

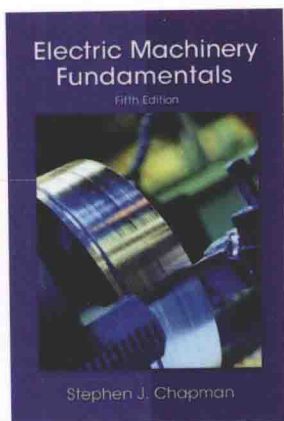
电气工程、自动化专业规划教材

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# 电机学 (第五版) (英文版)

Electric Machinery Fundamentals  
Fifth Edition

【美】 Stephen J. Chapman 著



电子工业出版社  
PUBLISHING HOUSE OF ELECTRONICS INDUSTRY

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Electric Machinery Fundamentals, Fifth Edition

[美] Stephen J. Chapman 著

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Publishing House of Electronics Industry

北京·BEIJING

## 内 容 简 介

本书是关于电机学的经典教材。全书内容涉及电机原理、变压器、交流电机、同步发电机、同步电动机、感应电动机、直流电机、直流电动机和发电机、单相及特种电动机等,附录内容包括三相电路、线圈节距及分布绕组、同步电机的凸极理论、常数及转换系数表。全书叙述清晰,图文并茂,例题、习题丰富,并结合 MATLAB 进行了大量的仿真分析。

本书可作为电气工程、自动化专业学生的教材,也可供从事该领域工作的工程技术人员学习参考。

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## ABOUT THE AUTHOR

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**Stephen J. Chapman** received a B.S. in Electrical Engineering from Louisiana State University (1975) and an M.S.E. in Electrical Engineering from the University of Central Florida (1979), and pursued further graduate studies at Rice University.

From 1975 to 1980, he served as an officer in the U.S. Navy, assigned to teach electrical engineering at the U.S. Naval Nuclear Power School in Orlando, Florida. From 1980 to 1982, he was affiliated with the University of Houston, where he ran the power systems program in the College of Technology.

From 1982 to 1988 and from 1991 to 1995, he served as a member of the technical staff of the Massachusetts Institute of Technology's Lincoln Laboratory, both at the main facility in Lexington, Massachusetts, and at the field site on Kwajalein Atoll in the Republic of the Marshall Islands. While there, he did research in radar signal processing systems. He ultimately became the leader of four large operational range instrumentation radars at the Kwajalein field site (TRADEX, ALTAIR, ALCOR, and MMW).

From 1988 to 1991, Mr. Chapman was a research engineer for Shell Development Company in Houston, Texas, where he did seismic signal processing research. He was also affiliated with the University of Houston, where he continued to teach on a part-time basis.

Mr. Chapman is currently manager of systems modeling and operational analysis for BAE Systems Australia, in Melbourne.

Mr. Chapman is a senior member of the Institute of Electrical and Electronic Engineers (and several of its component societies). He is also a member of Engineers Australia.

In the years since the first edition of *Electric Machinery Fundamentals* was published, there has been rapid advance in the development of larger and more sophisticated solid-state motor drive packages. The first edition of this book stated that dc motors were the method of choice for demanding variable-speed applications. That statement is no longer true today. Now, the system of choice for speed control applications is most often an ac induction motor with a solid-state motor drive. DC motors have been largely relegated to special-purpose applications where a dc power source is readily available, such as in automotive electrical systems.

The third edition of the book was extensively restructured to reflect these changes. The material on ac motors and generators is now covered in Chapters 3 through 6, before the material on dc machines. In addition, the dc machinery coverage was reduced compared to earlier editions. This edition continues with this same basic structure.

In addition, the former Chapter 3 on solid-state electronics has been deleted from the fifth edition. Feedback from users has indicated that that material was too detailed for a quick overview, and not detailed enough for a solid-state electronics course. Since very few instructors were using this material, it has been removed from this edition and added as a supplement on the book's website. Any instructor or student wishing to continue using the material in this chapter can freely download it.

Learning objectives have been added to the beginning of each chapter to enhance student learning.

