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Modal identification of output-only systems using frequency domain decomposition

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Abstract
In this paper a new frequency domain technique is introduced for the modal identification of output-only systems, i.e. in the case where the modal parameters must be estimated without knowing the input exciting the system. By its user friendliness the technique is closely related to the classical approach where the modal parameters are estimated by simple peak picking. However, by introducing a decomposition of the spectral density function matrix, the response spectra can be separated into a set of single degree of freedom systems, each corresponding to an individual mode. By using this decomposition technique close modes can be identified with high accuracy even in the case of strong noise contamination of the signals. Also, the technique clearly indicates harmonic components in the response signals.

(Some figures in this article are in colour only in the electronic version; see www.iop.org)

1. Introduction

Modal identification of output-only systems is normally associated with the identification of modal parameters from the natural responses of civil engineering structures, space structures and large mechanical structures. Normally, in these cases the loads are unknown and, thus the modal identification has to be carried out based on the responses only. Real case examples on some civil engineering structures can be found in Ventura and Horyna [1] or Andersen *et al* [2].

The present paper deals with a new way of identifying the modal parameters of a structure from the responses only when the structure is loaded by a broad-banded excitation.

The technique presented in this paper is an extension of the classical frequency domain approach often referred to as the basic frequency domain (BFD) technique, or the peak picking technique. The classical approach is based on simple signal processing using a discrete Fourier transform, and uses the fact that well separated modes can be estimated directly from the power spectral density matrix at the peak [3]. Other implementations of the technique make use of the coherence between channels [4].

The classical technique gives reasonable estimates of the natural frequencies and mode shapes if the modes are well separated. However, in the case of close modes, it can be difficult to detect the close modes, and, even in the case where close modes are detected, estimates becomes heavily biased. Furthermore, the frequency estimates are limited by the frequency resolution of the spectral density estimate and, in all cases, damping estimation is uncertain or impossible.

Even though the classical approach has limitations concerning accuracy in the identification process, the classical approach has important advantages when compared to other approaches. It is natural to compare it with classical two-stage time domain approaches such as the polyreference technique [5], the Ibrahim time domain technique [6] and the eigensystem realization algorithm [7], or to compare it with the new one-stage time domain identification techniques known as the stochastic subspace identification algorithms [8]. The main advantages compared to these other techniques is that the classical approach is much more user friendly, it is faster, simpler to use and gives the user a 'feeling' of the data he or she is dealing with. The fact that the user works directly with the spectral density functions helps the user in figuring out what is structural just by looking at the spectral density functions.

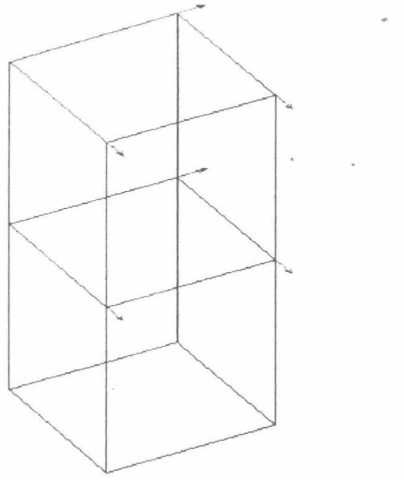

Figure 1. Geometry of two-storey building model. Measurement points are indicated by arrows.

This reinforces the users understanding of the physics and thus provides a valuable basis for a meaningful identification.

The technique presented in this paper is a frequency domain decomposition (FDD) technique. It removes all the disadvantages associated with the classical approach, but keeps the important features of user friendliness and even improves the physical understanding by dealing directly with the spectral density function. Furthermore, the technique gives a clear indication of harmonic components in the response signals.

In this paper it is shown that taking the singular value decomposition (SVD) of the spectral matrix, the spectral matrix is decomposed into a set of auto spectral density functions, each corresponding to a single degree of freedom (SDOF) system. This result is exact in the case where the loading is white noise, the structure is lightly damped and when the mode shapes of close modes are geometrically orthogonal. If these assumptions are not satisfied the decomposition into SDOF systems is approximate, but still the results are significantly more accurate than the results of the classical approach.

2. Theoretical background of frequency domain decomposition

The relationship between the unknown inputs $x(t)$ and the measured responses $y(t)$ can be expressed as [9]

$$G_{yy}(j\omega) = \bar{H}(j\omega)G_{xx}(j\omega)H(j\omega)^{T} \tag{1}$$

where $G_{xx}(j\omega)$ is the $(r \times r)$ power spectral density (PSD) matrix of the input, r is the number of inputs, $G_{yy}(j\omega)$ is the $(m \times m)$ PSD matrix of the responses, m is the number of responses, $H(j\omega)$ is the $(m \times r)$ frequency response function (FRF) matrix and the overbar and superscript T denote the complex conjugate and transpose, respectively.

The FRF can be written in partial fraction, i.e. pole/residue, form

$$H(j\omega) = \sum_{k=1}^{n} \frac{R_k}{j\omega - \lambda_k} + \frac{\bar{R}_k}{j\omega - \bar{\lambda}_k} \tag{2}$$

where n is the number of modes, λ_k is the pole and R_k is the residue:

$$R_k = \phi_k \gamma_k^{T} \tag{3}$$

where ϕ_k and γ_k are the mode shape vector and the modal participation vector, respectively. Suppose the input is white noise, i.e. its PSD is a constant matrix ($G_{xx}(j\omega) = C$), then equation (1) becomes

$$G_{yy}(j\omega) = \sum_{k=1}^{n}\sum_{s=1}^{n}\left[\frac{R_k}{j\omega - \lambda_k} + \frac{\bar{R}_k}{j\omega - \bar{\lambda}_k}\right] \times C\left[\frac{R_s}{j\omega - \lambda_s} + \frac{\bar{R}_s}{j\omega - \bar{\lambda}_s}\right]^{H} \tag{4}$$

where superscript H denotes a complex conjugate and transpose. Multiplying the two partial fraction factors and making use of the Heaviside partial fraction theorem, after some mathematical manipulations, the output PSD can be reduced to a pole/residue form as follows

$$G_{yy}(j\omega) = \sum_{k=1}^{n} \frac{A_k}{j\omega - \lambda_k} + \frac{\bar{A}_k}{j\omega - \bar{\lambda}_k} + \frac{B_k}{-j\omega - \lambda_k} + \frac{\bar{B}_k}{-j\omega - \bar{\lambda}_k} \tag{5}$$

where A_k is the kth residue matrix of the output PSD. As for the output PSD itself, the residue matrix is an $(m \times m)$ Hermitian matrix and is given by

$$A_k = R_k C\left(\sum_{s=1}^{n} \frac{\bar{R}_s^{T}}{-\lambda_k - \bar{\lambda}_s} + \frac{\bar{R}_s^{T}}{-\lambda_k - \bar{\lambda}_s}\right). \tag{6}$$

