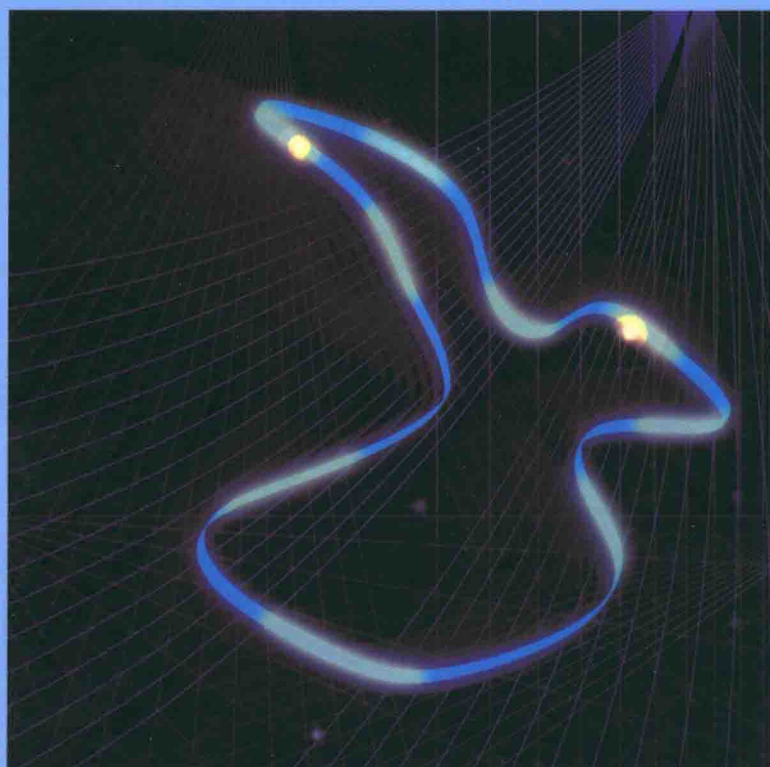


Elias Kiritsis

String Theory in a Nutshell

弦论



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Elias Kiritsis

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Preface

In the fall of 2003, it was suggested to me that a new textbook on the discipline that goes under the name of string theory was needed in order to accommodate several advances that have happened since Polchinski's books appeared.

Although I wrote a short textbook in 1997, I have not yet learned my lesson. Therefore, I embarked on a direct confrontation with deadlines that, although not as spectacular as the ones in Polchinski's book, managed to wreak havoc on my academic and personal schedule. I have not even put sufficient credence in a colleague's statement "it is never too late to stop writing a book." I can only hope now that the result was worth the effort.

This is a textbook on the collection of ideas categorized under the name of "string theory." This is a large domain that has as its central goal the unification of all interactions including gravity. There have been several surges of progress in different directions in the past twenty years, and this book tries to give the student of the field some of the salient ideas.

It is *not* the purpose of this book to provide deep insights into the theory. I can only leave this to more competent colleagues. Its purpose is to provide the fastest possible introduction to the basic formalism and structure of string theory and its main properties and ramifications that are on a reasonably solid footing today.

This effort is unlike writing a textbook about the Standard Model of the fundamental interactions. It is less clear here what will turn out to be just mathematics, what will transform into real physics, and what will be neither of the above. However, the scope and deep interest of the endeavor, namely, to understand the basic mysteries of the universe at the most fundamental level, has driven more than a generation of bright physicists and has provided breakthroughs in our theoretical understanding of both gravitational and gauge theories. It is this interest that drives researchers today, together with the hope that the theory will eventually be seriously confronted with experimental data.

There are several current and past areas of research in string theory that have not been treated in this book. The reasons were varied. They include the subjects of the

Green-Schwarz quantization of superstrings, the Berkovits quantization, string field theory, the whole issue of cosmological backgrounds in string theory, strings and branes at finite temperature and the issue of the Hagedorn transition, as well as the more recent studies of tachyon condensation and the development of the formalism to investigate compactifications with fluxes and moduli stabilization. For all these subjects a brief guide into the bibliography is given.

