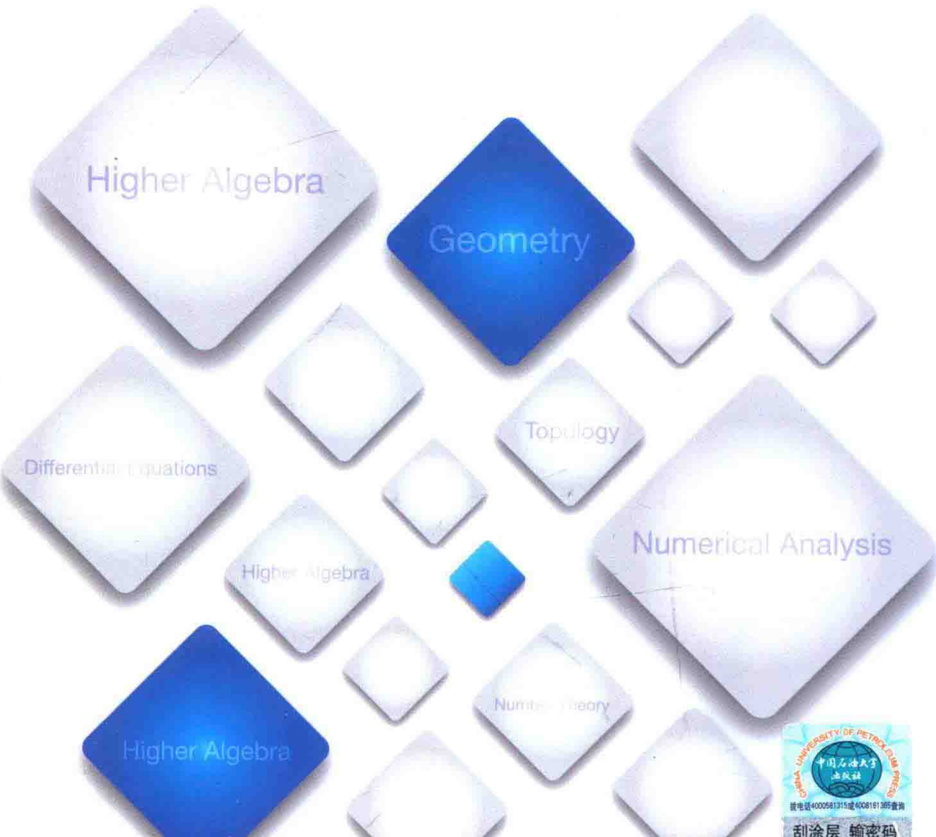
TEACHING MATERIALS
FOR COLLEGE STUDENTS
高等学校教材

数学专业英语

Mathematical English

沈 晨　李维国　编



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内容简介

本书是根据数学专业外语教学大纲，以本科生教学内容为基础编写的。全书共分14个单元。主要包括：解析几何、数学分析、高等代数、集合论、初等数论、抽象代数、微分方程、拓扑学、数值分析、数学史等分支中的基础性知识。

本书可作为高等院校数学与应用数学、信息与计算科学、统计学等专业的本科生和研究生的教学和参考用书，也可供相关科学技术人员参考。

前言 Preface

数学专业英语课程为学生在更广阔更深入的专业领域学习，提供了必要的基础和实践平台，是大学英语教学体系中培养、提高学生英语实用技能的一个关键环节。编者在多年讲授这门课的基础上，编写了这本教材。

本书以数学知识的复习、深化和提高为目标，以学生熟悉的数学分析、高等代数和解析几何等课程中相应内容为框架，循序渐进地引入数学专业词汇和英文版数学教材所特有的语言表达方式。注重选取多种题材的数学原文材料，力求反映当代语言特点，在侧重数学专业基础性词汇的同时，兼顾新兴数学分支中出现的专业词汇，以使学生积累起阅读专业数学书刊所必需的词汇，能够顺利阅读原文数学教科书以及外文期刊中一般语言难度的文章，并具备初步的专业写作能力。