The contribution to the residue from the kth mode is given by

$$A_k = \frac{R_k C \bar{R}_k^{T}}{2\alpha_k} \tag{7}$$

where α_k is minus the real part of the pole $\lambda_k = -\alpha_k + j\omega_k$. As it appears this term becomes dominating when the damping is light, and, thus, is the case of light damping; the residue becomes proportional to the mode shape vector

$$A_k \propto R_k C \bar{R}_k = \phi_k \gamma_k^{T} C \gamma_k \phi_k^{T} = d_k \phi_k \phi_k^{T} \tag{8}$$

where d_k is a scalar constant. At a certain frequency ω only a limited number of modes will contribute significantly, typically one or two modes. Let this set of modes be denoted by $Sub(\omega)$. Thus, in the case of a lightly damped structure, the response spectral density can always be written

$$G_{yy}(j\omega) = \sum_{k \in Sub(\omega)} \frac{d_k \phi_k \phi_k^{T}}{j\omega - \lambda_k} + \frac{\bar{d}_k \bar{\phi}_k \bar{\phi}_k^{T}}{j\omega - \bar{\lambda}_k}. \tag{9}$$

This is a modal decomposition of the spectral matrix. The expression is similar to the results one would get directly from equation (1) under the assumption of independent white noise input, i.e. a diagonal spectral input matrix.

3. Identification algorithm

In the FDD identification, the first step is to estimate the PSD matrix. The estimate of the output PSD $\hat{G}_{yy}(j\omega)$ known at

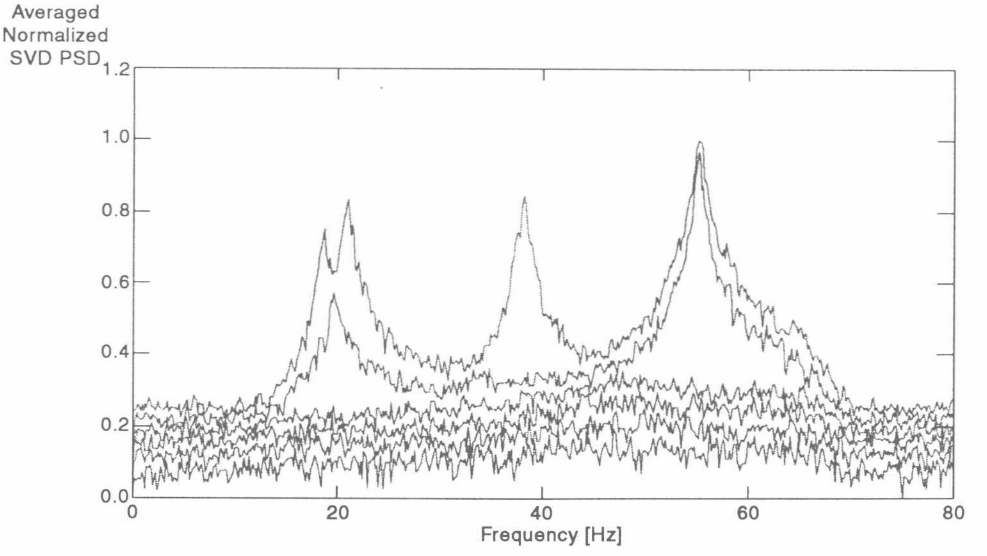

Figure 2. Singular values of the PSD matrix of the response.

discrete frequencies $\omega = \omega_i$ is then decomposed by taking the SVD of the matrix

$$\hat{G}_{yy}(j\omega_i) = U_i S_+ i U_i^H \tag{10}$$

where the matrix $U_i = [u_{i1}, u_{i2}, \ldots, u_{im}]$ is a unitary matrix holding the singular vectors u_{ij}, and S_i is a diagonal matrix holding the scalar singular values s_{ij}. Near a peak corresponding to the kth mode in the spectrum this mode, or maybe a possible close mode, will be dominating. If only the kth mode is dominating there will only be one term in equation (9). Thus, in this case, the first singular vector u_{i1} is an estimate of the mode shape

$$\hat{\phi} = u_{i1} \tag{11}$$

and the corresponding singular value is the auto-PSD function of the corresponding SDOF system, refer to equation (9). This PSD function is identified around the peak by comparing the mode shape estimate $\hat{\phi}$ with the singular vectors for the frequency lines around the peak. As long as a singular vector is found that has a high modal assurance criterion (MAC) value with ϕ, the corresponding singular value belongs to the SDOF density function.

From the piece of the SDOF density function obtained around the peak of the PSD, the natural frequency and the damping can be obtained. In this paper the piece of the SDOF PSD was taken back to the time domain by an inverse fast Fourier transform (IFFT), and the frequency and the damping was simply estimated from the crossing times and the logarithmic decrement of the corresponding SDOF autocorrelation function.

In the case where two modes are dominating, the first singular vector will always be a good estimate of the mode shape of the strongest mode. However, in the case where the two modes are orthogonal, the first two singular vectors are unbiased estimates of the corresponding mode shape vectors. In the case where the two modes are not orthogonal, the bias on the mode shape estimate of the dominant mode will typically be small, but the bias on the mode shape estimate of the weak mode will be strong. Thus, one has to estimate the mode shapes for the two close modes at two different frequency lines, one line where the first mode is dominant and another frequency line where the second mode is dominant.

4. Example, simulation of a two-storey building

In this example the response of a two-storey building is simulated using a lumped parameter system with six degrees of freedom. The measurements are assumed to be taken so that the rigid body motions of the floor slabs can be estimated. The geometry and the measurement points are shown in figure 1.