Chapter 1 provides an introduction to basic machinery concepts, and concludes by applying those concepts to a linear dc machine, which is the simplest possible example of a machine. Chapter 2 covers transformers, which are not rotating machines, but which share many similar analysis techniques.

After Chapter 2, an instructor may choose to teach either dc or ac machinery first. Chapters 3 through 6 cover ac machinery, and Chapters 7 and 8 cover dc machinery. These chapter sequences have been made completely independent of each other, so that an instructor can cover the material in the order which best suits his or her needs. For example, a one-semester course with a primary concentration in ac machinery might consist of parts of Chapters 1, 2, 3, 4, 5, and 6, with any remaining time devoted to dc machinery. A one-semester course with a primary concentration in dc machinery might consist of parts of Chapters 1, 2, 7, and 8, with any remaining time devoted to ac machinery. Chapter 9 is devoted to single-phase and special-purpose motors, such as universal motors, stepper motors, brushless dc motors, and shaded-pole motors.

The homework problems and the ends of chapters have been revised and corrected, and more than 70% of the problems are either new or modified since the last edition.

In recent years, there have been major changes in the methods used to teach machinery to electrical engineering and electrical technology students. Excellent analytical tools such as MATLAB<sup>®</sup>

have become widely available in university engineering curricula. These tools make very complex calculations simple to perform, and they allow students to explore the behavior of problems interactively. This edition of *Electric Machinery Fundamentals* makes selected use of MATLAB to enhance a student's learning experience where appropriate. For example, students use MATLAB in Chapter 6 to calculate the torque–speed characteristics of induction motors, and to explore the properties of double-cage induction motors.

This text does not teach MATLAB; it assumes that the student is familiar with it through previous work. Also, the book does *not* depend on a student having MATLAB. MATLAB provides an enhancement to the learning experience if it is available, but if it is not, the examples involving MATLAB can simply be skipped, and the remainder of the text still makes sense.

This book would never have been possible without the help of dozens of people over the past 25 years. It is gratifying for me to see the book still popular after all that time, and much of that is due to the excellent feedback provided by reviewers. For this edition, I would especially like to thank:

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# CHAPTER 1

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## INTRODUCTION TO MACHINERY PRINCIPLES

### OBJECTIVES

- Learn the basics of rotational mechanics: angular velocity, angular acceleration, torque, and Newton's law for rotation.
- Learn how to produce a magnetic field.
- Understand magnetic circuits.
- Understand the behavior of ferromagnetic materials.
- Understand hysteresis in ferromagnetic materials.
- Understand Faraday's law.
- Understand how to produce an induced force on a wire.
- Understand how to produce an induced voltage across a wire.
- Understand the operation of a simple linear machine.
- Be able to work with real, reactive, and apparent powers.

### 1.1 ELECTRICAL MACHINES, TRANSFORMERS, AND DAILY LIFE

An **electrical machine** is a device that can convert either mechanical energy to electrical energy or electrical energy to mechanical energy. When such a device is used to convert mechanical energy to electrical energy, it is called a *generator*. When it converts electrical energy to mechanical energy, it is called a *motor*. Since any given electrical machine can convert power in either direction, any machine can be used as either a generator or a motor. Almost all practical motors and generators convert energy from one form to another through the action of a magnetic field, and only machines using magnetic

fields to perform such conversions are considered in this book.

The *transformer* is an electrical device that is closely related to electrical machines. It converts ac electrical energy at one voltage level to ac electrical energy at another voltage level. Since transformers operate on the same principles as generators and motors, depending on the action of a magnetic field to accomplish the change in voltage level, they are usually studied together with generators and motors.

These three types of electric devices are ubiquitous in modern daily life. Electric motors in the home run refrigerators, freezers, vacuum cleaners, blenders, air conditioners, fans, and many similar appliances. In the workplace, motors provide the motive power for almost all tools. Of course, generators are necessary to supply the power used by all these motors.

