David N. DeJong Chetan Dave

# Structural Macroeconometrics

**Second Edition** 

结构宏观计量经济学 <sup>第2版</sup>



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#### Preface

Scratch where it itches.

—Alice Roosevelt Longworth

If it ain't broke, don't fix it.

—Bert Lance

The usual excuse for crafting the second edition of a text is rapidly changing subject matter. We could certainly invoke this excuse here: the empirical analysis of dynamic stochastic general equilibrium (DSGE) models has evolved considerably over the past five years. Instead, we confess that this excuse is only second-order important in this instance. First-order important is our graduation from the steepest portion of a critical learning curve.

In particular, when we set out to characterize likelihood evaluation and filtering for nonlinear model representations, our comprehension of applicable numerical methods did not extend beyond the standard particle filter. Moreover, our understanding of the particle filter was limited to the mechanics of its execution (which is exceedingly simple), and did not extend to a sufficient appreciation of its shortcomings (which, alas, can be significant in empirically relevant situations). Indeed, as we were putting the finishing touches on the first edition, one of us (DeJong) was developing plans to implement the particle filter in pursuit of a promising empirical application. Shortly thereafter, the particle filter was found to be not up to the task, and despite the passage of time, the application remains on the back burner (thank goodness for tenure).

The good news is that, with necessity serving as the mother of invention, we have come to better appreciate the strengths and weaknesses of the particle filter, as well as alternative methods. Even better, our emergence from the learning curve has led to the development (by DeJong, in collaboration with Hariharan Dharmarajan, Roman Liesenfeld, Guilherme Moura, and Jean-François Richard) of an alternative filtering method: the efficient importance sampling (EIS) filter. As we shall demonstrate, implementation of the EIS filter is well suited to DSGE models, and the payoff to its adoption can be high.

The emphasis here on filtering reflects our appreciation of the importance that departures from linearity hold for the future of empirical work involving DSGE models. Five years ago, the use of nonlinear model

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representations in empirical applications was exotic; five years from now, we expect this use to be routine. Accordingly, while in the first edition the treatment of nonlinear model representations was isolated to a pair of closing chapters, linear and nonlinear treatments are now characterized in tandem throughout the text.

Users of the first edition will notice additional changes as well: details are sketched in chapter 1. However, we remain satisfied with the bulk of the subjects covered in the first edition, and have heeded the wisdom of Bert Lance in their treatment. We hope we have done so judiciously.

Since the publication of the first edition, we have benefited from input on the text received from students we have taught at Carnegie-Mellon and Di Tella universities, the Kiel Institute for the World Economy, the University of Pittsburgh, and the University of Texas at Dallas. We have also been alerted to many errors of commission: we are particularly indebted to Roberto Chang, Andres Fernandez, Michael Hauser, Jan Christopher Ocampo, and Surach Tanboon for their careful reading. Certainly we committed many errors of omission in the first edition when it came to describing literatures related to the topics we covered. We have sought to update and expand our treatment of these literatures here, though in so doing we have surely left a great many excellent papers unmentioned: apologies in advance for this.

Dave DeJong and Chetan Dave

## Preface to the First Edition

Let me only say that what econometrics—aided by electronic computers—can do, is only to push forward by leaps and bounds the line of demarcation from where we have to rely on our intuition and sense of smell.

—Ragnar Frisch, Nobel Prize Lecture, June 1970.

It is a capital mistake to theorize before one has data. Insensibly one begins to twist facts to suit theories, instead of theories to suit facts.

-Sir Arthur Conan Doyle

A steady stream of conceptual and computational advances realized over the past three decades has helped to bridge the gulf that has historically separated theoretical from empirical research in macroeconomics. As a result, measurement is becoming increasingly aligned with theory. The purpose of this text is to provide guidance in bringing theoretical models to the forefront of macroeconometric analyses.

The text is suitable for use as a supplementary resource in introductory graduate courses in macroeconomics and econometrics, and as a primary textbook in advanced graduate courses devoted to the pursuit of applied research in macroeconomics. The lecture notes that ultimately gave rise to the text were designed for this latter purpose. The text's historical perspective, along with its unified presentation of alternative methodologies, should also make it a valuable resource for academic and professional researchers.

Readers of the text are assumed to have familiarity with multivariate calculus, matrix algebra, and difference equations, and cursory knowledge of basic econometric techniques. Familiarity with dynamic programming is also useful but not necessary. This is the tool used to map the class of models of interest here into the system of nonlinear expectational difference equations that serve as the point of departure for the empirical methodologies presented in the text. However, familiarity with dynamic programming is not needed to follow the text's presentation of empirical methodologies.