This structure has two sets of close modes. The first two modes are bending modes; the model was calibrated in such a way that these two bending modes were close, but not very close. The third mode is a torsion mode and the fourth and fifth modes are again close bending modes. The model was calibrated in such a way that the fourth and fifth modes were very close, nearly repeated poles.

The response was simulated using a vector ARMA model to ensure that the simulated responses were covariance equivalent [10]. The model was loaded by white noise, and the response was analysed using the identification technique introduced above. The simulated time series had a length of 10 000 data points and three cases were considered: no noise, 10% noise added and 20% noise added.

The singular values of the spectral density function matrix are shown in figure 2. As it appears, the close modes are clearly indicated in this plot. Using the FDD identification procedure described above, the natural frequencies and damping ratios were identified with high accuracy, see table 1 for the natural frequencies and table 2 for the damping values. As it appears, the technique is not sensitive to the noise. Also, the mode shape estimates were very close to the exact results. Note especially the mode shapes for the two nearly repeated modes (the fourth and the fifth) in figure 5.

5. Indication of harmonics

As explained above, the FDD technique presented in this paper decomposes the spectral density into a set auto spectral density

Table 1. Estimated natural frequencies (in hertz).

Exact	Without noise	10% noise	20% noise
18.686	18.676	18.661	18.665
21.054	20.930	20.927	20.938
38.166	38.188	38.188	38.206
55.055	55.036	55.011	54.999
55.121	55.129	55.133	55.125

Table 2. Estimated damping ratios (in per cent).

Exact	Without noise	10% noise	20% noise
2.13	2.22	2.19	2.33
1.89	1.97	1.98	1.97
1.04	1.12	1.11	1.13
0.72	0.61	0.61	0.55
0.72	0.76	0.76	0.77

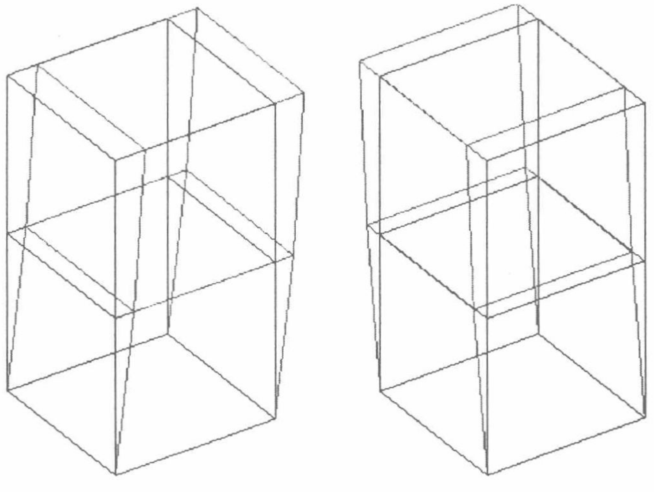

Figure 3. Estimated mode shapes for the first and the second modes (building bending).

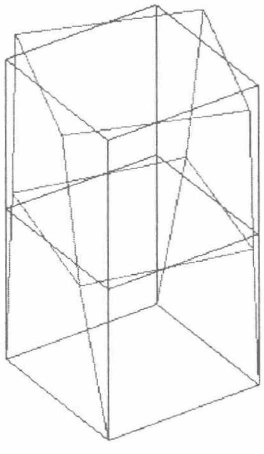

Figure 4. Estimated mode shape for the third mode (building torsion).

functions, each one corresponding to one of the SDOF system representing the corresponding mode. If a harmonic is present, this corresponds a local amplification of the auto spectral density function of all the SDOF systems, i.e. all, or nearly all, of the singular values in the spectral plot will show a peak

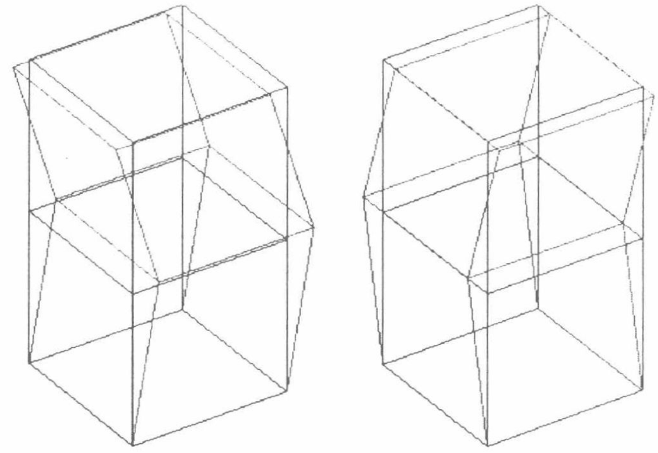

Figure 5. Estimated mode shapes for the fourth and the fifth modes (building bending).

at the frequency where the harmonic is present. This result also holds in the case of a quasi-stationary harmonic, i.e. in the case of a harmonic with a slowly varying frequency.

Thus, if one observes that not only the first singular value has a peak at a certain frequency, but most of the other singular also have a peak at that same frequency, then this is strong indication that the peak does not represent a structural response but a harmonic. An example of the application of this feature of the FDD technique is given in Brincker *et al* [15].

If a structural mode is close to the harmonic, the harmonic does not destroy the mode shape estimate. However, one should be careful not to use the amplified values of the SDOF bell (amplified by the harmonic) when using the inverse Fourier transform to estimate frequency and damping in the time domain. This will heavily bias the frequency and damping estimate.

6. Conclusions

In this paper a new frequency domain identification technique, FDD, has been introduced. The technique is based on decomposing the PSD function matrix using SVD. It has been shown that this decomposes the spectral response into a set of SDOF systems, each corresponding to one individual mode.

The technique has been illustrated on a simulation example with noise and close modes. The results clearly indicate that the present technique is able to estimate close modes with high accuracy and that the technique is not sensitive to noise. In the case of close modes that are not orthogonal, the mode shape of the dominant mode is still well estimated. However, if the other mode is not dominating any frequency, other ways of estimating the mode shape for such a mode must be introduced.

The technique has been applied successfully to several civil engineering cases [11, 12] and to several cases of identification in mechanical engineering where the structure was loaded by rotating machinery [13–15].

The technique clearly indicates the presence of harmonics in the response signal, i.e. without further indication the user directly separates harmonic peaks from structural response peaks.

