南京航空航天大學

论 文 集

（二〇〇五年）、 第2册

航空宇航学院

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航空宇航学院

012 系

013 系

航空宇航学院2005年学术论文清单（012）

序号	姓名	职称	单位	论文题目	刊物、会议名称	年、卷、期
1	杨岵生	正高	012	A Complete Boundary Integral Formulation for Compressible Navier-Stokes Equations	International Journal for Numerical Methods InFluids	2005.47.12
2	张　立 唐登斌	博士 正高	012 012	Numerical Simulation of Turbulent Spots in Inclined Open-channel Flow	Journal of Hydrodynamics Ser.B	2005.17.02
3	张　立 唐登斌	博士 正高	012 012	Nonlinear Evaluation of Turbulent Coherent Structure in Channel Flows	Modern Physics Letters B	2005.19.28-29
4	郭琳琳 唐登斌	博士 正高	012 012	带压力梯度的外平行流边界层稳定性研究	南京航空航天大学学报	2005.37.02
5	刘吉学 唐登斌	硕士 正高	012 012	On Nonparallel Stability of 3-D Compressible Boundary Layers	Modern Physics Letters B	2005.19.28-29
6	刘吉学 唐登斌	硕士 正高	012 012	机翼外平行边界层稳定性研究	南京航空航天大学学报	2005.37.06
7	朱　君 赵　宁 郑华盛	博士 正高 博士	012 012 012	HIGH ORDER LOCALIZED ENO SCHEMES ON UNSTRUCTURED MESHES FOR CONSERVATION LAWS	Modern Physics Letters B	2005.19.28-29
8	朱　君 赵　宁	博士 正高	012 012	非结构网格多区域动态加密及分布式并行计算	空气动力学报	2005.23.01
9	朱　君 赵　宁	博士 正高	012 012	一种MWENO格式的构造和应用	空气动力学报	2005.23.03
10	朱　君 赵　宁	博士 正高	012 012	二维非结构网格生成及其数值模拟	南京航空航天大学学报	2005.37.02
11	郑华盛 赵　宁	博士 正高	012 012	一个基于通量分裂的高精度MmB差分格式	空气动力学报	2005.23.01
12	郑华盛 赵　宁	博士 正高	012 012	一类高精度TVD差分格式及其应用	应用力学学报	2005.22.04
13	郑华盛 赵　宁	博士 正高	012 012	双曲型守恒律的一种高精度TVD差分格式	计算物理	2005.22.01
14	张学莹 赵　宁	博士 正高	012 012	基于体积分数形式的多介质流动数值模拟	南京航空航天大学学报	2005.37.04
15	陆志良 郭同庆 管　德	正高 博士 院士	012 012	复杂组合体颤振计算	第九届全国空气弹性学术交流会论文（优秀论文）	2005
16	郭同庆 陆志良	博士 正高	012 012	A CFD/CSD Model for Transonic Flutter	Computers Materials & Continua	2005.02.02
17	郭同庆 陆志良	博士 正高	012 012	Numerical Studies about Asymmetric Vortex Flow around a Slender Body At High Incidence	Modern Physics Letters B	2005.19.28-29
18	陈红全 舒　昌	正高 正高	012 012	An Efficient Implicit Mesh-Free Method to Solve Two-Dimensional Compressible Euler Equations	International Journal of Modern Physics C	2005.16.03
19	马志华 陈红全	博士 正高	012 012	SIMULATIONS OF TRANSONIC INVISCID FLOWS OVER AIRFOILS USING MESHFREE ADAPTIVE ALGORITHM	Modern Physics Letters B	2005.19.28-29
20	马志华 陈红全	博士 正高	012 012	Meshfree Adaptive Algorithm for sSolving Euler Equations On Structured Grid points	Transactions of Nanjing University of Aeronautics & Astronautics	2005.22.04
21	谢　强 陈红全	硕士 正高	012 012	一种基于块结构的非结构网格快速生成方法	南京航空航天大学学报	2005.37.02
22	江尖贤 陈红全	硕士 正高	012 012	多段翼型无网格算法及其布点技术研究	南京理工大学学报	2005.29.01
23	王江峰 伍贻兆	副高 正高	012 012	三维非结构混合网格高超声速流场并行算法	Transactions of Nanjing University of Aeronautics & Astronautics	2005.22.03
24	王江峰 伍贻兆	副高 正高	012 012	Prallel Computation of 3D Hypersonic Flows with Chemical Non-equilibrium Effects on Unstructured Hybrib Meshes	East West High Speed Flow Field Conferernce	2005
25	余奇华 王江峰 伍贻兆	硕士 副高 正高	012 012 012	高超声速化学非平衡绕流分布式并行算法	南京航空航天大学学报	2005.37.04
26	黄　达 吴根兴	副高 正高	012 012	Investigation of The Suitability for The Linear Superposition Model	Modern Physics Letters B	2005.19.28-29
27	黄　达 吴根兴	副高 正高	012 012	飞机大振幅滚转运动动态气动特性实验研究	空气动力学报	2005.23.02
28	黄　达 吴根兴	副高 正高	012 012	飞机偏航-滚转耦合运动非定常空气动力实验	南京航空航天大学学报	2005.37.04