全书共分 14 个单元。第 1、2 单元的题材为 Precalculus。对于课文中出现的数学词汇，以评注的形式，给予汉、英双重解释。尤其，用英语给出了较为详尽的释义、定义和有关历史背景，以作为进一步阅读的补充，拓宽学生专业视野，并使他们通过对初等数学中数学词汇及语言表达的学习，做好向大学数学内容过渡的准备。第 3～6、第 9～11 单元安排 Mathematical Analysis, Higher Algebra, Set Theory, Differential Equations 等相应于数学专业基础课内容的英文材料。所选材料以“数学教程”模式为主，语言、语法难度适中，为学生阅读原文数学教材在词汇、语篇等方面做了准备。第 7、8、12、13 单元安排了与数学的专业课程有关的英文材料，如 Number Theory, Abstract Algebra, Topology, Numerical Analysis 等内容，逐渐加大语言和专业知识方面

的难度。书中还安排了“非数学教程”式的学习材料，包括著名数学家传记（主要涉及人物的数学经历、工作和成就）、访谈、数学书刊的评论、数学新闻报道等，数学词汇涉及面广泛，语言表达丰富。

最后，对中国石油大学（华东）理学院费祥历教授、孙清滢教授和谭尚旺教授的细心审阅表示衷心感谢，并对中国石油大学出版社的大力支持表示诚挚的谢意。

编　者

2011 年 10 月于青岛

Contents

Unit 1

Precalculus

In this unit, we'll review topics such as functions, graphs, logarithms, and trigonometry. We will assume that you're comfortable with the laws of exponents and radicals as well as with basic arithmetic operations involving real numbers, complex numbers, and general algebraic expressions.

Text A Functions

Various fields of human endeavor have to do with relationships that exist between one collection(集合) of objects and another. Graphs, charts, curves, and formulas are familiar to everyone who reads the newspapers. These are merely devices for describing special relations in a quantitative fashion. Mathematicians refer to certain types of these relations as functions.

Example (a) The force F necessary to stretch a steel spring a distance x beyond its natural length is proportional to x. That is, $F=cx$, where c is a number independent of x called the spring constant. This formula, discovered by Robert Hooke in the mid-17th century, is called Hooke's law, and it is said to express the force as a function of the displacement.

Example (b) The volume of a cube is a function of its edge-length. If the edges have length x, the volume V is given by the formula $V=x^3$.

Example (c) A prime is any integer $n>1$ that can't be expressed in the form $n=ab$, where a and b are positive integers, both less than n. The first few primes are 2, 3, 5, 7, 11, 13, 17, 19. For a given real number $x>0$, it is possible to count the number of primes less then or equal to x. This number is said to be a function of x even though no simple algebraic formula is known for computing it (without counting) when x is known.

The word "function" was introduced into mathematics by Leibniz, who used the term primarily to refer to certain kinds of mathematical formulas. It was later realized that Leibniz's idea of function was much too limited in its scope, and the meaning of the word has since undergone many stages of generalization. Today, the meaning of function is essentially this: Given two sets, say X and Y, a function is a correspondence which associates with each element of X one and only one element of Y. The set X is called the domain of the function. Those elements of Y associated with the elements in X form a set called the range of the function. (This may be all of Y, but need not be.)

Letters of the English and Greek alphabets are often used to denote functions. The particular letters f, g, h, F, G, H, and φ are frequently used for this purpose. If f is a given function and if x is an object of its domain, the notation $f(x)$ is used to designate that object in the range which is associated to x by the function f, and it is called the value of f at x or the image of x under f. One often writes $x \mapsto f(x)$ to denote that x is mapped to the element $f(x)$. The symbol $f(x)$ is read as "f of x." We shall use the notation $f: X \to Y$ to denote the fact that X and Y are two sets and f is a function from X to Y.

Although the function idea places no restriction on the nature of the objects in the domain X and in the range Y, in elementary calculus we are primarily interested in functions whose domain and range are sets of real numbers. Such functions are called real-valued functions of a real variable, or, more briefly, real functions(实函数), and they may be illustrated geometrically by a graph in the x-y plane. We plot the domain X on the x-axis, and above each point x in X we plot the point (x, y), where $y=f(x)$. The totality of such points (x, y) is called the graph of the function.

