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A Bilingual Brief Course in University Physics

Part One Mechanics

大学物理双语简明教程

第一册 力学

LIANG Jiachang ZHANG Liping LI Li

梁家昌 张丽平 李 立

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Preface

In order to make as early as possible the lower-grade university students raise their ability of listening to the special English, of reading the text books and papers in English and of writing in English, a bilingual brief course in university physics was completed on the base of many year's teaching experiences.

Throughout this course the fundamental principles (including superposition principle, symmetries, conservation laws and dynamic equations) and their applications are emphasized.

Some of new ideas in physical developments have been mentioned in the summaries of every chapter. It is of benefit to learn creatively for students.

The exercises in the course were selected and completed by Masters ZHANG Liping and LI Li. Professor CHEN Yuanzhen and vice professor LIU guangxuan read and corrected English and Chinese proofs, respectively. The design and typesetting of this course were completed by Mr. CHEN Jun and Mr. CHEN Zhicong, index was selected by Master BAI Cuiqin and figures were drawn by Masters BAI Cuiqin and LIU Tieju . Master CHEN Yifei designed The cover ,Vice professor SHI Xinhua and Ms. LIANG Jing acted as editors。

In writing this course, the authors have made reference to the various university textbooks lately published both at home and abroad, including:

1. ZHANG Sanhui et al. University Physics. Second ed. Beijing : Tsinghua Univeristy Press,1999.

2. CHENG Shouzhu et al. General Physics. Fifth ed. Beijing : China High Education Press,1998.

3. Dexin Lu. University Physics. First ed. Beijing and Heidelberg : China High Education Press

前言

为了尽早提高大学低年级学生对专业英语的听力、阅读英语教材、文章与用英语书写的能力，在作者多年教学经验的基础上编著了这套大学物理双语简明教程。

在本教程中，始终强调物理学中的基本原理（包括迭加原理、对称性、守恒定律、动力学方程）及其应用。

在每章的小结中，提到了物理学发展的某些新思想。这有利于学生创造性地学习。

书中的习题是由张丽平、李立两位硕士选择和完成的。陈远震教授与刘广瑄副教授分别对本书的英文与中文稿作了校对与改错。本书的版面设计与排版是由陈军与陈治璁两位先生完成的，索引为白翠琴硕士所作，制图则由白翠琴及刘铁驹硕士承担。陈逸飞硕士设计了封面，石新华副教授与梁晶女士担任编辑。

在编著本教程时，我们参改了近年来国内外出版的大学物理教材，包括：

1. 张三慧 等。大学物理。第二版。北京：清华大学出版社，1999。

2. 程守洙 等。普通物理。第五版。北京：高等教育出版社，1998。

3. 卢德馨。大学物理。第一版。北京与海德堡：高等教育出版

and Spring-Verlag, 1999.

4. D. Halliday et al. Fundamentals of Physics. Sixth ed. New York : John Wiley & Sons, Inc,2001.

5. H.D.Young et al. Sears and Zemansky's University Physics. Tenth ed. Beijing: China Machine Press,2002.

The authors have the honor to pay special sincere thanks to the authors of the above textbooks. They also would like to deeply express their thanks to Mr. TIAN Guilin and Ms. LI Meirong for their assistance.

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社与 Spring- Verlag 出版社，1999。

4. D.Halliday 等。基础物理。第六版。纽约：John Wiley & Sons 出版社，2001。

5. H.D.Young 等。Sear 与 Zemansky 大学物理。第十版。北京：中国机械出版社，2002。

本书作者对上述教材的作者特别致以诚挚的感谢。他们也对田桂林先生与李美荣女士的协助深表感谢。

作者
于中国民用航空学院

Contents　　　　目录

Chapter 1 Kinematics in Classical Mechanics

Mechanics studies on the motion of bodies. It is divided into two parts: kinematics and dynamics. Kinematics is of the studies on motion without regard to the cause of the motion. In kinematics some of physical quantities are defined and relationships among these quantities established.

1.1 Physical quantities in kinematics

In order to describe the phenomena in kinematics in mechanics, it is necessary to define some of appropriate quantities:

(1) the position vector $\boldsymbol{r}(t)$ of a body,

(2) the displacement $\Delta \boldsymbol{r}(t) = \boldsymbol{r}_2(t) - \boldsymbol{r}_1(t)$ of a body,

(3) the velocity $\boldsymbol{v}(t) = \dfrac{\mathrm{d}\boldsymbol{r}(t)}{\mathrm{d}t}$ of a body,

(4) the acceleration $\boldsymbol{a}(t) = \dfrac{\mathrm{d}\boldsymbol{v}(t)}{\mathrm{d}t} = \dfrac{\mathrm{d}^2\boldsymbol{r}(t)}{\mathrm{d}t^2}$ of a body.

Now we explain these quantities in detail.