References

[1] Ventura C E and Horyna T 1997 Structural assessment by modal analysis in Western Canada *Proc. 15th Int. Modal Analysis Conf. (Orlando, FL, 1997)*

[2] Andersen P, Brincker R, Peeters B, De Roeck G, Hermans L and Krämer C 1999 Comparison of system identification methods using ambient bridge test data *Proc. 17th Int. Modal Analysis Conf. (Kissimee, FL, 1999)*

[3] Bendat J S and Piersol A G 1993 *Engineering Applications of Correlation and Spectral Analysis* (New York: Wiley)

[4] Felber A J 1993 Development of a hybrid bridge evaluation system *PhD Thesis* Department of Civil Engineering, University of British Columbia, Vancouver, Canada

[5] Vold H, Kundrat J, Rocklin G T and Russel R 1982 A multi-input modal estimation algorithm for mini-computer *(SAE Technical Paper Series, No 820194)*

[6] Ibrahim S R and Milkulcik E C 1976 The experimental determination of vibration test parameters from time responses *Shock Vib. Bull.* **46** 187–96

[7] Juang J-N and Pappa R S 1985 An eigensystem realization algorithm for modal parameter identification and modal reduction *J. Guidance, Control Dynam.* **8** 620–7

[8] Van Overschee P and De Moor B 1996 *Subspace Identification for Linear Systems* (Dordrecht: Kluwer)

[9] Bendat J S and Piersol A G 1986 *Random Data, Analysis and Measurement Procedures* (New York: Wiley)

[10] Andersen P, Brincker R and Kirkegaard P H 1996 Theory of covariance equivalent ARMAV models of civil engineering structures *Proc. 14th Int. Modal Analysis Conf. (Dearborn, MI)*

[11] Brincker R and Andersen P 2000 Ambient response analysis of the Heritage Court Tower building structure *Proc 18th Int. Modal Analysis Conf. (San Antonio, TX, 7–10 February, 2000)*

[12] Brincker R, Frandsen J and Andersen P 2000 Ambient response analysis of the Great Belt Bridge *Proc 18th Int. Modal Analysis Conf. (San Antonio, TX, 7–10 February, 2000)*

[13] Brincker R, Andersen P and Møller N 2000 Output-only modal testing of a car body subject to engine excitation *Proc. 18th Int. Modal Analysis Conf. (San Antonio, TX, 7–10 February, 2000)*

[14] Møller N, Brincker R and Andersen P 2000 Modal extraction on a diesel engine in operation *Proc 18th Int. Modal Analysis Conf., (San Antonio, TX, 7–10 February, 2000)*

[15] Brincker R, Andersen P and Møller N 2000 An indicator for separation of structural and harmonic modes in output-only modal testing *Proc. Eur. COST F3 Conf. on System Identification and Structural Health Monitoring (Universidad Politecnica de Madrid, Spain, 6–9 June, 2000)*

Modal Indicators for Operational Modal Identification

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Abstract

Modal validation is of paramount importance for all two-stage time domain modal identification algorithms. However, due to a higher noise/signal ratio in operational/ambient modal analysis, being able to determine the right model order and to distinguish between structural modes and computational modes become more significant than in traditional modal analysis. The two major modal indicators, i.e. Modal Confidence Factor (MCF) and Modal Amplitude Coherence (MAmC) are extended to two-stage time domain modal identification algorithms, together with a newly developed indicator, named as Modal Participation Indicator (MPI). The application of the three indicators is illustrated on different cases of operational/ambient modal identification. Three major time domain modal identification algorithms are used, the Polyreference Complex Exponential (PRCE), Extended Ibrahim Time Domain (EITD), Eigensystem Realization Algorithm (ERA). The three identification algorithms are implemented from a unified point-of-view with the modal indicators. Numerical simulations are conducted on a two-story building structure and on an aircraft model and it is investigated how the modal indicators work to distinguish the physical modes from the computational modes.

Introduction

Operational modal identification has attracted great attention in civil, aerospace and mechanical engineering in recent years. Compared to traditional modal analysis, which is normally conducted in the lab environment making use of both input-output data, operational/ambient modal analysis has many advantages:

- No artificial excitation needed and no boundary condition simulation required;
- Dynamic characteristics of the whole system, instead of component, can be obtained;
- For all or part of measurement coordinates can be used as references, the operational modal identification is always Multi-Input (Reference)-Multi-Output MIMO algorithm. The closed-spaced or even repeated modes can easily be handled, and, therefore, suitable for real world complex structures;
- The model identified under real loading will be linearized, due to broad band ambient/random excitation, for much more representative working points;
- Operational modal identification can not only be utilized for structural dynamics analysis and design, but also In-situ vibration based structural health monitoring and damage identification.

Many time domain MIMO modal identification algorithms such as Polyreference Complex Exponential (PRCE), Extended Ibrahim Time Domain (EITD), Eigensystem Realization Algorithm (ERA) and its extension [1]-[6], etc. have been developed in 1980's. Impulse Response Functions (IRF) is measured at first, normally via inverse FFT from FRF, and then modal parameters are identified via above-mentioned algorithms using IRF data. The 2-stage modal identification techniques have been successfully used for traditional modal analysis. However, they can also be adopted for operational modal analysis. In the 1990's a Natural Excitation Technique (NExT) was proposed [7]. NExT is based on the principle that Correlation Function (CF) measured under natural excitation (or operational/ambient condition) can be expressed as a sum of decaying sinusoids. Each decaying sinusoid has a damped natural frequency, damping ratio and mode shape coefficient that is identical to the one of the corresponding structural mode. According to this principle, all the 2-stage time domain MIMO identification techniques can be adopted for operational/ambient modal identification by using CFs instead of IRFs.

However, all the time domain (TD) modal identification algorithms have a serious problem on model order determination. When extracting physical or structural modes, the TD modal identification algorithm always generates spurious or computational modes to account for unwanted effects, such as noise, leakage, residuals and non-linearity's, etc. The computational modes fulfill an important role in that they permit more accurate modal estimation by supplying statistical DOF to absorb these effects. In the traditional modal identification IRF can be obtained via inverse FFT of Frequency Response Function (FRF), and may have less computational modes. For operational modal identification, which makes use of correlation function calculated from random response data, the model order determination and structural modes distinguishing become much more significant. Therefore, it is extremely important to determine the correct number of model order or total number of modes at first, and then to distinguish structural modes from computational ones. In order to accomplish this important task, many modal validation approaches have been developed.