Functions constitute a very important notion in mathematics, and the definition of a function becomes increasingly sophisticated as your mathematical education progresses.

Example Consider the real-valued function f, defined by the equation $f(x) = -\sqrt{\dfrac{x+1}{|2x-1|}}$.

(a) Determine the largest subset of $\mathbf{R}$ that can serve as the domain of f.

(b) A point x_0 in the domain of f is called a fixed point of f if $f(x_0)=x_0$. This function f has exactly one rational fixed point. What is it?

Solution: (a)First, the expression in the denominator can't equal 0, so $x=1/2$ has to be excluded from the domain. Next, the fraction under the square root sign must be nonnegative. Since the denominator is positive for all $x\neq 1/2$, we need only ensure that the numerator is nonnegative; thus, it must be true that $x\geqslant -1$. Therefore, the domain of f is equal to $[-1, 1/2)\cup(1/2,\infty)$.

(b)To determine the fixed point of f, we need to solve the equation $f(x)=x$. Because of the presence of the absolute value sign, this leads to the two equations $(x+1)/(2x-1)=x^2$ or $(x+1)/(1-2x)=x^2$, which are equivalent to the equations $2x^3-x^2-x-1=0$ or $2x^3-x^2+x+1=0$.

By the rational roots theorem, the only possible rational roots of these equations are ± 1 or $\pm 1/2$. We can disregard $x=1/2$, because it's not in the domain of f, and simply check the remaining three possible rational roots. None of them satisfies the first equation, but $x=-1/2$ satisfies the second. Therefore, the function f has $x=-1/2$ as a fixed point, as this calculation shows: $f(-1/2)=-(1/4)^{1/2}=-1/2$.

(Extract taken from Chapter 1 of *CALCULUS*, *VOLUME I* by Tom M. Apostol, Chapter 1 of *Cracking the GRE Math Subject Test* by Steven A. Leduc.)

Words and Expressions

function 函数(quantity whose value depends on the varying values of other)

graph 图形(pictorial representation of numerical data, or a method of showing the mathematical relationship between two or more variables by drawing a diagram)

logarithm 对数. Let a be any positive number, not equal to 1. Then, for any real number x, the meaning of a^x can be defined. The logarithmic function to base a, denoted by $\log_a$, is defined as the inverse function of this function. So $y=\log_a x$ if and only if $x=a^y$. Thus $\log_a x$ is the index to which a must be raised in order to get x. Since any power a^y of a is positive, x must be positive for $\log_a x$ to be defined, and so the domain of the function $\log_a x$ is the set of positive real numbers. The value $\log_a x$ is called simply the logarithm of x to base a. The notation $\log x$ may be used when the base intended is understood. ([5]p. 165)

trigonometry 三角学. The study of angles and of the angular relationships of planar and three-dimensional figures is known as trigonometry.

assume 假定；假设（accept sth as true before there is proof）

law of exponents 指数定律

radical 根式. The symbol $\sqrt[n]{x}$ used to indicate a root is called a radical. The expression $\sqrt[n]{x}$ is therefore read "x radical n," or "the nth root of x." In the radical symbol, the horizontal line is called the vinculum, the quantity under the vinculum is called the radicand(被开方数), and the quantity n written to the left is called the index.

arithmetic 算术的(adj.); 算术(n.) **arithmetic operation** 算术运算

real number 实数. The field of all rational and irrational numbers is called the real numbers, or simply the "reals", and denoted **R**. The set of real numbers is also called the continuum(连续统), denoted c. The real numbers can be extended with the addition of the imaginary number i, equal to $\sqrt{-1}$. Numbers of the form $x+\mathrm{i}y$, where x and y are both real, are called complex numbers, which also form a field.

complex number 复数 **algebraic** 代数的

algebraic expression 代数式

chart 图表(a display of information in the form of a diagram, graph, etc.)

curve 曲线. In topology, a curve is a one-dimensional continuum. In analytic geometry, a curve is continuous map from a one-dimensional space to an n-dimensional space.