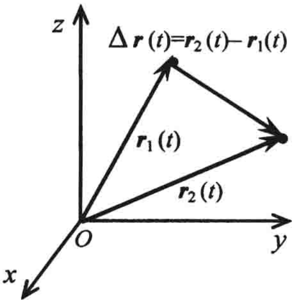

Fig.1.1.1 Motion of a body in three-dimensional Cartesian coordinates

第一章 经典力学中的运动学

力学研究物体的运动。它可划分为两部分：运动学和动力学。运动学研究运动而不涉及运动的起因。在运动学中，定义了一些物理量，并且建立起这些物理量间的关系。

1.1 运动学中的物理量

为了描述力学中运动学的现象，需要定义一些恰当的物理量：

(1) 物体的位置矢量 $\boldsymbol{r}(t)$,

(2) 物体的位移矢量

$$\Delta \boldsymbol{r}(t) = \boldsymbol{r}_2(t) - \boldsymbol{r}_1(t),$$

(3) 物体的速度 $\boldsymbol{v}(t) = \dfrac{\mathrm{d}\boldsymbol{r}(t)}{\mathrm{d}t}$,

(4) 物体的加速度

$$\boldsymbol{a}(t) = \frac{\mathrm{d}\boldsymbol{v}(t)}{\mathrm{d}t} = \frac{\mathrm{d}^2\boldsymbol{r}(t)}{\mathrm{d}t^2}$$

现在我们来详细解释这些物理量。

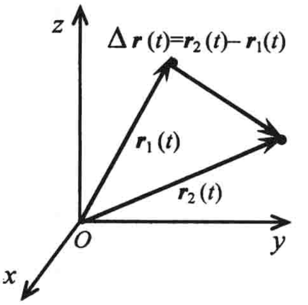

图 1.1.1 三维笛卡尔坐标系中物体的运动

1. The position vector $\boldsymbol{r}(t)$ of a body

In Cartesian coordinates the position vector $\boldsymbol{r}(t)$ of a body should be expressed as

$$\boldsymbol{r}(t)=x(t)\boldsymbol{i}+y(t)\boldsymbol{j}+z(t)\boldsymbol{k} \qquad (1.1.1)$$

where x, y and z are the components of $\boldsymbol{r}(t)$ at three coordinate axes and $\boldsymbol{i}, \boldsymbol{j}$ and $\boldsymbol{k}$ the unit vectors of these three coordinate axes, respectively, as shown in Fig. 1.1.1.

2. The displacement $\Delta\boldsymbol{r}$ of a body

Fig.1.1.1 shows that when a body from its position $\boldsymbol{r}_1$ moves to another position $\boldsymbol{r}_2$, vector $\Delta\boldsymbol{r}(t)$ describes the displacement of this body. In Cartesian coordinates $\Delta\boldsymbol{r}(t)=\boldsymbol{r}_2(t)-\boldsymbol{r}_1(t)$ can be expressed as

$$\boldsymbol{r}_1=x_1\boldsymbol{i}+y_1\boldsymbol{j}+z_1\boldsymbol{k},$$
$$\boldsymbol{r}_2=x_2\boldsymbol{i}+y_2\boldsymbol{j}+z_2\boldsymbol{k},$$

and

$$\begin{aligned}\Delta\boldsymbol{r}&=\boldsymbol{r}_2-\boldsymbol{r}_1\\&=(x_2-x_1)\boldsymbol{i}+(y_2-y_1)\boldsymbol{j}+(z_2-z_1)\boldsymbol{k}\\&=\Delta x\boldsymbol{i}+\Delta y\boldsymbol{j}+\Delta z\boldsymbol{k}. \qquad (1.1.2)\end{aligned}$$

Using $\boldsymbol{i}$ to do scalar product with Eq.(1.1.1), we have

$$\boldsymbol{r}\cdot\boldsymbol{i}=x\boldsymbol{i}\cdot\boldsymbol{i}+y\boldsymbol{j}\cdot\boldsymbol{i}+z\boldsymbol{k}\cdot\boldsymbol{i}=x,$$

$$i.e.\quad x=|\boldsymbol{r}|\cos(\boldsymbol{r},\boldsymbol{i}).$$

Similarly, we have

$$y=|\boldsymbol{r}|\cos(\boldsymbol{r},\boldsymbol{j}), z=|\boldsymbol{r}|\cos(\boldsymbol{r},\boldsymbol{k}) \qquad (1.1.3)$$

And $$|\boldsymbol{r}|=\sqrt{x^2+y^2+z^2}. \qquad (1.1.4)$$

The calculus indicates that the instantaneous velocity $\boldsymbol{v}(t)$ of the body's motion should be expressed as

$$\boldsymbol{v}(t)=\lim_{\Delta t\to 0}\frac{\Delta\boldsymbol{r}(t)}{\Delta t}=\frac{\mathrm{d}\boldsymbol{r}(t)}{\mathrm{d}t},$$

which is the change rate of its position $\boldsymbol{r}$ with time t. Derivating Eq.(1.1.1) with respect to t, we obtain

$$\begin{aligned}\boldsymbol{v}(t)&=\frac{\mathrm{d}\boldsymbol{r}(t)}{\mathrm{d}t}=\frac{\mathrm{d}x(t)}{\mathrm{d}t}\boldsymbol{i}+\frac{\mathrm{d}y(t)}{\mathrm{d}t}\boldsymbol{j}+\frac{\mathrm{d}z(t)}{\mathrm{d}t}\boldsymbol{k}\\&=v_x(t)\boldsymbol{i}+v_y(t)\boldsymbol{j}+v_z(t)\boldsymbol{k}, \qquad (1.1.5)\end{aligned}$$

where $v_x(t)$, $v_y(t)$ and $v_z(t)$ are the components of velocity $\boldsymbol{v}(t)$, respectively.