Modal validation can be performed via three kind approaches: visual inspection, modal indicator and diagram. Visual inspection of mode shapes and comparing measured data with those synthesized from the estimated modal parameters are typical examples of these qualitative approaches. The second kind of approaches make use of quantitative modal indicators, such as Modal Assurance Criterion (MAC), Modal Confidence Factor (MCF), Modal Amplitude Coherence (MAmC), etc. Graphical validation involves tracking the model error, or rank of the data matrix, or estimating frequency, damping as a function of model order. The resulting Error Chart, Rank Chart or Stability Diagram is then utilized for modal validation.

In this paper two modal indicators, MCF and MAmC, are extended and a new one named as Modal Participation Indicator (MPI) is developed for major 2-stage time domain modal identification algorithms. Numerical simulations via a two-story building and an aircraft model are conducted to show the performance of the three modal indicators for operational modal identification algorithms—PRCE, EITD and ERA.

Modal Indicators

1. Modal Assurance Criterion (MAC).

Modal Scale Factor (MSF) and MAC are used widely to compare two modal vectors. The MSF gives a least squares estimate of the ratio between two vectors

$$MSF(\phi_r,\phi_s)=\frac{\phi_r^H\phi_s}{\phi_r^H\phi_r} \qquad (1)$$

MAC is defined as [8]

$$MAC(\phi_r,\phi_s)=\frac{\left|\phi_r^H\phi_s\right|^2}{\left[\phi_r^H\phi_r\right]\left[\phi_s^H\phi_s\right]} \qquad (2)$$

Which is actually the square of the correlation coefficient of the two modal vectors. If MAC is unity the two modal vectors are identical within modal scale factor. Therefore, the MAC can be utilized as a modal indicator for different modal estimates.

2. Modal Confidence Factor (MCF)

Ibrahim introduced the concept of MCF by generating pseudo-measurements in the ITD modal identification algorithm [9]. These pseudo-measurements are actually delayed physical time signals. MCF exploits redundant phase relationships that are satisfied by physical modes, but which are meaningless for computational modes. The MCF has been extended for the PRCE [10] For r-th mode MCF can be calculated by the following formula

$$MCF_r=\frac{\phi_r^H\bar{\phi}_r}{\phi_r^H\phi_r}e^{-\lambda_r p\Delta t} \qquad (3)$$

Where λ is the eigenvalue, Δt is the sampling time interval and p is a positive integer. For a physical mode, the MCF would be unity, whereas computational modes would have a MCF of arbitrary phase and amplitude. A MCF close to one is thus a necessary, but not sufficient reason for an eigenvector to be associated with a physical mode.

It is obvious that MCF can also be used for other 2-stage TD modal identification algorithms, such as EITD, ERA, etc. MCF is a complex number. For simplicity only the norm can be used for modal indicator. The main drawback of the method is that the amount of the data is doubled.

3. Modal Amplit6ude Coherence (MAmC)

MAmC was proposed by the authors of ERA [11] for distinguishing structural modes from noise modes with ERA. We have extended the MAmC to all 2-stage TD modal identification algorithms (PRCE, EITD, ERA, etc). The basic formulation for MAmC is derived as follows.

For a linear system, the map from input to output can be described by Markov parameter (Impulse Response Function in traditional modal analysis or Covariance Functions in Ambient modal analysis) sequence

$$Y=\left[Y_0\ Y_1\ Y_2\cdots Y_{Nt-1}\right] \qquad (4)$$

Where Nt is the number of the data points. In the modal coordinate the Markov parameter can be expressed as

$$Y_k=\sum_{r=1}^{n}\phi_r\lambda_r^{k-1}\gamma_r^H \qquad (5)$$

Where ϕ_r, λ_r and γ_r are r-th modal vector, eigenvalue and modal participation factor, respectively. Define the sequence

$$\hat{q}_r=\left[\hat{\gamma}_r^H\ \hat{\lambda}_r\hat{\gamma}_r^H\ \hat{\lambda}_r^2\hat{\gamma}_r^H\cdots\hat{\lambda}_r^{Nt-1}\hat{\gamma}_r^H\right] \qquad (6)$$

Which represents the time series reconstructed from the identified eigenvalue and modal participation factor. The Markov parameter becomes

$$\hat{Y}_k = \sum_{r=1}^{n} \hat{\phi}_r \hat{q}_r \qquad (7)$$

It can be seen that the sequence q_r is associated with mode shape ϕ_r, and is called the identified Modal Amplitude time history for the r-th mode. The modal amplitude can also be calculated directly from measured Markov parameters via SVD of Hankel matrix, and denoted as $\bar{q}_r$. With noise-polluted data and nonzero singular values truncated, the identified modal amplitude is an approximation of the one calculated directly from measured Markov parameters. The MAmC can then be defined as correlation coefficient or coherence function of the two modal amplitude vectors as

$$MAmC = \frac{\left|\hat{q}_r^H \bar{q}_r\right|}{\left(\left|\bar{q}_r^H \bar{q}_r\right|\left|\hat{q}_r^H \hat{q}_r\right|\right)^{1/2}} \qquad (8)$$

4. Modal Participation Indicator (MPI)

In the ambient/operational modal analysis, Correlation or Covariance Function can be measured as Markov parameter, and expressed via eigenvalue, modal vector (mode shape) and modal participation factor:

$$Y_k = \sum_{r=1}^{n} \phi_r \lambda_r^{k-1} \gamma_r^T \qquad (9)$$

Choosing all the measurement coordinates as references, the dimension of modal partition vector is then equal to corresponding mode shape. We can therefore define Modal Participation Scale (MPS) α_r as

$$\gamma_r = \alpha_r \phi \qquad (10)$$

The contribution of the r-th mode to the covariance matrix can then be expressed as

$$Y_k^r = \alpha_r \phi_r \phi_r^H \lambda_r^{k-1} \qquad (11)$$

MPI represents a kind of "kinetic energy" in time domain, and can be adopted as a modal indicator to distinguish structural and computational modes. MPI can be calculated via least square solution of the two vectors as the following formula

$$MPI_r = \alpha_r = \frac{\left|\phi_r^H \gamma_r\right|}{\phi_r^H \phi_r} \qquad (12)$$

When implementing, r-th modal participation indicator MPI_r is normalized as the percentage of the "total energy".