Why are electric motors and generators so common? The answer is very simple: Electric power is a clean and efficient energy source that is easy to transmit over long distances, and easy to control. An electric motor does not require constant ventilation and fuel the way that an internal-combustion engine does, so the motor is very well suited for use in environments where the pollutants associated with combustion are not desirable. Instead, heat or mechanical energy can be converted to electrical form at a distant location, the energy can be transmitted over long distances to the place where it is to be used, and it can be used cleanly in any home, office, or factory. Transformers aid this process by reducing the energy loss between the point of electric power generation and the point of its use.

### 1.2 A NOTE ON UNITS AND NOTATION

The design and study of electric machines and power systems are among the oldest areas of electrical engineering. Study began in the latter part of the nineteenth century. At that time, electrical units were being standardized internationally, and these units came to be universally used by engineers. Volts, amperes, ohms, watts, and similar units, which are part of the metric system of units, have long been used to describe electrical quantities in machines.

In English-speaking countries, though, mechanical quantities had long been measured with the English system of units (inches, feet, pounds, etc.). This practice was followed in the study of machines. Therefore, for many years the electrical and mechanical quantities of machines have been measured with different systems of units.

In 1954, a comprehensive system of units based on the metric system was adopted as an international standard. This system of units became known as the *Système International* (SI) and has been adopted throughout most of the world. The United States is practically the sole holdout—even Britain and Canada have switched over to SI.

The SI units will inevitably become standard in the United States as time goes by, and professional societies such as the Institute of Electrical and Electronics Engineers (IEEE) have standardized on metric units for all work. However, many people have grown up using English units, and this system will remain in daily use for a long time. Engineering students and working engineers in the United States today must be familiar with both sets of units, since they will encounter both throughout their professional lives. Therefore, this book includes problems and examples using both SI and English units. The emphasis in the examples is on SI units, but the older system is not entirely neglected.

## Notation

In this book, vectors, electrical phasors, and other complex values are shown in bold face (e.g.,  $\mathbf{F}$ ), while scalars are shown in italic face (e.g.,  $R$ ). In addition, a special font is used to represent magnetic quantities such as magnetomotive force (e.g.,  $\mathcal{F}$ ).

## 1.3 ROTATIONAL MOTION, NEWTON'S LAW, AND POWER RELATIONSHIPS

Almost all electric machines rotate about an axis, called the *shaft* of the machine. Because of the rotational nature of machinery, it is important to have a basic understanding of rotational motion. This section contains a brief review of the concepts of distance, velocity, acceleration, Newton's law, and power as they apply to rotating machinery. For a more detailed discussion of the concepts of rotational dynamics, see References 2, 4, and 5.

In general, a three-dimensional vector is required to completely describe the rotation of an object in space. However, machines normally turn on a fixed shaft, so their rotation is restricted to one angular dimension. Relative to a given end of the machine's shaft, the direction of rotation can be described as either *clockwise* (CW) or *counterclockwise* (CCW). For the purpose of this volume, a counterclockwise angle of rotation is assumed to be positive, and a clockwise one is assumed to be negative. For rotation about a fixed shaft, all the concepts in this section reduce to scalars.

Each major concept of rotational motion is defined below and is related to the corresponding idea from linear motion.

### Angular Position $\theta$

The angular position  $\theta$  of an object is the angle at which it is oriented, measured from some arbitrary reference point. Angular position is usually measured in radians or degrees. It corresponds to the linear concept of distance along a line.

### Angular Velocity $\omega$

Angular velocity (or speed) is the rate of change in angular position with respect to time. It is assumed positive if the rotation is in a counterclockwise direction. Angular velocity is the rotational analog of the concept of velocity on a line. One-dimensional linear velocity along a line is defined as the rate of change of the displacement along the line ( $r$ ) with respect to time.

$$v = \frac{dr}{dt} \quad (1-1)$$

Similarly, angular velocity  $\omega$  is defined as the rate of change of the angular displacement  $\theta$  with respect to time.

$$\omega = \frac{d\theta}{dt} \quad (1-2)$$

If the units of angular position are radians, then angular velocity is measured in radians per second.