At the end of each chapter a section of bibliography is provided. Its purpose is to guide the student to the starting points of the subject's literature. In particular (s)he is guided to specialized reviews and original papers when they are deemed to have a pedagogical value. Obviously, the choice of references reflects the knowledge and taste of the author, and I apologize in advance for possible omissions or errors.

A serious effort was made to provide many exercises after each chapter. There are 460 exercises in this book and they are of several types. The simplest guide the student to complete calculations that are sketched but not done in detail in the text. Such exercises are typically easy although sometimes they can be labor intensive. Exercises are given where the student is invited to work on other issues that are not directly dealt with in the text, but which are nevertheless useful in understanding the issues involved. Finally, there are exercises that initiate an exploration of areas not treated in this book. Some exercises are hard and they have been the subject of full-blown research articles in the past. The intervention of the instructor in the choice of exercises is therefore important.

A website dedicated to this book will be in service: <http://hep.physics.uoc.gr/~kiritsis/string-book/>. It will collect information that might be useful to readers, including the omnipresent corrections of errors and misprints. The author welcomes suggestions for corrections at the address kiritsis@physics.uoc.gr

I am indebted to many people who have indirectly contributed to the present effort. They include my teachers who gave me the first tools to do science, colleagues who shared their knowledge with me, collaborators who shared knowledge and patience, and students who pushed me several times to clarify my understanding of subjects. They are too many to mention by name. I would like to thank, however, P. Anastasopoulos, M. Bianchi, R. Casero, U. Gursoy, F. Nitti, A. Paredes, S. Wadia, and especially L. Alvarez-Gaume and B. Pioline for reading the manuscript at various stages and providing corrections and constructive criticism. Extra thanks go to P. Anastasopoulos for his help with the figures. I would also like to thank the Marie Curie program of the European Commission for funding my research in the past and during the period this book was written. Finally and most importantly, I would like to thank my life partner Takoui for understanding and support during this difficult endeavor.

Elias Kiritsis
January 2006

Abbreviations

ADM	Arnowitt-Deser-Misner
AdS	Anti-de Sitter
ALE	Asymptotically locally Euclidean
BCFT	Boundary conformal field theory
BF	Breitenlohner-Freedman (bound)
BFSS	Banks-Fischler-Shenker-Susskind
BH	Bekenstein-Hawking
BPS	Bogomolnyi-Prasad-Sommerfield
BRST	Becchi-Rouet-Stora-Tyutin
BTZ	Banados-Teitelboim-Zanelli
CFT	Conformal field theory
CP	Chan-Paton
CP	Charge conjugation \times parity
CP^n	Complex projective space
CY	Calabi-Yau (manifold)
DBI	Dirac-Born-Infeld
DD	Dirichlet-Dirichlet (boundary conditions)
DDF	Di Vecchia-Del Giudice-Fubini
DLCQ	Discrete light-cone quantization
DN	Dirichlet-Neumann (boundary conditions)
DNT	Dirac-Nepomechie-Teitelboim
EFT	Effective field theory
FI	Fayet-Iliopoulos
GH	Gibbons-Hawking
GS	Green-Schwarz
GSO	Gliozzi-Scherk-Olive

\mathcal{H}_2	Upper half plane
HMS	Hypermultiplet moduli space
HRG	Holographic renormalization group
HW	Highest weight
KK	Kaluza-Klein
KN	Kerr-Newman
MW	Majorana-Weyl
MQM	Matrix quantum mechanics
NN	Neumann-Neumann (boundary conditions)
NS	Neveu-Schwarz
NSR	Neveu-Schwarz-Ramond
OPE	Operator product expansion
PB	Poisson bracket
QFT	Quantum field theory
R	Ramond
RG	Renormalization group
RN	Reissner-Nordström
\mathbb{RP}_2	Real projective plane
RS	Randall-Sundrum
SCFT	SuperConformal field theory
SM	Standard Model
ST	String theory
SUGRA	Supergravity
SUSY	Supersymmetry
SYM	Super Yang-Mills
vev(s)	vacuum expectation value(s)
VMS	Vector moduli space
WZ	Wess-Zumino
WZW	Wess-Zumino-Witten

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1.1 Prehistory

The quest in physics has been historically dominated by unraveling the simplicity of the physical laws, moving more and more toward the elementary. Although this is not guaranteed to succeed indefinitely, it has been vindicated so far. The other organizing tendency of the human mind is toward “unification”: finding a unique framework for describing seemingly disparate phenomena.