Numerical Simulations for Operational Modal Identification

The MAmC, MPF and MCF are applied in major 2-stage time domain modal identification algorithms and applied to operational modal identification as modal indicators.

Three major time domain modal identification algorithms, PRCE, EITD and ERA, are implemented via unified point-of-view as follows:

- Establish Hankel matrices H_0 and H_1 from measured covariance functions;
- Calculate system matrix via least squares solution from Hankel matrices for PRCE or EITD;
- Calculate system input and measurement matrices via singular value decomposition for ERA;
- Eigenvalue solution of system matrix to obtain eigenvalues and mode shapes for EITD or eigenvalues and modal participation vectors for PRCE;
- Least squares solution to obtain modal participation vectors for EITD, and mode shapes for PRCE;
- For ERA, eigenvalue solution of the system matrix to obtain eigenvalues and mode shapes together with measurement matrix, and modal participation vectors from input matrix;

Two examples with closely spaced modes are used to show the performance of the different modal indicators.

1. Two-story Building

The first numerical example is a two-story building, which is simulated by a lumped parameter system with 6 degrees of freedom. The measurements are assumed to be taken so that the rigid body motions of the floor slaps can be estimated. The geometry and the measurement points are shown in Figure 1. This structure has two sets of close modes. The first two modes are first bending modes, and these two bending modes are close, but not very close. The third mode is a torsion mode. The fourth and fifth modes are very closed second bending modes. Figure 2 depicts first 5 modes. The response was simulated using a vector ARMA model to ensure that the simulated responses were covariance equivalent [13]. The model was loaded by white noise, and the response was analyzed using the 2-stage time domain identification techniques introduced above. The simulated time series had a length of 10,000 data points with 20 % noise added.

Computer simulations of operational modal identification were conducted using PRCE, EITD and ERA with MAmC, MPI and MCF as modal indicators. Table 1 to 3 present MAmC and MPI results via PRCE, EITD and ERA identification, respectively. Tables 4 and 5 show the results of MCF via EITD and PRCE separately for double data are needed in order to compute MCF. The main parameters to be selected in the numerical simulation are the number of total modes (n) and the number of data points. In the Tables "*" denotes the target modal frequencies. The range of damping ratio, 0-5 %, is used as the first "filter" to eliminate computational modes. It can be seen that all three modal indicators work pretty well in distinguishing structural modes from computation modes. Compared to MAmC and MCF, the newly proposed MPI has better performance.

2. GARTEUR Aircraft Model

An aircraft model called GARTEUR developed by the

Group of Aeronautical Research and Technology in EURope is adopted as the second example [14]. The model represents the dynamic characteristics of real world aircraft, and is widely used in Europe. The main requirement for the GARTEUR model is to simulate dynamic characteristics of real world aircraft. GARTEUR model has the following features: (1) A group of 3 very closely spaced modes, (2) Frequency range from 5 to 60 Hertz, (3) Special damping Treatment via adding visco-elastic materials on the wing surface; (4) A joint at the wing/fuselage connection for transportation with model dimension of 2 by 2 meters

The Finite Element Model (FEM) of Garteur consists of 51 three-dimensional beam elements and 68 nodal points with altogether 408 DOF model. Figure 3 presents the first 6 modes of GARTEUR model. The first six natural frequencies are: 6.09Hz, 15.80Hz, 33.01Hz, 33.66Hz, 35.14Hz and 49.79 Hz.

Markov parameters are synthesized from the modal parameters calculated from FEM with 1.00% damping ration added. Altogether 24 DOFs are selected as measurement locations. To simulate the noise-pollution test data, 10% Gaussian distributed noise is added to synthesized Markov parameters. Sampling frequency is 150Hz with 1024 sampling points

As for the 2-story building case, all three modal identification algorithms are used for GARTEUR example. However, the simulation data are synthesized using 2-input 24-output measurement, therefore, only MAmC and MCF are adopted for modal indication. Tables 6 to 10 show the performance of MAmC and MCF for operational modal identification algorithms PRCE, EITD and ERA. It is observed that the two modal indicators exhibit favorable performance

Concluding Remarks

1. Two modal indicators, Modal Confidence Factor (MCF) and Modal Amplitude Coherence (MAmC) are extended to major 2-stage time domain operational modal identification algorithms;
2. A new modal indicator named as Modal Participation Indicator (MPI) is developed and implemented;
3. Three major operational/ambient modal identification algorithms, Polyreference Complex Exponential (PRCE), Extended Ibrahim Time Domain (EITD) and Eigensystem Realization Algorithm (ERA), are implemented from unified point-of-view together with three modal indicators;
4. Numerical simulations are conducted using two examples: 2-story building and an aircraft model. The results show that all three modal indicators work pretty well in distinguishing structural modes from computational ones;
5. MCF needs double data, and hence more computing intensive and time consuming; MAmC often results with the number closed to unity and some times is hardly to separate noise modes from the structural ones;
6. Newly proposed Modal Participation Indicator (MPI) can clearly indicate the structural modes in most cases, and performs better than the other two indicators;
7. The identification results are normally depending on the parameter selection for most of 2-stage time domain modal identification. To finally determine the true structural modes Stability Diagram is suggested together with modal indicators.

Acknowledgement

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References

[1]. Vold, H. et. al., A Multi-Input Modal Estimation Algorithm for Mini-Computers, SAE Technical Paper No. 820194, 1982

[2]. Leuridan, J. and Vold, H., A Time Domain Linear Modal Estimation Technique for Global Modal Parameter Identification, Proc. of the 2-nd IMAC, Orlando, FL, USA, Feb. 6-9,1984

[3]. Juang, J.-N. and Pappa, R., An Eigensystem Realization Algorithm (ERA) for Modal Parameter Identification, NASA/JPL Workshop on Identification and Control of Flexible Space Structures, Pasadena, CA, USA, 1984

[4]. Ibrahim, S., Modal Identification Techniques: Assessment and Comparison, Proc. of the 3-rd IMAC, Orlando, FL, USA, Jan. 28-31, 1985,

[5]. Zhang, L.-M., Yao, Y.-X. And Lu, M.-F., An Improved Time Domain Polyreference Method for Modal Identification, Mechanical System and Signal Processing, 1(4), 1987

[6]. Juang, J. –N., Cooper, J. E. And Wright, J. R., An Eigensystem Realization Algorithm Using Data Correlation (ERA/DC) for Modal Parameter Identification, Control-Theory and Advanced Technology, 4(1), 1988.

