1-1.218\xi^{1/2}-0.359\xi+0.361\xi^{3/2}\right)\left[1-\left|\frac{z}{h}\right|^{0.94\xi^{-0.58}}\right]^2 \tag{22}$$

Substituting (22) into (21), 3D stress and strain fields near the tip of a real crack can be predicted.

3.4 Effect of 3D Constraint on Interface Crack Tip Fields

From (22) and (3) it can be seen that *Tz* will change with materials. Then in the case of crack on the interface of two different materials, crack tip fields may be affected by the change of *Tz* from one material to another.

Our investigation on the near tip fields of cracks lying on the interface of two strain hardening materials under 3D stress constraints has shown strong effects of *Tz* on the continuities and singularities of the radial and equivalent stresses.

4. J-A_2-Tz THEORY

4.1 Out-of-plane and In-plane Constraint of Plane Strain Cracks

In plane strain state, $\varepsilon_{zz}=0$. So from (5) and (7) it can be get that

$$Tz = \upsilon_{ep} = \frac{1}{2} - \left(\frac{1}{2} - \upsilon\right)\frac{1}{\alpha}\left(\frac{\sigma_0}{\sigma_e}\right)^{n-1} \tag{23}$$

Therefore, the in-plane stress-strain fields are coupled with Tz. This coupling relation is hard to be revealed properly by the asymptotic solutions. Based on the higher order solution which can match the in-plane constraint very well and the above analysis we propose the following J-A_2-Tz theory

$$\begin{aligned} \sigma_{ij} &= \left[\frac{J}{\alpha\varepsilon_0\sigma_0 I r}\right]^{\frac{n}{n+1}} \tilde{\sigma}_{ij}(\theta) + A_2\left[r^{s_2}\hat{\sigma}_{ij}(\theta) + A_2 r^{s_2}\bar{\sigma}_{ij}(\theta)\right], \quad (i,j=1,2) \\ \sigma_{33} &= Tz(\sigma_{11} + \sigma_{22}), \qquad (Tz = \upsilon_{ep}) \\ \sigma_e &= \left[(1 - Tz + Tz^2)(\sigma_{11}^2 + \sigma_{22}^2) - (1 - 2Tz - 2Tz^2)\sigma_{11}\sigma_{22} + 3\sigma_{12}^2\right]^{\frac{1}{2}}. \end{aligned} \tag{24}$$

When a J-Q representation is used to replace the J-A_2 solution in (24), a J-Q_T-Tz theory can be obtained.

4.2 J-A_2-Tz Theory for General 3D Cracks

For general three-dimensional cracks. the constraints consist of the in-plane constraint as well as the out-of-plane constraint.

i) Out-of-plane constraint

The out-of-plane constraint is defined as the stress constraint out of the plane of the sheet element in consideration. It can be represented by Tz. For general 3D cracks the real distribution of Tz of the cracked body should be used. For through-thick cracks, Tz is given by eq.(22).

ii) In-plane constraint

In-plane constraint is defined as the constraint caused by the boundary conditions of the sheet element. The J-Q representation or higher order solutions can be used to match the in-plane stresses for various in-plane constraints.

Considering both of the in-plane and out-of-plane constraints, a J-A_2-Tz or J-Q_T-Tz theory can also be proposed for general 3D crack problems:

i) In-plane stresses σ_{ij} ($i,j=1,2$): described by J-A_2 or J-Q theory.

ii) The equivalent stress σ_e and σ_{zz}: determined by eq.(22) and the last two equations of (24).

iii) Strain field and other parameters should be calculated on the basis of i) and ii).

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The Unified Description of the Three-dimensional Fields at Notches and Cracks

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ABSTRACT

The relationship of notch fields and crack fields is concerned in this paper. Based on theoretical studies and 3D numerical simulations, some common features of the 3D elastic-plastic notch fields and the crack fields are analyzed and the unified description of the fields is discussed. It is surprising to find that the 3D stress constraints at a circular hole, a notch and a crack can be described uniformly. Fracture of notches is discussed and some interesting topics in this direction are listed to unify fatigue mechanics and fracture mechanics.