1. 物体的位置矢量 $\boldsymbol{r}(t)$

在笛卡尔坐标中，物体的位置矢量 $\boldsymbol{r}(t)$ 应表成

$$\boldsymbol{r}(t)=x(t)\boldsymbol{i}+y(t)\boldsymbol{j}+z(t)\boldsymbol{k} \qquad (1.1.1)$$

如图 1.1.1 所示，其中 x, y, z 分别是 $\boldsymbol{r}(t)$ 在三个坐标轴上的分量，$\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$ 分别是三个坐标轴的单位矢量。

2. 物体的位移矢量 $\Delta\boldsymbol{r}$

图 1.1.1 展示，当一物体从位置 $\boldsymbol{r}_1$ 位移到位置 $\boldsymbol{r}_2$ 时，矢量 $\Delta\boldsymbol{r}(t)$ 描述物体的位移。在笛卡尔坐标中，$\Delta\boldsymbol{r}(t)=\boldsymbol{r}_2(t)-\boldsymbol{r}_1(t)$ 可表示成

$$\boldsymbol{r}_1=x_1\boldsymbol{i}+y_1\boldsymbol{j}+z_1\boldsymbol{k},$$
$$\boldsymbol{r}_2=x_2\boldsymbol{i}+y_2\boldsymbol{j}+z_2\boldsymbol{k},$$

所以

$$\begin{aligned}\Delta\boldsymbol{r}&=\boldsymbol{r}_2-\boldsymbol{r}_1\\&=(x_2-x_1)\boldsymbol{i}+(y_2-y_1)\boldsymbol{j}+(z_2-z_1)\boldsymbol{k}\\&=\Delta x\boldsymbol{i}+\Delta y\boldsymbol{j}+\Delta z\boldsymbol{k}. \qquad (1.1.2)\end{aligned}$$

将 $\boldsymbol{i}$ 与(1.1.1)式作标积，可得

$$\boldsymbol{r}\cdot\boldsymbol{i}=x\boldsymbol{i}\cdot\boldsymbol{i}+y\boldsymbol{j}\cdot\boldsymbol{i}+z\boldsymbol{k}\cdot\boldsymbol{i}=x,$$

即 $x=|\boldsymbol{r}|\cos(\boldsymbol{r},\boldsymbol{i})$

同样，可得

$$y=|\boldsymbol{r}|\cos(\boldsymbol{r},\boldsymbol{j}), z=|\boldsymbol{r}|\cos(\boldsymbol{r},\boldsymbol{k}) \qquad (1.1.3)$$

及 $$|\boldsymbol{r}|=\sqrt{x^2+y^2+z^2}. \qquad (1.1.4)$$

微积分表明，物体运动的瞬时速度 $\boldsymbol{v}(t)$应表示成

$$\boldsymbol{v}(t)=\lim_{\Delta t\to 0}\frac{\Delta\boldsymbol{r}(t)}{\Delta t}=\frac{\mathrm{d}\boldsymbol{r}(t)}{\mathrm{d}t},$$

它是位移随时间的变化率。把（1.1.1）式对时间 t 求导，可得

$$\begin{aligned}\boldsymbol{v}(t)&=\frac{\mathrm{d}\boldsymbol{r}(t)}{\mathrm{d}t}=\frac{\mathrm{d}x(t)}{\mathrm{d}t}\boldsymbol{i}+\frac{\mathrm{d}y(t)}{\mathrm{d}t}\boldsymbol{j}+\frac{\mathrm{d}z(t)}{\mathrm{d}t}\boldsymbol{k}\\&=v_x(t)\boldsymbol{i}+v_y(t)\boldsymbol{j}+v_z(t)\boldsymbol{k}, \qquad (1.1.5)\end{aligned}$$

其中 $v_x(t)$, $v_y(t)$和 $v_z(t)$分别是速度 $\boldsymbol{v}(t)$ 的三个分量。

In a similar way to Eq.(1.1.3), we have

$$v_x = |\boldsymbol{v}|\cos(\boldsymbol{v},\boldsymbol{i}),\quad v_y = |\boldsymbol{v}|\cos(\boldsymbol{v},\boldsymbol{j}),\quad v_z = |\boldsymbol{v}|\cos(\boldsymbol{v},\boldsymbol{k}), \tag{1.1.6}$$

And $$|\boldsymbol{v}| = \sqrt{{v_x}^2 + {v_y}^2 + {v_z}^2}. \tag{1.1.7}$$

3. The acceleration of a body $\boldsymbol{a}(t)$

The calculus also indicates that the instantaneous acceleration $\boldsymbol{a}(t)$ of a body motion should be expressed as

$$\boldsymbol{a}(t) = \lim_{\Delta t\to 0}\frac{\Delta \boldsymbol{v}(t)}{\Delta t} = \frac{\mathrm{d}\boldsymbol{v}(t)}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mathrm{d}\boldsymbol{r}(t)}{\mathrm{d}t}\right) = \frac{\mathrm{d}^2\boldsymbol{r}(t)}{\mathrm{d}t^2}.$$