[7]. James, G. H., Carne, T.G., Lauffer, J.P. Nard, A. R., Modal Testing Using Natural Excitation, Proc. of the 10-th IMAC, San Diego, CA, USA, Feb. 3-7. 1992

[8]. Allemang, R.J., Brown, D.L., "A Correlation Coefficient for Modal Vector Analysis", Proceedings, International Modal Analysis Conference.

[9]. Ibrahim, S., Modal Confidence Factor in Vibration Testing, AIAA Journal of Spacecraft and Rockets, 15(5), 1978

[10]. Vold, H., Crowley, J., A Modal Confidence Factor for the Polyreference Method, Proc. the 3rd IMAC, Orlando, FL, USA, Jan. 28-31, 1985

[11]. Juang, J.-N. and Pappa, R.S., "An Eigensystem Realization Algorithm for Modal Parameter Identification and Model Reduction", *AIAA, Journal of Guidance, Control, and Dynamics,* 8 (4), 1985

[12]. Brincker, R., Zhang, L.-M. and Anderson, P., Modal

Identification from Ambient Response using Frequency Domain Decomposition, Proc. of the 18-th IMAC, San Antonio, TX, USA, Feb. 7-10, 2000
[13] Andersen, P., R. Brincker, and P.H. Kirkegaard: "Theory of Covariance Equivalent ARMAV Models of Civil Engineering Structures", Proc of the 14th Dearborn, MI, USA, Feb. 1996.

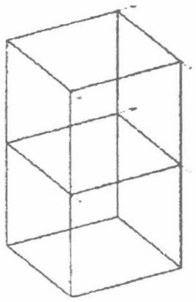
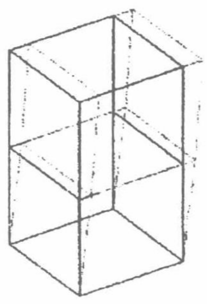
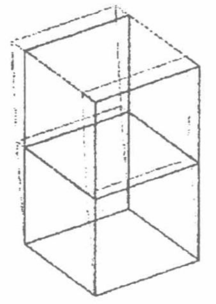


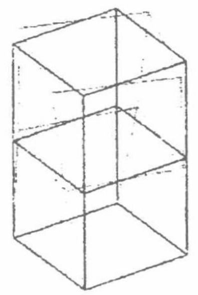
Fig. 1 2-story building



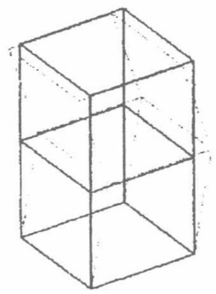
(a) f_1=18.69, ξ_1=0.0213



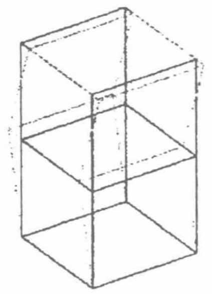
(b) f_2=21.05, ξ_2=0.0189



(c) f_3=38.17, ξ_3=0.0104



(d) f_4=55.06, ξ_4=0.0072



(e) f_5=55.12, ξ_5=0.0072

Fig. 2 First Five Modes of 2-story building

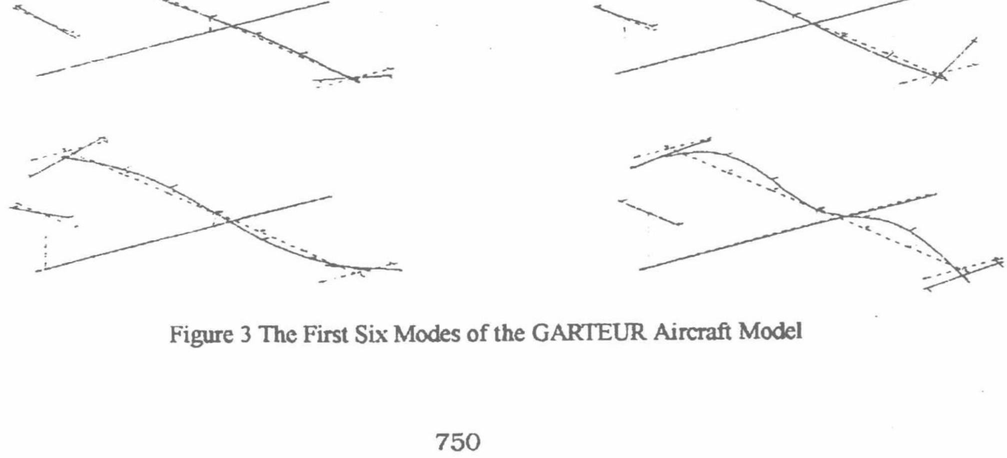


Figure 3 The First Six Modes of the GARTEUR Aircraft Model

Table 1 Results of EITD for Two-Story Building (n=12)

Mode	Freq (Hz)	Damp.(%)	MamC	MPL(%)
1	18.69*	2.46	1.00	3.02
2	20.97*	2.14	1.00	7.11
3	38.08	1.86	1.00	0.01
4	38.14*	0.89	1.00	7.64
5	55.03*	0.65	1.00	26.05
6	55.08*	0.64	1.00	55.67
7	55.13	2.30	0.99	0.11
8	66.64	4.33	0.90	0.38

Table 2. Results of PRCE for Two-Story Building (n=15)

Mode	Freq.(Hz)	Damp.(%)	MamC	MPL(%)
1	18.76*	2.61	1.00	1.89
2	20.88*	1.87	1.00	4.32
3	38.14	0.79	0.98	0.60
4	38.15*	0.99	1.00	4.84
5	54.95	2.26	0.86	0.20
6	55.02*	0.61	0.81	42.68
7	55.09*	0.67	1.00	44.92
8	62.01	2.22	0.67	0.40
9	69.39	0.23	0.24	0.15