The physics of the late nineteenth and twentieth centuries is a series of discoveries and unifications. Maxwell unified electricity and magnetism. Einstein developed the general theory of relativity that unified the principle of relativity and gravity. In the late 1940s, there was a culmination of two decades’ efforts in the unification of electromagnetism and quantum mechanics. In the 1960s and 1970s, the theory of weak and electromagnetic interactions was also unified. Moreover, around the same period there was also a wider conceptual unification. Three of the four fundamental forces known were described by gauge theories. The fourth, gravity, is also based on a local invariance, albeit of a different type, and so far stands apart.¹ The combined theory, containing the quantum field theories of the electroweak and strong interactions together with the classical theory of gravity, formed the Standard Model of fundamental interactions. It is based on the gauge group $SU(3) \times SU(2) \times U(1)$. Its spin-1 gauge bosons mediate the strong and electroweak interactions. The matter particles are quarks and leptons of spin $\frac{1}{2}$ in three copies (known as generations and differing widely in mass), and a spin-0 particle, the Higgs boson, still experimentally elusive, that is responsible for the spontaneous breaking of the electroweak gauge symmetry.

¹ Today, we have some intriguing evidence that even gravity may be a strong-coupling facet of an extra underlying four-dimensional gauge theory.

The Standard Model has been experimentally tested and has survived thirty years of accelerator experiments.² This highly successful theory, however, is not satisfactory:

- A classical theory, namely, gravity, described by general relativity, must be added to the Standard Model in order to agree with experimental data. This theory is not renormalizable at the quantum level. In other words, new input is needed in order to understand its high-energy behavior. This has been a challenge to the physics community since the 1930s and (apart from string theory) very little has been learned on this subject since then.
- The three SM interactions are not completely unified. The gauge group is semisimple. Gravity seems even further from unification with the gauge theories. A related problem is that the Standard Model contains many parameters that look *a priori* arbitrary.
- The model is unstable as we increase the energy (hierarchy problem of mass scales) and the theory loses predictivity as one starts moving far from current accelerator energies and closer to the Planck scale. Gauge bosons are protected from destabilizing corrections because of gauge invariance. The fermions are equally protected due to chiral symmetries. The real culprit is the Higgs boson.

Several attempts have been made to improve on the problems above.

The first attempts focused on improving on unification. They gave rise to the grand unified theories (GUTs). All interactions were collected in a simple group $SU(5)$ in the beginning, but also $SO(10)$, E_6 , and others. The fermions of a given generation were organized in the (larger) representations of the GUT group. There were successes in this endeavor, including the prediction of $\sin^2 \theta_W$ and the prediction of light right-handed neutrinos in some GUTs. However, there was a need for Higgs bosons to break the GUT symmetry to the SM group and the hierarchy problem took its toll by making it technically impossible to engineer a light electroweak Higgs.

The physics community realized that the focus must be on bypassing the hierarchy problem. A first idea attacked the problem at its root: it attempted to banish the Higgs boson as an elementary state and to replace it with extra fermionic degrees of freedom. It introduced a new gauge interaction (termed technicolor) which bounds these fermions strongly; one of the techni-hadrons should have the right properties to replace the elementary Higgs boson as responsible for the electroweak symmetry breaking. The negative side of this line of thought is that it relied on the nonperturbative physics of the technicolor interaction. Realistic model building turned out to be difficult and eventually this line of thought was mostly abandoned.

A competing idea relied on a new type of symmetry, supersymmetry, that connects bosons to fermions. This property turned out to be essential since it could force the bad-mannered spin-0 bosons to behave as well as their spin- $\frac{1}{2}$ partners. This works well, but supersymmetry stipulated that each SM fermion must have a spin-0 superpartner with equal mass. This being obviously false, supersymmetry must be spontaneously broken at

² With the exception of the neutrino sector that was suspected to be incomplete and is currently the source of interesting discoveries.