Deducing the first derivative of $\boldsymbol{v}(t)$ in Eq.(1.1.5) or second derivative of $\boldsymbol{r}(t)$ in Eq.(1.1.1) with respect to time t, we have

$$\begin{aligned}\boldsymbol{a}(t) &= \frac{\mathrm{d}\boldsymbol{v}(t)}{\mathrm{d}t} = \frac{\mathrm{d}^2\boldsymbol{r}(t)}{\mathrm{d}t^2}\\ &= \frac{\mathrm{d}v_x(t)}{\mathrm{d}t}\boldsymbol{i} + \frac{\mathrm{d}v_y(t)}{\mathrm{d}t}\boldsymbol{j} + \frac{\mathrm{d}v_z(t)}{\mathrm{d}t}\boldsymbol{k}\\ &= \frac{\mathrm{d}^2x(t)}{\mathrm{d}t^2}\boldsymbol{i} + \frac{\mathrm{d}^2y(t)}{\mathrm{d}t^2}\boldsymbol{j} + \frac{\mathrm{d}^2z(t)}{\mathrm{d}t^2}\boldsymbol{k}\\ &= a_x(t)\boldsymbol{i} + a_y(t)\boldsymbol{j} + a_z(t)\boldsymbol{k},\end{aligned} \tag{1.1.8}$$

where a_x, a_y and a_z are the components of acceleration $\boldsymbol{a}$, respectively. In a similar way to Eq.(1.1.3) and Eq.(1.1.6), we have

$$a_x = |\boldsymbol{a}|\cos(\boldsymbol{a},\boldsymbol{i}),\quad a_y = |\boldsymbol{a}|\cos(\boldsymbol{a},\boldsymbol{j}),\quad a_y = |\boldsymbol{a}|\cos(\boldsymbol{a},\boldsymbol{k}) \tag{1.1.9}$$

and $$|\boldsymbol{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}. \tag{1.1.10}$$

Eqs.(1.1.1), (1.1.5) and (1.1.8) indicate that deducing

用与得到式（1.1.3）相似的方法可得

$$v_x = |\boldsymbol{v}|\cos(\boldsymbol{v},\boldsymbol{i}),\quad v_y = |\boldsymbol{v}|\cos(\boldsymbol{v},\boldsymbol{j}),\quad v_z = |\boldsymbol{v}|\cos(\boldsymbol{v},\boldsymbol{k}), \tag{1.1.6}$$

及 $$|\boldsymbol{v}| = \sqrt{{v_x}^2 + {v_y}^2 + {v_z}^2}. \tag{1.1.7}$$

3. 物体的加速度 $\boldsymbol{a}(t)$

微积分也表明，一物体运动的瞬时加速度$\boldsymbol{a}(t)$应表成

$$\boldsymbol{a}(t) = \lim_{\Delta t\to 0}\frac{\Delta \boldsymbol{v}(t)}{\Delta t} = \frac{\mathrm{d}\boldsymbol{v}(t)}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mathrm{d}\boldsymbol{r}(t)}{\mathrm{d}t}\right) = \frac{\mathrm{d}^2\boldsymbol{r}(t)}{\mathrm{d}t^2}.$$

将式（1.1.5）对时间 t 求一次导数或者将式（1.1.1）对时间 t 求二次导数，可得

$$\begin{aligned}\boldsymbol{a}(t) &= \frac{\mathrm{d}\boldsymbol{v}(t)}{\mathrm{d}t} = \frac{\mathrm{d}^2\boldsymbol{r}(t)}{\mathrm{d}t^2}\\ &= \frac{\mathrm{d}v_x(t)}{\mathrm{d}t}\boldsymbol{i} + \frac{\mathrm{d}v_y(t)}{\mathrm{d}t}\boldsymbol{j} + \frac{\mathrm{d}v_z(t)}{\mathrm{d}t}\boldsymbol{k}\\ &= \frac{\mathrm{d}^2x(t)}{\mathrm{d}t^2}\boldsymbol{i} + \frac{\mathrm{d}^2y(t)}{\mathrm{d}t^2}\boldsymbol{j} + \frac{\mathrm{d}^2z(t)}{\mathrm{d}t^2}\boldsymbol{k}\\ &= a_x(t)\boldsymbol{i} + a_y(t)\boldsymbol{j} + a_z(t)\boldsymbol{k},\end{aligned} \tag{1.1.8}$$

其中 a_x，a_y 和 a_z 分别是加速度 $\boldsymbol{a}$ 的三个分量，用与得到式（1.1.3）以及式（1.1.6）相似的方法，可得

$$a_x = |\boldsymbol{a}|\cos(\boldsymbol{a},\boldsymbol{i}),\quad a_y = |\boldsymbol{a}|\cos(\boldsymbol{a},\boldsymbol{j}),\quad a_y = |\boldsymbol{a}|\cos(\boldsymbol{a},\boldsymbol{k}) \tag{1.1.9}$$

及 $$|\boldsymbol{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}. \tag{1.1.10}$$

式（1.1.1），（1.1.5）以及（1.1.8）

the first and second derivatives of motion equation $\boldsymbol{r} = \boldsymbol{r}(t)$ of a body with respect to time t, its motion state, including velocity $\frac{\mathrm{d}\boldsymbol{r}(t)}{\mathrm{d}t}$ and acceleration $\frac{\mathrm{d}^2\boldsymbol{r}(t)}{\mathrm{d}t^2}$, can be obtained. By contrast, given the instantaneous velocity $\frac{\mathrm{d}x(t)}{\mathrm{d}t}$, $\frac{\mathrm{d}y(t)}{\mathrm{d}t}$ and $\frac{\mathrm{d}z(t)}{\mathrm{d}t}$ or instantaneous acceleration $\frac{\mathrm{d}^2x(t)}{\mathrm{d}t^2}$, $\frac{\mathrm{d}^2y(t)}{\mathrm{d}t^2}$ and $\frac{\mathrm{d}^2z(t)}{\mathrm{d}t^2}$ of a body at any time with definite initial conditions, including

$$\boldsymbol{r}(t_0) = \boldsymbol{r}_0 = x_0\boldsymbol{i} + y_0\boldsymbol{j} + z_0\boldsymbol{k}$$