1. INTRODUCTION

Both notches and cracks are stress raisers in structures. They have the same importance in strength analysis and safe design. For sharp cracks, the fracture mechanics method of stress-strain analyses has been shown to be effective and successful. However, the stress-strain fields at notches with blunt ends are much more complicated and difficult to deal with. Even for two-dimensional (2D) notch problems, there is no rational description available for the elastic-plastic fields near the notch-root. In the three-dimensional (3D) frame, the additional scale of the finite notch-root radius will introduce several mechanics parameters which are necessary to be considered in stress analyses, such as the ratio of root radius to thickness, root radius to the size of the plastic zone, *etc.* On the other hand, notches and cracks have many common features. Physically, there is no ideally sharp crack. When the depth to root radius ratio and the size of interesting region to the radius ratio become larger, the effect of the notch-root radius become less important and a notch can be treated as a crack. Another typical case is circular holes for which 2D as well as 3D elastic theoretical solutions can be found in the literatures.

As shown by Fig.1, the geometrical variation from a hole to a notch and to a crack is a continuous process so that there is no determined distinction between notches and cracks. In the figure, ρ is the notch-root radius, a is the notch depth, B is the thickness of a plate.

It is well known that the elastic solution of a 2D crack can be obtained by the solution of a elliptical hole as ρ/a is approaching zero. The Creager and Paris solution [1] and the recent work by Kuang [2] and Lazzarin and Tovo [3] have built a relation between the 2D notch-root and crack-tip elastic stress fields. The elastic solution for a circular hole in a finite thickness plate has been obtained by Chang and Guo [4]. For general notches in finite thickness plates, the in-plane stress distributions are insensitive to the notch radius to thickness ratio ρ/B [5] while the stress concentration factor K_t is a function of ρ/B and changes through the thickness. In elastic-plastic situation, theoretical solutions can only be obtained for 2D circular holes under specific loading conditions [6] and sharp cracks. The 3D fields and the elastic-plastic deformation near general notches which are more interesting in engineering are poorly understood, thus it will be the focus of the following discussions.

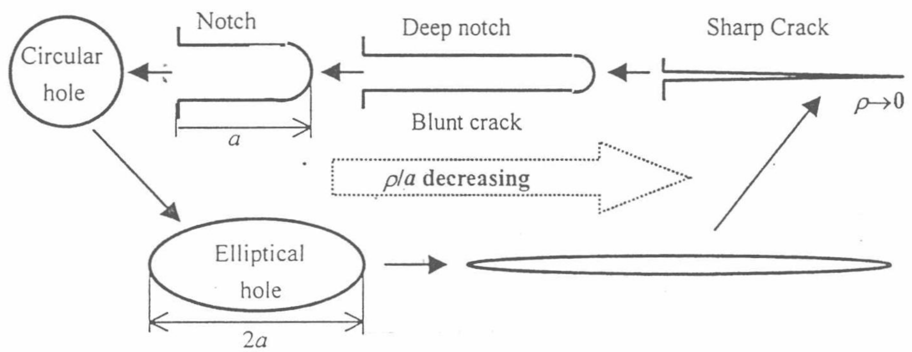

Fig.1. The geometrical relationship between notches and cracks

2. ELASTIC STRESS ANALYSIS

2.1. A Unified Description of Notch and Crack Tip Fields

When the stress solution is formulated for a general notch, the stress function expressed in variable separated form of r and θ,

$$\varphi = A_0 r^{\mu} \tilde{\varphi}_0(\theta) + \sum_{k=1}^{m} A_k r^{\lambda_k} \tilde{\varphi}_k(\theta) \tag{1}$$

should satisfy the governing equation of the problem,

$$\nabla^2 \sigma_{ij} + \frac{1}{1+\nu} \sigma_{kk,ij} = 0. \tag{2}$$

Where the second and higher terms in (1) are the solution for a sharp crack or a sharp V-notch which can be solved easily by the fracture theory, the first term represents the influence of blunt notch-root and $\mu<\lambda_1<\lambda_2<\ldots$. Once the terms for a sharp crack or V-notch are obtained by solving the standard two-point boundary value problem, the first term can not be solved accurately along the notch boundary, as shown in [3], but μ and $\tilde{\varphi}_0, \tilde{\varphi}_0', \tilde{\varphi}_0'', \tilde{\varphi}_0''', \tilde{\varphi}_0''''$ can be solved accurately on the line of θ=0 by the boundary conditions at the notch-root and eq.(2) [7]. Thus the stress solution for a deep blunt notch can be written as