In dealing with ordinary electric machines, engineers often use units other than radians per second to describe shaft speed. Frequently, the speed is given in revolutions per second or revolutions per minute. Because speed is such an important quantity in the study of machines, it is customary to use

## 4 ELECTRIC MACHINERY FUNDAMENTALS

different symbols for speed when it is expressed in different units. By using these different symbols, any possible confusion as to the units intended is minimized. The following symbols are used in this book to describe angular velocity:

$\omega_m$	angular velocity expressed in radians per second
$f_m$	angular velocity expressed in revolutions per second
$n_m$	angular velocity expressed in revolutions per minute

The subscript  $m$  on these symbols indicates a mechanical quantity, as opposed to an electrical quantity. If there is no possibility of confusion between mechanical and electrical quantities, the subscript is often left out.

These measures of shaft speed are related to each other by the following equations:

$$n_m = 60f_m \quad (1-3a)$$

$$f_m = \frac{\omega_m}{2\pi} \quad (1-3b)$$

### Angular Acceleration $\alpha$

Angular acceleration is the rate of change in angular velocity with respect to time. It is assumed positive if the angular velocity is increasing in an algebraic sense. Angular acceleration is the rotational analog of the concept of acceleration on a line. Just as one-dimensional linear acceleration is defined by the equation

$$a = \frac{dv}{dt} \quad (1-4)$$

angular acceleration is defined by

$$\alpha = \frac{d\omega}{dt} \quad (1-5)$$

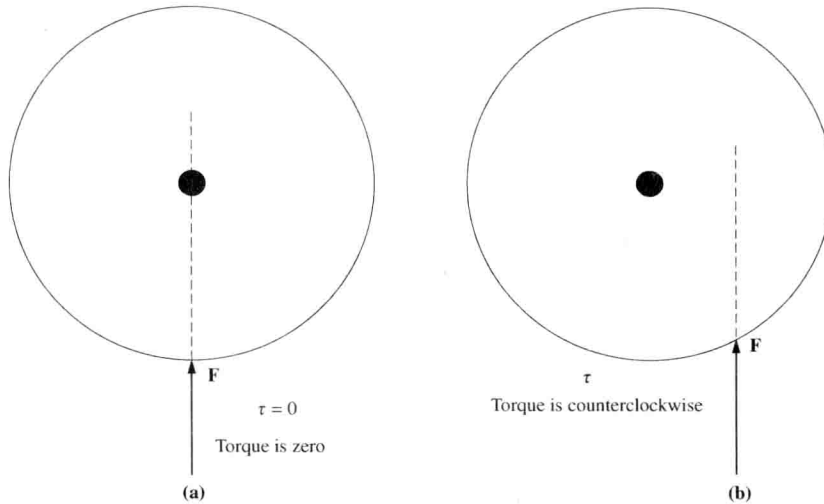
If the units of angular velocity are radians per second, then angular acceleration is measured in radians per second squared.

### Torque $\tau$

In linear motion, a *force* applied to an object causes its velocity to change. In the absence of a net force on the object, its velocity is constant. The greater the force applied to the object, the more rapidly its velocity changes.

There exists a similar concept for rotation. When an object is rotating, its angular velocity is constant unless a *torque* is present on it. The greater the torque on the object, the more rapidly the angular velocity of the object changes.

What is torque? It can loosely be called the “twisting force” on an object. Intuitively, torque is fairly easy to understand. Imagine a cylinder that is free to rotate about its axis. If a force is applied to the cylinder in such a way that its line of action passes through the axis (Figure 1-1a), then the cylinder will not rotate. However, if the same force is placed so that its line of action passes to the right of the axis (Figure 1-1b), then the cylinder will tend to rotate in a counterclockwise direction. The torque or twisting action on the cylinder depends on (1) the magnitude of the applied force and (2) the distance between the axis of rotation and the line of action of the force.



**FIGURE 1-1** (a) A force applied to a cylinder so that it passes through the axis of rotation.  $\tau = 0$ . (b) A force applied to a cylinder so that its line of action misses the axis of rotation. Here  $\tau$  is counterclockwise.