and $\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}t}|_{t=t_0} = \boldsymbol{v}_0 = v_{0x}\boldsymbol{i} + v_{0y}\boldsymbol{j} + v_{0z}\boldsymbol{k}$ at time t_0, through integrating $\frac{\mathrm{d}x(t)}{\mathrm{d}t}$, $\frac{\mathrm{d}y(t)}{\mathrm{d}t}$, $\frac{\mathrm{d}z(t)}{\mathrm{d}t}$, $\frac{\mathrm{d}^2x(t)}{\mathrm{d}t^2}$, $\frac{\mathrm{d}^2y(t)}{\mathrm{d}t^2}$ and $\frac{\mathrm{d}^2z(t)}{\mathrm{d}t^2}$, the motion equation $x = x(t)$, $y = y$(t) and $z = z(t)$ or $\boldsymbol{r} = \boldsymbol{r}(t)$ can be obtained.

表明，一物体的运动方程 $\boldsymbol{r} = \boldsymbol{r}(t)$，对时间 t 进行一次和二次求导，就能确定该物体的运动状态，包括它的速度 $\frac{\mathrm{d}\boldsymbol{r}(t)}{\mathrm{d}t}$ 及加速度 $\frac{\mathrm{d}^2\boldsymbol{r}(t)}{\mathrm{d}t^2}$. 相反，若给定物体的瞬时速度 $\frac{\mathrm{d}x(t)}{\mathrm{d}t}$，$\frac{\mathrm{d}y(t)}{\mathrm{d}t}$，$\frac{\mathrm{d}z(t)}{\mathrm{d}t}$ 或者瞬时加速度 $\frac{\mathrm{d}^2x(t)}{\mathrm{d}t^2}$，$\frac{\mathrm{d}^2y(t)}{\mathrm{d}t^2}$，$\frac{\mathrm{d}^2z(t)}{\mathrm{d}t^2}$ 及确定的初始条件，包括 t_0 时刻的 $\boldsymbol{r}(t_0) = \boldsymbol{r}_0 = x_0\boldsymbol{i} + y_0\boldsymbol{j} + z_0\boldsymbol{k}$ 以及 $\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}t}|_{t=t_0} = \boldsymbol{v}_0 = v_{0x}\boldsymbol{i} + v_{0y}\boldsymbol{j} + v_{0z}\boldsymbol{k}$，通过对 $\frac{\mathrm{d}x(t)}{\mathrm{d}t}$，$\frac{\mathrm{d}y(t)}{\mathrm{d}t}$，$\frac{\mathrm{d}z(t)}{\mathrm{d}t}$，$\frac{\mathrm{d}^2x(t)}{\mathrm{d}t^2}$，$\frac{\mathrm{d}^2y(t)}{\mathrm{d}t^2}$ 以及 $\frac{\mathrm{d}^2z(t)}{\mathrm{d}t^2}$ 求积分，就能得到物体的运动方程 $x = x(t)$, $y = y$(t) 和 $z = z(t)$或 $\boldsymbol{r} = \boldsymbol{r}(t)$.

Example 1 Given $a_x(t)$ with initial conditions $x(t_0) = x_0$ and $v_x(t_0) = v_{0x}$ at time t_0, write the expressions for $v_x(t)$ and $x(t)$.

[solution] Because $\frac{\mathrm{d}v_x(t)}{\mathrm{d}t} = a_x(t)$, therefore

$$\int_{v_{0x}}^{v_x(t)} \mathrm{d}v_x = \int_{t_0}^{t} a_x(t)\mathrm{d}t, \quad \text{i.e.}$$

$$v_x(t) = v_{0x} + \int_{t_0}^{t} a_x(t)\mathrm{d}t. \tag{1}$$

Because $\frac{\mathrm{d}x(t)}{\mathrm{d}t} = v_x(t) = v_{0x} + \int_{t_0}^{t} a_x(t)\mathrm{d}t$,

therefore

$$\int_{x_0}^{x(t)} \mathrm{d}x = \int_{t_0}^{t} v_{0x}\mathrm{d}t + \int_{t_0}^{t}[\int_{t_0}^{t} a_x(t')\mathrm{d}t']\mathrm{d}t,$$

例 1 给定 $a_x(t)$ 以及在时间 t_0 时的初始条件 $x(t_0) = x_0$，$v_x(t_0) = v_{0x}$,写出 $v_x(t)$和 $x(t)$的表达式。

解：因为 $\frac{\mathrm{d}v_x(t)}{\mathrm{d}t} = a_x(t)$,所以

$$\int_{v_{0x}}^{v_x(t)} \mathrm{d}v_x = \int_{t_0}^{t} a_x(t)\mathrm{d}t, \text{即}$$

$$v_x(t) = v_{0x} + \int_{t_0}^{t} a_x(t)\mathrm{d}t. \tag{1}$$

因 $\frac{\mathrm{d}x(t)}{\mathrm{d}t} = v_x(t) = v_{0x} + \int_{t_0}^{t} a_x(t)\mathrm{d}t$,

所以

$$\int_{x_0}^{x(t)} \mathrm{d}x = \int_{t_0}^{t} v_{0x}\mathrm{d}t + \int_{t_0}^{t}[\int_{t_0}^{t} a_x(t')\mathrm{d}t']\mathrm{d}t,$$

i.e.