$$\sigma_{ij} = A_0 r^{\mu-2}\tilde{\sigma}_{ij}^0 + [\sigma_{ij}]_{sharp} \tag{3}$$

and μ and $\tilde{\sigma}_{ij}^0$ can be determined completely by the solution of the sharp notch. It is important to find that not only the singular term of the sharp notch field, but also the higher term(s) have strong influence on μ and $\tilde{\sigma}_{ij}^0$. Therefore, μ-2=-3/2 as obtained by Creager & Paris for a blunt crack [1] and Hui & Ruina for a small hole at the crack tip [8] is only a specific value under certain in-plane constraint. For example, μ is a function of the T-stress for a blunt crack as shown in Fig.2 where W is the width of the plate.

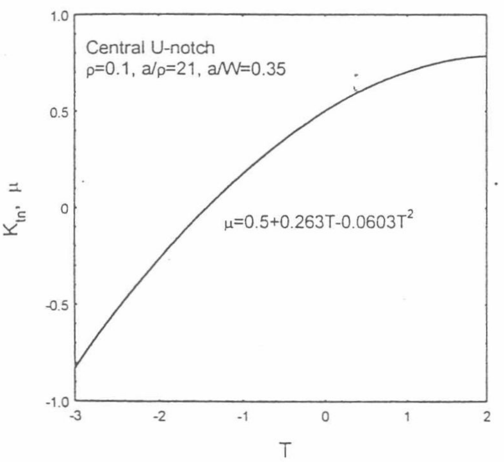

Fig.2 The effect of T-stress on the notch-root stress solution.

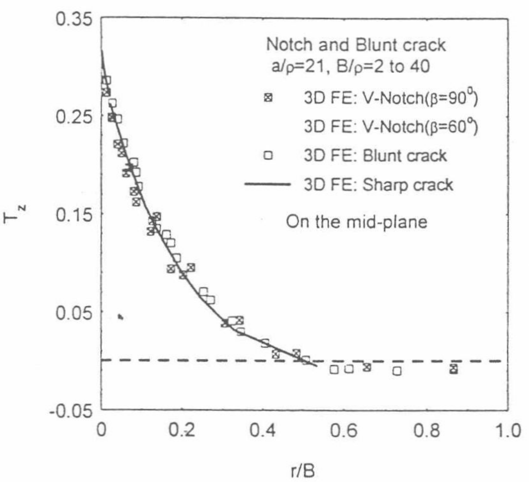

Fig.3 The out-of-plane constraint ahead of cracks and notches in finite thickness plates

2.2. 3D Constraints

3D finite element analyses [5] show that in front of a opening mode notch in finite thickness plate, the distribution of the normalized stress $\sigma_{yy}/\sigma_{yymax}$ and the in-plane stress ratio $T_x=\sigma_{xx}/\sigma_{yy}$ with r/ρ is nearly independent of the plate thickness and can be predicted well with the corresponding 2D solutions. Therefore, the out-of-plane stresses become an important parameter in describing the 3D notch-root fields. For convenience, the out-of-plane constraint factor $T_z=\sigma_{zz}/(\sigma_{xx}+\sigma_{yy})$ used by Guo [9] for 3D cracks is introduced. For elastic through-thickness cracks, T_z can be expressed approximately by

$$T_z = \nu\left[1 - 1.79\left(\frac{r}{B}\right)^{1/2} + 0.113\left(\frac{r}{B}\right) + 0.631\left(\frac{r}{B}\right)^{3/2}\right] f(z/B). \tag{4}$$

When the origin of the coordinate r setting at the middle point of the center of the notch root arc and the notch-root, the variation of T_z with r/B in front of a blunt crack, a shallow notch and a hole can collapse to a unified curve, as shown by Fig.3. Therefore, the solution for a circular hole or for a through-thickness crack can be used to estimate T_z for general notches. It can be seen from the figure that when $B/\rho<1$, T_z will remain less than $T_{zmax}/4$ and merit less attention in practical application.