formula 公式. In mathematics, a formula is a fact, rule, or principle that is expressed in terms of mathematical symbols. Examples of formulas include equations, equalities, identities, inequalities, and asymptotic expressions. The term "formula" is also commonly used in the theory of logic to mean sentential formula (命题公式, also called a propositional formula), i. e. , a formula in propositional calculus. The correct Latin plural form of formula is "formulae," although the less pretentious-sounding "formulas" is more commonly used.

displacement 位移；刚体运动(n.)

volume 体积. The volume of a solid body is the amount of "space" it occupies.

cube 立方体. The cube is the Platonic solid P_1(also called the regular hexahedron). It is composed of six square faces that meet each other at right angles and has eight vertices and 12 edges.

prime 素数(质数) **integer** 整数

correspondence 对应(the relation between two sets where an operation on the members of one set maps some or all of them on to one or more members of the other)

domain 定义域. Let R be a binary relation. Then the set of all x such that $x\mathrm{R}y$ is called the domain of R. That is, the domain of R is the set of all first coordinates of the ordered pairs in R.

range 值域　　**calculus** 微积分[学]

real-valued function 实值函数. A function whose range is in the real numbers is said to be a real function, also called a real-valued function.

geometry 几何[学]

mathematics 数学. Mathematics is a broad-ranging field of study in which the properties and interactions of idealized objects are examined. Whereas mathematics began merely as a calculational tool for computation and tabulation(列表) of quantities, it has blossomed into an extremely rich and diverse set of tools, terminologies, and approaches which range from the purely abstract to the utilitarian. The term "mathematics" is often shortened to "math" in informal American speech and "maths" in British English. According to the *Oxford English Dictionary*, the term "math" has been in usage in London and the United States since 1890. The first usage of "maths" first occurred as a colloquialism(口语体) 21 years later. The term "math" also means a Hindu convent, so since the introduction of the variant "maths" corresponds to the period where Queen Victoria was empress of India, perhaps the trailing "s" was added to avoid confusion in India.

subset 子集

fixed point 不动点; [固]定点. A fixed point is a point that does not change upon application of a map, system of differential equations, etc.

rational 有理的　　**rational root** 有理根

denominator 分母

fraction 分数; 分式. A rational number expressed in the form a/b (in-line notation) or $\frac{a}{b}$(traditional "display" notation), where a is called the numerator and b is called the denominator. When written in-line, the slash(短斜线) "/" between numerator and denominator is called a solidus. A lowest terms fraction is a fraction with common terms canceled out of the numerator and denominator.

square root 平方根. A square root, also called a radical or surd(根式), of x is a number r such that $r^2=x$. The function $\sqrt{x}$ is therefore the inverse function of $f(x)=x^2$ for $x\geqslant 0$.

absolute value 绝对值

equivalent 等价的. If $A\Rightarrow B$ and $B\Rightarrow A$(where $\Rightarrow$ denotes implies), then A and B are said to be equivalent, a relationship which is written symbolically as $A \Leftrightarrow B$.

calculation 计算

Reading Material A

1. Function

A function is a relation that uniquely associates members of one set with members of another set. More formally, a function from A to B is an object f such that every $a \in A$ is uniquely associated with an object $f(a) \in B$. A function is therefore a many-to-one (or sometimes one-to-one) relation. The set A of values at which a function is defined is called its domain, while the set B of values that the function can produce is called its range. The term "map" is synonymous(同义的) with function.

Unfortunately, the term "function" is also used to refer to relations that map single points in the domain to possibly multiple points in the range. These "functions" are called multivalued functions (or multiple-valued functions), and arise prominently in the theory of complex functions, where the presence of multiple values engenders the use of so-called branch cuts.

Several notations are commonly used to represent (non-multivalued) functions. The most rigorous notation is $f: x \to f(x)$, which specifies that f is function acting upon a single number x (i. e., f is a univariate, or one-variable, function) and returning a value $f(x)$. To be even more precise, a notation like "$f: \mathbf{R} \to \mathbf{R}$, where $f(x) = x^2$" is sometimes used to explicitly specify the domain and range of the function. The slightly different "maps to" notation $f: x \mapsto f(x)$ is sometimes also used when the function is explicitly considered as a "map".