The torque on an object is defined as the product of the force applied to the object and the smallest distance between the line of action of the force and the object's axis of rotation. If  $\mathbf{r}$  is a vector pointing from the axis of rotation to the point of application of the force, and if  $\mathbf{F}$  is the applied force, then the torque can be described as

$$\begin{aligned}\tau &= (\text{force applied})(\text{perpendicular distance}) \\ &= (F)(r \sin \theta) = rF \sin \theta\end{aligned}\quad (1-6)$$

where  $\theta$  is the angle between the vector  $\mathbf{r}$  and the vector  $\mathbf{F}$ . The direction of the torque is clockwise if it would tend to cause a clockwise rotation and counterclockwise if it would tend to cause a counterclockwise rotation (Figure 1-2).

The units of torque are newton-meters in SI units and pound-feet in the English system.

### Newton's Law of Rotation

Newton's law for objects moving along a straight line describes the relationship between the force applied to an object and its resulting acceleration. This relationship is given by the equation

$$F = ma \quad (1-7)$$

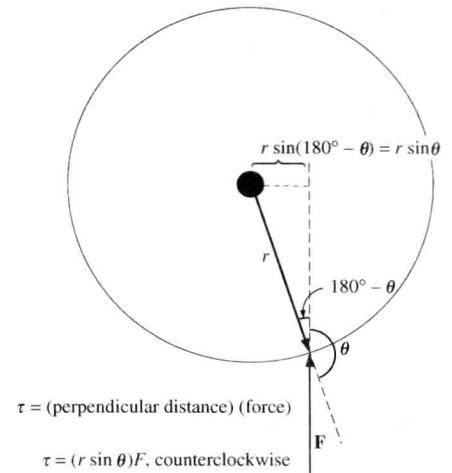
where

$F$  = net force applied to an object

$m$  = mass of the object

$a$  = resulting acceleration

In SI units, force is measured in newtons, mass in kilograms, and acceleration in meters per second



**FIGURE 1-2** Derivation of the equation for the torque on an object.



squared. In the English system, force is measured in pounds, mass in slugs, and acceleration in feet per second squared.

A similar equation describes the relationship between the torque applied to an object and its resulting angular acceleration. This relationship, called *Newton's law of rotation*, is given by the equation

$$\tau = J\alpha \quad (1-8)$$

where  $\tau$  is the net applied torque in newton-meters or pound-feet and  $\alpha$  is the resulting angular acceleration in radians per second squared. The term  $J$  serves the same purpose as an object's mass in linear motion. It is called the *moment of inertia* of the object and is measured in kilogram-meters squared or slug-feet squared. Calculation of the moment of inertia of an object is beyond the scope of this book. For information about it see Ref. 2.

### Work $W$

For linear motion, work is defined as the application of a *force* through a *distance*. In equation form,

$$W = \int F dr \quad (1-9)$$

where it is assumed that the force is collinear with the direction of motion. For the special case of a constant force applied collinearly with the direction of motion, this equation becomes just

$$W = Fr \quad (1-10)$$

The units of work are joules in SI and foot-pounds in the English system.

For rotational motion, work is the application of a *torque* through an *angle*. Here the equation for work is

$$W = \int \tau d\theta \quad (1-11)$$

and if the torque is constant,

$$W = \tau\theta \quad (1-12)$$

### Power $P$

Power is the rate of doing work, or the increase in work per unit time. The equation for power is

$$P = \frac{dW}{dt} \quad (1-13)$$

It is usually measured in joules per second (watts), but also can be measured in foot-pounds per second or in horsepower.

By this definition, and assuming that force is constant and collinear with the direction of motion, power is given by

$$P = \frac{dW}{dt} = \frac{d}{dt}(Fr) = F \left( \frac{dr}{dt} \right) = Fv \quad (1-14)$$

Similarly, assuming constant torque, power in rotational motion is given by

$$P = \frac{dW}{dt} = \frac{d}{dt}(\tau\theta) = \tau \left( \frac{d\theta}{dt} \right) = \tau\omega \quad (1-15)$$