$$x = x_0 + v_{0x}(t - t_0) + \int_{t_0}^{t}\left[\int_{t_0}^{t} a_x(t')\mathrm{d}t'\right]\mathrm{d}t. \quad (2)$$

if a_x =const, we have

$$v_x(t) = v_{0x} + a_x(t - t_0) \quad (3)$$

and

$$x = x_0 + v_{0x}(t - t_0) + \frac{1}{2}a_x(t - t_0)^2. \quad (4)$$

If $a_x = a \cdot t^2$, we have

$$v_x(t) = v_{0x} + \frac{1}{3}a(t - t_0)^3 \quad (5)$$

And

$$x = x_0 + v_{0x}(t - t_0) + \frac{1}{12}a(t - t_0)^4. \quad (6)$$

Example 2 The river width is *d*, the flow speed of its water is proportional to the width and reaches maximum v_0 at the middle of the river. At two banks of the river the flow speed is zero. A boat moves with a constant velocity $\boldsymbol{u}$ perpendicular to the flow direction. Find the motion equation of the boat.

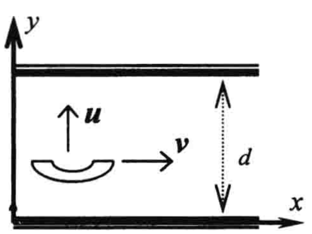

Fig.1.1.2 The motion of a boat in river

[Solution] At first take the earth surface as reference frame and x, y as two-dimensional

即

$$x = x_0 + v_{0x}(t - t_0) + \int_{t_0}^{t}\left[\int_{t_0}^{t} a_x(t')\mathrm{d}t'\right]\mathrm{d}t. \quad (2)$$

假设a_x =常数，可得

$$v_x(t) = v_{0x} + a_x(t - t_0) \quad (3)$$

以及

$$x = x_0 + v_{0x}(t - t_0) + \frac{1}{2}a_x(t - t_0)^2. \quad (4)$$

假设$a_x = a \cdot t^2$,可得

$$v_x(t) = v_{0x} + \frac{1}{3}a(t - t_0)^3 \quad (5)$$

以及

$$x = x_0 + v_{0x}(t - t_0) + \frac{1}{12}a(t - t_0)^4. \quad (6)$$

例 2 河宽为 *d*，水流速率与河宽成正比，并且在河中央达到最大值 v_0。在河的两岸水流速率为 0。一船以不变的速度 $\boldsymbol{u}$ 沿与水流方向垂直的方向运动，求船的运动方程。

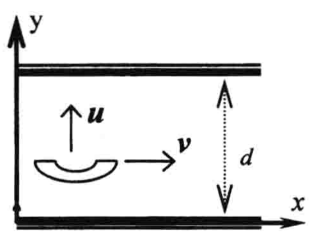

图 1.1.2 船在河中的运动

[解] 如图 1.1.2 所示，首先取地球表面为参照系，取 x，y 为二维

coordinates, as shown in Fig.1.1.2. According to the condition of flow speed, we have

$$v_x=\begin{cases}(\frac{v_0}{d/2})y, & \text{when } 0\le y\le\frac{d}{2} \quad (1)\\ (\frac{v_0}{d/2})(d-y), & \text{when } \frac{d}{2}\le y\le d \quad (2)\end{cases}$$

And $u=u_y=\text{const.}$ (3)

Calculate the motion equation at $0\le y\le\frac{d}{2}$:

Because $\mathrm{d}y/\mathrm{d}t=u_y=\text{const},$

therefore $\int_0^y \mathrm{d}y=u_y\int_0^t \mathrm{d}t$, i.e. $y=u_y t,$

or $t=y/u_y.$ (4)

Because $\mathrm{d}x/\mathrm{d}t=v_x=(\frac{v_0}{d/2})y,$

therefore

$$\int_0^x \mathrm{d}x=\frac{2v_0}{d}\int_0^t y\mathrm{d}t. \quad (5)$$

Differentiating Eq.(4) and substituting it in Eq.(5), we have

$$x=\frac{2v_0}{d}\int_0^y \frac{y\mathrm{d}u}{u_y}=\frac{v_0y^2}{u_yd}. \quad (6)$$

Continue to calculate the motion equation at

$d/2\le y\le d$:

Under this condition we have

$$\mathrm{d}y=u_y\mathrm{d}t \quad (4)'$$

And $\frac{\mathrm{d}x}{\mathrm{d}t}=(\frac{v_0}{d/2})(d-y),$ therefore

坐标。根据水流速率条件，我们得到：

$$v_x=\begin{cases}(\frac{v_0}{d/2})y, & 当\ 0\le y\le\frac{d}{2} \quad (1)\\ (\frac{v_0}{d/2})(d-y), & 当\ \frac{d}{2}\le y\le d \quad (2)\end{cases}$$

及 $u=u_y=$ 常数。 （3）

计算当$0\le y\le\frac{d}{2}$时的运动方程：

由于 $\mathrm{d}y/\mathrm{d}t=u_y=$常数，

因此$\int_0^y \mathrm{d}y=u_y\int_0^t \mathrm{d}t$，即 $y=u_y t,$

或 $t=y/u_y.$ （4）

由于$\mathrm{d}x/\mathrm{d}t=v_x=(\frac{v_0}{d/2})y,$

因此

$$\int_0^x \mathrm{d}x=\frac{2v_0}{d}\int_0^t y\mathrm{d}t. \quad (5)$$

将式(4)微分并代入式(5)，可得

$$x=\frac{2v_0}{d}\int_0^y \frac{y\mathrm{d}u}{u_y}=\frac{v_0y^2}{u_yd}. \quad (6)$$