Generally speaking, the symbol f refers to the function itself, while $f(x)$ refers to the value taken by the function when evaluated at a point x. However, especially in more introductory texts, the notation $f(x)$ is commonly used to refer to the function f itself (as opposed to the value of the function evaluated at x). In this context, the argument x is considered to be a dummy variable whose presence indicates that the function f takes a single argument (as opposed to $f(x, y)$, etc.). While this notation is deprecated by professional mathematicians, it is the more familiar one for most nonprofessionals. Therefore, unless indicated otherwise by context, the notation $f(x)$ is taken in this work to be a shorthand for the more rigorous $f: x \to f(x)$.

multivalued 多值的
multivalued function 多值函数
multiple-valued function 多值函数
complex function 复函数
multiple value 多值
branch cut 分支切割
univariate 单变量的，一元的
argument 自变量，变元
dummy variable 哑变量

2. Graph

The word "graph" has (at least) two meanings in mathematics.

In elementary mathematics, "graph" refers to a function graph or "graph of a function," i. e. , a plot.

In a mathematician's terminology, a graph is a collection of points and lines connecting some (possibly empty) subset of them. The points of a graph are most commonly known as graph vertices, but may also be called "nodes" or simply "points". Similarly, the lines connecting the vertices of a graph are most commonly known as graph edges, but may also be called "arcs" or "lines".

The study of graphs is known as graph theory (图论), and was first systematically investigated by D. König(柯尼希(1884—1944)) in the 1930s. Unfortunately, as M. Gardner (加德纳(1914—2010)) notes, "The confusion of this term [i. e. , the term "graph" to describe a network of vertices and edges] with the 'graphs' of analytic geometry [i. e. , plots of functions] is regrettable, but the term has stuck." Some educators use the term "vertex-edge graph" for a connected set of nodes in an attempt to preserve the common usage of "graph" to mean the plot of a function.

plot 图(n.)
node 结点
arc 弧
line 边，线，直线

3. Arithmetic

Arithmetic is the branch of mathematics dealing with integers or, more generally, numerical computation. Arithmetical operations include addition, congruence calculation, division, factorization, multiplication, power computation, root extraction, and subtraction. Arithmetic was part of the quadrivium taught in medieval universities. A mnemonic for the spelling of "arithmetic" is "a rat in the house may eat the ice cream".

The branch of mathematics known as number theory is sometimes known as higher arithmetic.

Modular arithmetic is the arithmetic of congruences.

Floating-point arithmetic is the arithmetic performed on real numbers by computers or other automated devices using a fixed number of bits.

The fundamental theorem of arithmetic, also called the unique factorization theorem, states that any positive integer can be represented in exactly one way as a product of primes.

numerical 数值的;数字的
numerical computation 数值计算
arithmetical 算术的,算术上的
congruence 同余,同余式
extraction 求根,开方
extraction of root 求根法,开方法
modular arithmetic 模算术
floating-point arithmetic 浮点运算
bit 比特,位,二进制位

4. Imaginary Unit

The imaginary unit is often loosely referred to as the "square root of -1", however care should be taken as there are in fact two square roots of -1 (namely i and $-$i). A naive use of this idea thus may lead to difficulties.

By definition, the imaginary unit i is one solution (the other solution is $-$i) of the quadratic equation(二次方程) $x^2+1=0$ or equivalently $x^2=-1$.

Since there is no real number that produces a negative real number when squared, we imagine such a number and assign to it the symbol i. It is important to realize, though, that i is as well-defined a mathematical construct as the real numbers, despite its formal name and being less than immediately intuitive.

Real number operations can be extended to imaginary and complex numbers by treating i as an unknown quantity while manipulating an expression, and then using the definition to replace any occurrence of i^2 with -1.

With its second order polynomial with no multiple real root, the defining equation $x^2+1=0$ has two distinct solutions, which are equally valid and which happen to be additive and multiplicative inverses of each other. More precisely, once a solution i of the equation has been fixed, the value $-$i (which is not equal to i) is also a solution. Since the equation is the only definition of i, it appears that the definition is ambiguous (more precisely, not well-defined). However, no ambiguity results as long as one of the solu-