We decided to write this text because through our own teaching and research, we have sought to contribute towards the goal of aligning theory

with empirical analysis in macroeconomics; this text is a natural extension of these efforts. We set out to accomplish two objectives in writing it. First, we wished to provide a unified overview of this diverse yet interrelated area of active research. Second, we wanted to equip students with a set of tools that would expedite their own entry into the field.

The content of this text reflects much that we have learned over many years spent pursuing collaborative research with a fabulous group of coauthors: Sheryl Ball, Patty Beeson, Dan Berkowitz, Stefan Dodds, Scott Dressler, Catherine Eckel, Emilio Espino, Steve Husted, Beth Ingram, Roman Leisenfeld, John Nankervis, Jean-François Richard, Marla Ripoll, Gene Savin, Werner Troesken, and Chuck Whiteman. We are deeply indebted to them for their implicit and explicit contributions to this project. We have also benefited from input provided by Charlie Evans, Jim Feigenbaum, Jesús Fernández-Villaverde, Peter Ireland, Naryana Kocherlakota, Chris Otrok, Barbara Rossi, Juan Rubio-Ramírez, Tom Sargent, Thomas Steinberger, Yi Wen, and Tao Zha on various aspects of the text. Finally, Hariharan Dharmarajan provided valuable research assistance on the empirical application presented in chapter 11.

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### Part I

# Introduction

#### Background and Overview

Science is facts; just as houses are made of stones, so is science made of facts; but a pile of stones is not a house and a collection of facts is not necessarily science.

-Henri Poincaré

#### 1.1 Background

The seminal contribution of Kydland and Prescott (1982) marked a sea change in the way macroeconomists conduct empirical research. Under the empirical paradigm that remained predominant at the time, the focus was either on purely statistical (or reduced-form) characterizations of macroeconomic behavior, or on systems-of-equations models that ignored both general-equilibrium considerations and forward-looking behavior on the part of purposeful decision makers. But the powerful criticism of this approach set forth by Lucas (1976), and the methodological contributions of, for example, Sims (1972) and Hansen and Sargent (1980), sparked a transition to a new empirical paradigm. In this transitional stage, the formal imposition of theoretical discipline on reduced-form characterizations became established. The source of this discipline was a class of models that have come to be known as dynamic stochastic general equilibrium (DSGE) models. The imposition of discipline most typically took the form of "cross-equation restrictions," under which the stochastic behavior of a set of exogenous variables, coupled with forward-looking behavior on the part of economic decision makers, yield implications for the endogenous stochastic behavior of variables determined by the decision makers. Nevertheless, the imposition of such restrictions was indirect, and reduced-form specifications continued to serve as the focal point of empirical research.

Kydland and Prescott turned this emphasis on its head. As a legacy of their work, DSGE models no longer serve as indirect sources of theoretical discipline to be imposed upon statistical specifications. Instead, they serve directly as the foundation upon which empirical work may be conducted. The methodologies used to implement DSGE models as foundational empirical models have evolved over time and vary considerably. The same

is true of the statistical formality with which this work is conducted. But despite the characteristic heterogeneity of methods used in pursuing contemporary empirical macroeconomic research, the influence of Kydland and Prescott remains evident today.

This book details the use of DSGE models as foundations upon which empirical work may be conducted. It is intended primarily as an instructional guide for graduate students and practitioners, and so contains a distinct how-to perspective throughout. The methodologies it presents are organized roughly following the chronological evolution of the empirical literature in macroeconomics that has emerged following the work of Kydland and Prescott; thus it also serves as a reference guide. Throughout, the methodologies are demonstrated using applications to three benchmark models: a real-business-cycle model (fashioned after King, Plosser, and Rebelo, 1988); a monetary model featuring monopolistically competitive firms (fashioned after Ireland, 2004a); and an asset-pricing model (fashioned after Lucas, 1978).

#### 1.2 Overview

The empirical tools outlined in the text share a common foundation: a system of nonlinear expectational difference equations representing a DSGE model. The solution to a given system takes the form of a collection of policy functions that map a subset of the variables featured in the model—state variables—into choices for the remaining subset of variables—control variables; the choices represent optimal behavior on the part of the decision makers featured in the model, subject to constraints implied by the state variables. Policy functions, coupled with laws of motion for the state variables, collectively represent the solution to a given DSGE model; the solution is in the form of a state-space representation.

Policy functions can be calculated analytically for only a narrow set of models, and thus must typically be approximated numerically. The text presents several alternative methodologies for achieving both model approximation and the subsequent empirical implementation of the corresponding state-space representation. Empirical implementation is demonstrated in applications involving parameter estimation, assessments of fit and model comparison, forecasting, policy analysis, and measurement of unobservable facets of aggregate economic activity (e.g., measurement of productivity shocks).

This book is divided into five parts. Part I contains three chapters. Following this brief introduction, chapter 2 establishes the consistent set of notation used throughout the text, and presents an important preliminary