接下来计算当$d/2\le y\le d$时的运动方程：

在该条件下有

$$\mathrm{d}y=u_y\mathrm{d}t \quad (4)'$$

以及$\frac{\mathrm{d}x}{\mathrm{d}t}=(\frac{v_0}{d/2})(d-y)$，因此

$$\int_{x_0}^{x} \mathrm{d}x = \frac{2v_0}{u_y}\int_{d/2}^{y}(1-\frac{y}{d})\mathrm{d}y, \text{ i.e.}$$

$$x - x_0 = \frac{2v_0 y}{u_y} - \frac{v_0 d}{u_y} - \frac{v_0}{u_y d}y^2 + \frac{v_0 d}{4u_y}. \quad (7)$$

Under the condition of $x = x_0$ at $y = d/2$, using Eq.(6), x_0 is obtained

$$x_0 = \frac{v_0}{u_y d}(\frac{d}{2})^2. \quad (8)$$

Substituting Eq.(8) into Eq.(7), the motion equation at $d/2 \le y \le d$ is obtained as follows:

$$x = \frac{2v_0 y}{u_y} - \frac{v_0 y^2}{u_y d} - \frac{v_0 d}{2u_y}. \quad (9)$$

1.2 Projectile motion

A projectile is an object in flight after being launched or thrown. First we assume that the distance traveled is much smaller than the radius of the earth so that the acceleration due to gravity remains constant. Secondly, we assume that the air resistance is negligible. We take the earth surface as reference frame and x, y as two-dimensional coordinates, as shown in Fig.1.2.1.

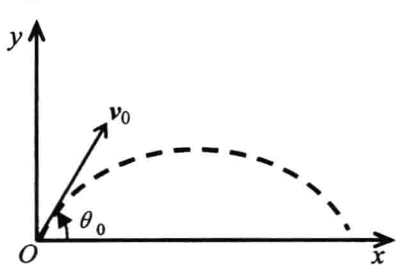

Fig 1.2.1 Projectile motion

$$\int_{x_0}^{x} \mathrm{d}x = \frac{2v_0}{u_y}\int_{d/2}^{y}(1-\frac{y}{d})\mathrm{d}y, \text{ 即}$$

$$x - x_0 = \frac{2v_0 y}{u_y} - \frac{v_0 d}{u_y} - \frac{v_0}{u_y d}y^2 + \frac{v_0 d}{4u_y}. \quad (7)$$

将 $y = d/2$ 时，$x = x_0$ 的条件应用到式(6)中，可得

$$x_0 = \frac{v_0}{u_y d}(\frac{d}{2})^2. \quad (8)$$

将式(8)代入式(7)得到当 $d/2 \le y \le d$ 时的运动方程：

$$x = \frac{2v_0 y}{u_y} - \frac{v_0 y^2}{u_y d} - \frac{v_0 d}{2u_y}. \quad (9)$$

1.2 抛体运动

抛体是指被发射或者抛出后的飞行中的物体。首先假设其运行的距离远小于地球的半径，所以其重力加速度保持不变；其次，假设空气阻力可忽略。取地球表面为参照系，x, y 为二维空间坐标轴，如图 1.2.1 所示。

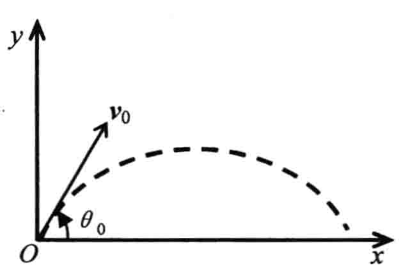

图 1.2.1 抛体运动

According to $\frac{\mathrm{d}v_x}{\mathrm{d}t}=0$, $\frac{\mathrm{d}v_y}{\mathrm{d}t}=-g$ and integrating $\int_{v_0\sin\theta}^{v_y}\mathrm{d}v_y=-\int_0^t g\mathrm{d}t$, we have

$$v_x=v_0\cos\theta_0, \quad (1.2.1)$$

$$v_y=v_0\sin\theta_0-gt, \quad (1.2.2)$$

where g is the gravitational acceleration.

Because $v_x=\frac{\mathrm{d}x}{\mathrm{d}t}$ and $v_y=\frac{\mathrm{d}y}{\mathrm{d}t}$, therefore,

$$\int_0^x\mathrm{d}x=\int_0^t v_x\mathrm{d}t=\int_0^t v_0\cos\theta_0\mathrm{d}t \quad \text{and}$$

$$\int_0^y\mathrm{d}y=\int_0^t v_y\mathrm{d}t=\int_0^t v_0\left(\sin\theta_0-gt\right)\mathrm{d}t$$

i.e.

$$x=v_0\cos\theta_0\cdot t, \quad (1.2.3)$$

and

$$y=v_0\sin\theta_0\cdot t-\frac{1}{2}gt^2. \quad (1.2.4)$$

Eliminating t using Equs. (1.2.3) and (1.2.4), an equation for the path or trajectory without parameter t can be found as follows:

$$y=x\tan\theta_0-\frac{gx^2}{2(v_0\cos\theta_0)^2}. \quad (1.2.5)$$

This is a parabola.