3. Elastic-plastic Stress-Strain Fields

The elastic-plastic deformation near notches is much more complicated than elastic one, theoretical solution can only be found for some simple notch geometry and loading configurations. Recently Guo [6] obtained the solution for an equal-biaxial stressed infinite plate with a circular hole. He found that the elastic-plastic solution of the problem can be obtained from the elastic solution by a simple replacement of variable. If the elastic solution of the equivalent strain ahead of the notch is known as

$$\frac{\varepsilon_{eq}}{\varepsilon_{ys}} = f(r), \tag{5}$$

then the corresponding elastic-plastic solution can be obtained as

$$\frac{\varepsilon_{eq}}{\varepsilon_{ys}} = f(r'), \quad r' = r \times \frac{r_{p0}}{r_p} \tag{6}$$

where ε_{ys} is the yield strain of the material, r_{p0} is defined by $f(r_{p0})=1$ and r_p is the size of the plastic zone ahead of the notch. This is the so called Strain-Equivalent-Rule (SER).

Finite element analyses show that the SER can not only be applied to general 2D and 3D notches, but can also be applied to short cracks. Some typical results are given in Figs.4 and 5.

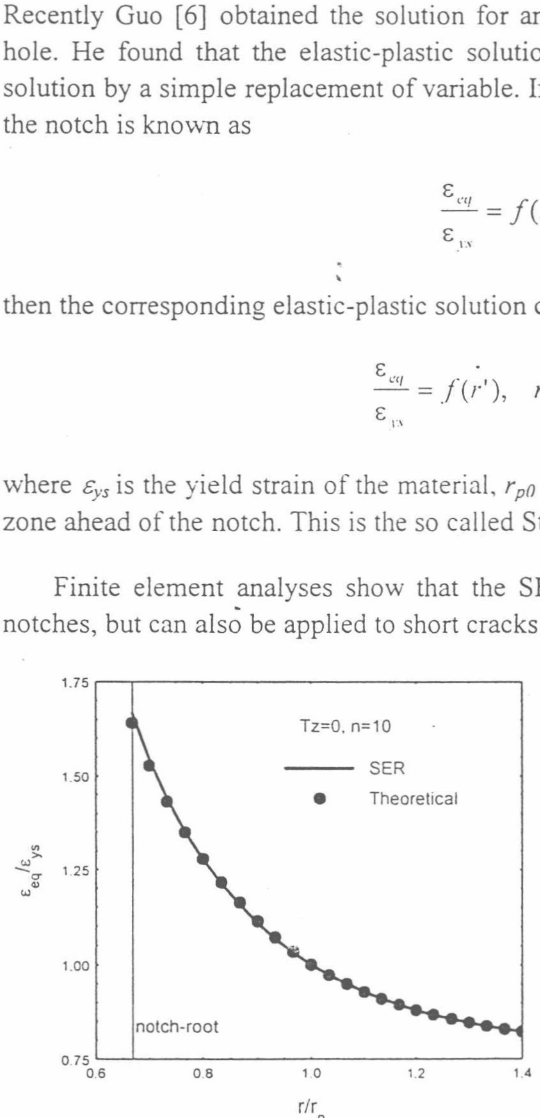

Fig.4. Strain distribution at a circular hole in plane stress state

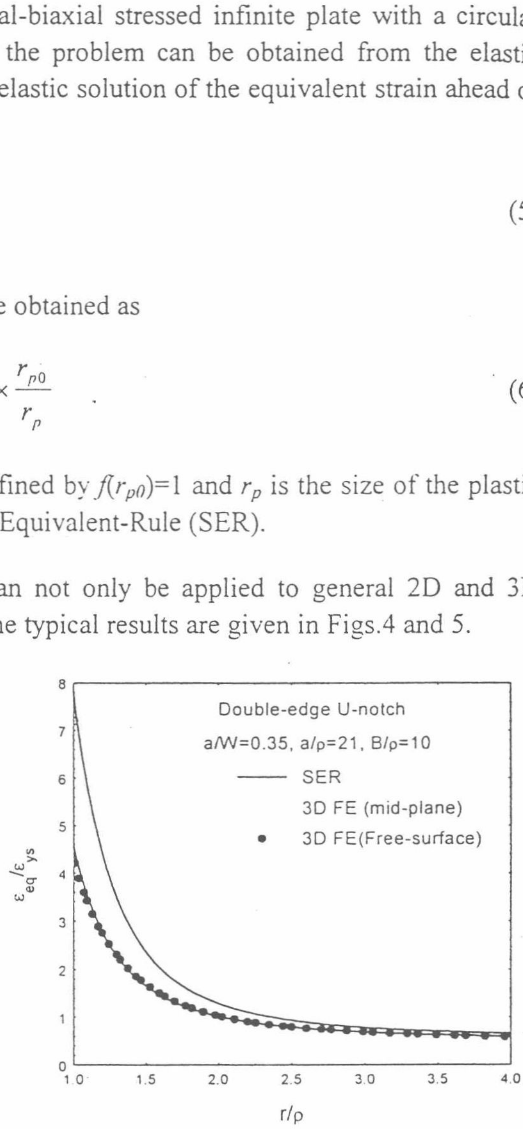

Fig.5. Strain distribution at a deep U-notch in a finite thickness plate.

For long cracks the SER can not hold good, but the plastic zone size is still a useful controlling parameter of the elastic-plastic fields.

4. FRACTURE CRITERIONS FOR NOTCHES AND CRACKS