Table 3. Results of ERA for Two-Story Building (n=8)

Mode	Freq.(Hz)	Damp.(%)	MAmC	MPL(%)
1	18.68*	2.19	1.00	18.62
2	20.93*	1.88	1.00	18.56
3	38.16*	1.03	1.00	18.36
4	54.62	1.40	0.96	6.22
5	55.01*	0.55	1.00	15.59
6	55.17*	0.54	1.00	15.94
7	59.19	3.71	0.54	6.70

Table 4. Results of EITD for Two-Story Building (n=15)

Mode	Freq.(Hz)	Damp.(%)	MCF
1	18.70*	2.15	0.98
2	20.91*	1.80	0.94
3	37.31	1.75	0.21
4	38.12*	1.32	0.53
5	38.19*	0.86	0.93
6	55.00*	0.74	0.89
7	55.03	1.11	0.71
8	55.16*	0.54	0.93
9	55.49	3.80	0.06
10	61.10	2.37	0.20

Table 5. Results of PRCE for Two-Story Building (n=15)

Mode	Freq.(Hz)	Damp.(%)	MCF
1	18.65*	2.20	0.92
2	21.01*	1.65	0.93
3	22.78	3.29	0.71
4	35.05	1.56	0.43
5	38.15*	1.00	0.93
6	38.38	1.22	0.92
7	53.57	0.46	0.92
8	54.98*	0.61	0.94
9	55.13*	0.57	0.94
10	55.23	1.03	0.89
11	55.37	3.06	0.72
12	58.29	0.34	0.80
13	61.22	0.02	0.33
14	62.41	4.21	0.36
15	66.52	4.32	0.72

Table 6. Results of EITD for GARTEUR (n=48)

Mode	Freq.(Hz)	Damp.(%)	MAmC
1	6.11*	1.10	1.00
2	13.52	0.27	0.95
3	15.80*	0.99	1.00
4	17.64	0.77	0.75
5	19.81	4.87	0.98
6	20.58	1.05	0.89
7	26.66	0.82	0.90
8	28.16	0.31	0.92
9	29.40	0.65	0.90
10	30.16	2.45	0.97
11	33.00*	1.20	1.00
12	33.51*	1.31	1.00
13	33.84	3.78	0.99
14	35.09*	0.91	1.00
15	37.01	4.06	0.99
16	37.01	4.06	0.99

PROJECTION-BASED SENSOR PLACEMENT FOR IN-OPERATION MODAL IDENTIFICATION

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ABSTRACT

The performance of a modal identification algorithm depends profoundly on the measured data. To obtain high quality measurements, especially in-operation measurements, careful sensor placement is of great importance. A newly developed sensor placement method for in-operation modal identification based on time domain data is presented in this paper. The new method utilizes the projection theory to compare the contribution of the candidate DOFs to the rank of Hankel matrix of the measured data. Firstly, the Hankel matrix with all candidate DOFs, which may be based on a priori Finite-Element Model of the structure, is constructed. Then an orthogonal projector matrix of the Hankel matrix can be extracted. The trace of the projector matrix provides a convenient representation of its rank. Therefore, the candidate DOFs can be sorted by the diagonal elements of the projector matrix, and DOFs corresponding to the smallest ones are eliminated. To avoid an ill-conditioned Hankel matrix, a modified condition number is introduced as a constraint. If the removal of some DOF augments this condition number, it should be remained. The new Hankel matrix with the remaining DOFs is then re-constructed. This process is repeatedly executed until the desired number of sensors remain. Finally, the numerical simulation is conducted to demonstrate the effectiveness of the new sensor placement method.

NOMENCLATURE

A	system matrix
B	input matrix
C	output matrix
x	state vector
y	output vector
u	input vector
$Rank(.)$	matrix rank
$Trace(.)$	matrix trace
A^+	pseudo-inverse of matrix A
A^T	transpose of matrix A
Y	Markov-parameter
k	k^{th} sampling interval
H	Hanklel matrix
S	singular-value
U, V	see Eq. 8
α, β	Hankel matrix row/column shift
DOF	Number Of Freedom
s	number of candidate measurement positions
m	number of inputs
N	number of modes to be identified
n	number of the desired sensors
R_n	n dimensional space
S_1, S_2	sub-spaces of space R_n
A_{S_1,S_2}	projection transformation of S_1 along S_2
$\oplus$	orthogonal direct sum
P	projector matrix
Ph	diagonal elements of projector matrix
E_i	contribution of the i^{th} DOF to the rank of the Hankel matrix
RS	ratio $S(1)/S(N)$

1 INTRODUCTION

Modal identification using dynamic tests has been extensively incorporated into the design development. Since the performance of modal parameter identification algorithms depends on the measured data, a systematic placement of sensor positions to obtain ideal measurements is necessary. And the limitations in the number of sensors especially for in-operation modal identification further necessitate the optimal placement of sensors.

A large number of sensor placement methods appeared in the last two decades[1],[2]. The simplest sensor placement method is visual inspection. This method just visually inspects certain dynamic parameters of the structures, and selects the DOFs with high/low values as the measurement locations. The object to be inspected in this method may be the mode shapes of interest. Modal Kinetic Energy(MKE), Driving Point Residues(DPR) and etc. Although efficient, this kind of methods is only practical for simple structures.

Recently, some new iterative sensor placement methods have been developed. The most practical and popular method should be the Effective Independence(EI)[3] method proposed by Kammer. The idea of this EI method is that those DOFs resulting in the most independent target modal matrix are the best candidates to locate the sensors. It eliminates DOFs that do not contribute to the independence of the target mode shapes. Also in this paper, the projection theory was introduced to rank the contribution of each DOF to the rank of the target modal matrix. Obviously, EI is a Frequency Domain(FD) method.

For Time Domain modal parameter identification algorithms, such as Ibrahim Time Domain method(ITD), Extended Ibrahim Time Domain method(EITD), Ploy-reference Complex Exponential method(PRCE), Eigensystem Realization Algorithm(ERA) and so on, Hankel matrix of the Markov-parameter plays the most important roles in the identification process. A column/row full rank and well-conditioned Hankel matrix is the precondition of a successful identification. Since singular-value truncation technique