按$\frac{\mathrm{d}v_x}{\mathrm{d}t}=0$，$\frac{\mathrm{d}v_y}{\mathrm{d}t}=-g$，并求积分$\int_{v_0\sin\theta}^{v_y}\mathrm{d}v_y=-\int_0^t g\mathrm{d}t$，可得

$$v_x=v_0\cos\theta_0, \quad (1.2.1)$$

$$v_y=v_0\sin\theta_0-gt, \quad (1.2.2)$$

其中g为重力加速度。

因为$v_x=\frac{\mathrm{d}x}{\mathrm{d}t}$和$v_y=\frac{\mathrm{d}y}{\mathrm{d}t}$，所以

$$\int_0^x\mathrm{d}x=\int_0^t v_x\mathrm{d}t=\int_0^t v_0\cos\theta_0\mathrm{d}t \quad 及$$

$$\int_0^y\mathrm{d}y=\int_0^t v_y\mathrm{d}t=\int_0^t v_0\left(\sin\theta_0-gt\right)\mathrm{d}t$$

即

$$x=v_0\cos\theta_0\cdot t, \quad (1.2.3)$$

和

$$y=v_0\sin\theta_0\cdot t-\frac{1}{2}gt^2. \quad (1.2.4)$$

用式(1.2.3)及式(1.2.4)消去t，便可得不含参数t的路径方程或轨迹：

$$y=x\tan\theta_0-\frac{gx^2}{2(v_0\cos\theta_0)^2}. \quad (1.2.5)$$

这是一条抛物线。

1.3 Circular motion

When a body travels around the circle, it is called circular motion.

1. The description of circular motion with angular magnitude

The position of a body can also be determined by an angle relative to a reference axis ox, as shown in Fig.1.3.1.

Its position change can be expressed with angle displacement:

1.3 圆周运动

物体沿一圆周运动时，称为圆周运动。

1.用角量对圆周运动的描述

物体的位置也可以用相对于参考轴 ox 的角度来确定，如图 1.3.1 所示。

位置的改变可以用角位移来表示：

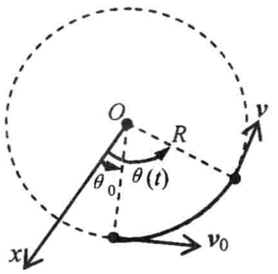

Fig 1.3.1 The circular motion

$$\Delta\theta(t)=\theta(t)-\theta_0, \qquad (1.3.1)$$

where θ_0 is the initial position relative to the reference axis at $t=t_0$. Put S to represent arc length passing by the body, and then

$$\Delta S = R\cdot\Delta\theta, \qquad (1.3.2)$$

Differentiating S with respect to t, we have

$$|\boldsymbol{v}|=\lim_{\Delta t\to 0}\left|\frac{\Delta S}{\Delta t}\right|=\left|\frac{\mathrm{d}S}{\mathrm{d}t}\right|=R\frac{\mathrm{d}\theta}{\mathrm{d}t}=R\omega, \qquad (1.3.3)$$

where ω is angular speed.

Differentiating $\boldsymbol{v}$ in Eq.(1.3.3) with respect to t, we obtain

$$\frac{\mathrm{d}|\boldsymbol{v}|}{\mathrm{d}t}=R\frac{\mathrm{d}\omega}{\mathrm{d}t}=R\alpha, \qquad (1.3.4)$$

where α is the angular acceleration of a body.

2. The tangent- and normal-acceleration in circular motion

Put $\boldsymbol{t}^0$ and $\boldsymbol{t}'^0$ represent the unit vectors at tangent direction, respectively, and

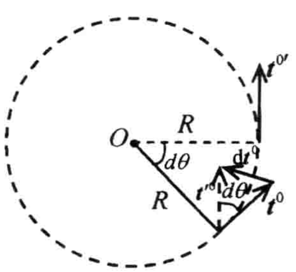

Fig 1.3.2 Circular motion

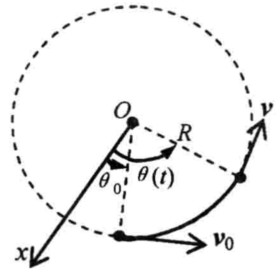

图 1.3.1 圆周运动

$$\Delta\theta(t)=\theta(t)-\theta_0, \qquad (1.3.1)$$

其中 θ_0 是 $t=t_0$ 时，相对于参考轴的初位置。用 S 表示物体所经过的弧长，则有

$$\Delta S = R\cdot\Delta\theta, \qquad (1.3.2)$$

S 对 t 微分，可得

$$|\boldsymbol{v}|=\lim_{\Delta t\to 0}\left|\frac{\Delta \boldsymbol{S}}{\Delta t}\right|=\left|\frac{\mathrm{d}\boldsymbol{S}}{\mathrm{d}t}\right|$$

$$=R\frac{\mathrm{d}\theta}{\mathrm{d}t}=R\omega, \qquad (1.3.3)$$

其中 ω 为角速度。

式(1.3.3)中的 $\boldsymbol{v}$ 对 t 微分，可得

$$\frac{\mathrm{d}|\boldsymbol{v}|}{\mathrm{d}t}=R\frac{\mathrm{d}\omega}{\mathrm{d}t}=R\alpha, \qquad (1.3.4)$$

式中 α 为物体的角加速度。

2. 切向和法向上的加速度

分别用 $\boldsymbol{t}^0$ 和 $\boldsymbol{t}'^0$ 代表切向上的两个单位矢量，因而

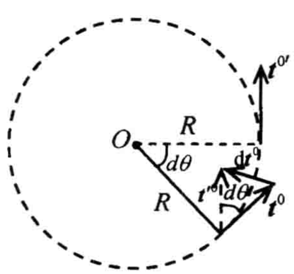

图 1.3.2 圆周运动