It is well known that the fracture of a material element is strongly dependent on the 3D stress state. With increasing a/ρ, the stress triaxiality R_σ which is equal to the ratio of the mean stress to the equivalent stress will increase. For a smooth bar R_σ is about 3/4 and for notched bar R_σ=1 to 2. For sharp cracks R_σ is about 3 on the mid-plane. Figures 6 and 7 give the simulation results by use of 3D finite element and a cell model [10]. It is shown clearly that both the critical fraction of void volume and the V_{GC} are not constant and do not change monotonously with R_σ. For notched bar (R_σ is about 1 to 2), V_{GC} is nearly constant as has been investigated in many experiments. However, in front of a crack V_{GC} may be much higher. For lower R_σ as in a smooth bar V_{GC} is obviously higher than in notched bars as well. Therefore, with proper initial values of the material parameters the 3D cell model simulation can provide us a more complete picture of fracture of notched and cracked bodies.

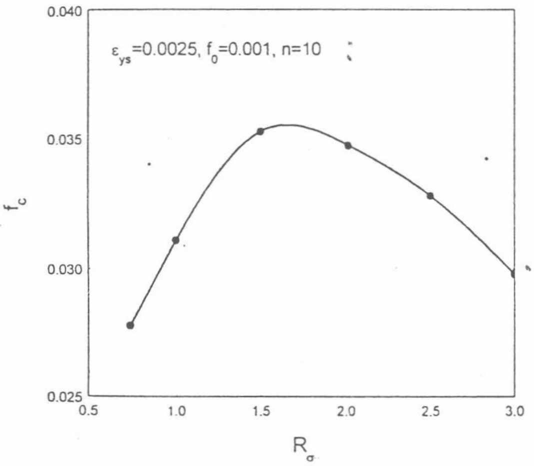

Fig.6. Critical fraction of void volume against stress triaxiality

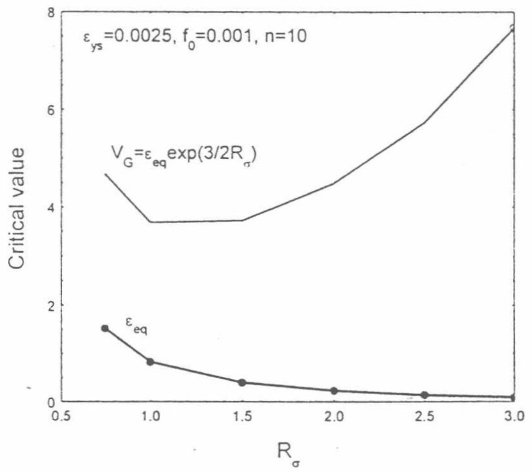

Fig.7. Variation of the critical strain and V_{GC} with stress triaxiality

5. DISCUSSIONS

Rational prediction of the life of structural components has long been the main objective of fatigue mechanics and fracture mechanics. The traditional fatigue mechanics determines the life by stress-strain concentration analysis of notches and mainly concerns the crack initiation life. In contrast, fracture mechanics is based on crack analysis and always provides the life of crack growth. To predict the whole life of a component rationally, confusion of fatigue mechanics and fracture mechanics is necessary. The following topics will be interesting in this direction:

i) Mechanics behavior in the crack blunting zone in which the fracture process occurs.

ii) Unified description of 3D elastic-plastic notch fields and crack fields.

iii) Fatigue damage mechanism of material elements at notches and cracks.

iv) Unified description of the whole process of crack initiation from material defects or pits

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P25

金属机件在复杂环境下的三维疲劳断裂

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摘要 以三维弹塑性断裂理论为基础，对复杂载荷、复杂环境作用下的金属材料和结构的疲劳、断裂的若干关键问题进行了概要分析。给出了由材料性能试验的标准试样结果预测结构中一般形态缺陷的三维破坏的最新结果，获得了对不同载荷条件下腐蚀疲劳裂纹扩展的统一描述，介绍了由裂纹扩展基准曲线预测谱载腐蚀疲劳裂纹扩展寿命的最新进展，对结构服役寿命/日历寿命研究方法作了探讨，最后提出了复杂环境下疲劳断裂研究的方向和问题。

关键词 三维疲劳断裂，随机谱，腐蚀疲劳，服役寿命

引 言

疲劳断裂是金属结构最主要的破坏形式。人们对此已进行了几十年的研究，积累了较丰富的疲劳断裂知识，制定了一系列材料疲劳断裂性能试验标准，发展了结构疲劳、耐久性和损伤容限设计方法，研制出一批高韧性、高疲劳抗力的新型结构材料，形成了一系列抗疲劳断裂材料制备和加工工艺，因疲劳断裂导致的破坏事故已大幅度减少。加之全球的科技、经济在世纪交替时期迅猛发展，疲劳断裂学科已过了发展的巅峰时期而进入稳定发展时期。然而，仍有许多重大问题令人困扰。例如，从实验室的标准材料性能试验到实际复杂结构，疲劳断裂行为的差异仍难以预测，在过去的十余年中，国际学术界以约束理论为突破口试图解决这种差异，但主要工作局限于面内约束或二维问题，对更实际的三维问题研究尚未全面展开。另一方面，随着经济性问题的日益突出，复杂环境对设备服役寿命（Service life）或日历寿命（Calendric life）的影响问题近年来越来越受到普遍关注[1]，在国内腐蚀破坏已列入“973”项目。但因问题的复杂性，目前的研究主要集中在实验现象和机理认识阶段，对复杂环境下寿命的预测只有一些经验规律可用。

寻求从实验室的标准试样的破坏到复杂结构的三维破坏、从实验室给定载荷条件下的破坏到实际任意载荷条件下的破坏、从实验室有限载荷环境组合和有限周期下的破坏到复杂的全服役期的破坏等之间规律的认识将是疲劳断裂研究需要解决的重大问题。先进材料和技术的开发利用，也涉及大量的复杂环境三维破坏问题。本文结合作者及其课题组近年的工作，对复杂环境中材料和结构的三维破坏的一些重要问题进行讨论。

1 三维断裂理论基础

1.1 三维缺陷和弹塑性分析

材料缺陷、加工缺陷、环境蚀坑、疲劳裂纹等应力应变集中部位是材料和结构破坏的关键细节，其附近的三维应力应变分布是研究材料与结构破坏的力学基础。

对理想的二维尖锐裂纹，其端部渐近应力应变场可由 HRR 解和近年发展的 J-Q 理论或高阶解给出

$$\sigma_{ij}=\left[\frac{J}{\alpha\varepsilon_0\sigma_0 I(n)r}\right]^{1/(n+1)}\tilde{\sigma}_{ij}(\theta)+Q\delta_{ij} \tag{1}$$