29	陈永亮 沈宏良 刘　昶	博士 副高 正高	012 012 012	机翼摇晃预测与抑制	航空学报	2005.26.03
30	郭　龙 沈宏良	本科 副高	012 012	飞机容冰技术的研究进展	飞行力学	2005.23.01
31	顾蕴松 明　晓	副高 正高	012 012	应用PIV技术研究“零质量”射流的非定常流场特性	实验流体力学	2005.19.01
32	顾蕴松 明　晓	副高 正高	012 012	大迎角细长体非对称空间流场特性的试验研究	实验流体力学	2005.19.02
33	史志伟 宾　斌 李甘牛 明　晓	副高 硕士 副高 正高	012 012 012 012	非定常自由流中的翼面动态压力测量	流体力学实验与测量	2004.18.4
34	史志伟 明　晓	副高 正高	012 012	非定常自由来流对飞机模型气动特性的影响	空气动力学学报	2004.22.4
35	史志伟 明　晓	副高 正高	012 012	基于模糊聚类的模糊神经网络在非定常气动力建模中的应用	空气动力学报	2005.23.01
36	史志伟 符　澄 明　晓	副高 硕士 正高	012 012 012	俯仰振荡三角翼在非定常自由流中运动的实验	南京航空航天大学学报	2005.37.04
37	周春华 姚永峰	副高 硕士	012 012	A Fictitious Domain/Domain Decomposition Method and its Application	Modern Physics Letters B	2005.19.28-29
38	周春华	副高	012	不可压Navier-Stokes方程基于误差估算的网格自适应解法	计算力学学报	2005.22.06
39	周春华	副高	012	虚拟区域法及其在流体力学中的应用	中国科技大学学报	2005.35.04
40	夏　健	副高	012	Truncation error reduction method for Poisson equation	Modern Physics Letters B	2005.19.28-29
41	夏　健	副高	012	基于混合网格的三维Navier-Stokes方程并行算法	航空学报	2005.26.03
42	夏　健	副高	012	基于聚合多重网格方法的跨音速颤振的数值模拟	计算力学学报	2005.22.01
43	夏　健	副高	012	各向同性紊流能量衰减的大涡模拟	计算物理	2005.22.01
44	王焕瑾 高　正	副高 正高	012 0111	转换式高速直升机RD15方案	航空学报	2005.26.01
45	王焕瑾 高　正	副高 正高	012 0111	高速直升机方案研究	飞行力学	2005.23.01
46	王焕瑾 高　正	副高 正高	012 0111	Model Test of Unmanned High-Speed Helicopter RD15 Scheme	Proceeding of the 2nd International Basic Research Conference on Rotorcraft Technology	2005
47	唐智礼	中级	012	Multi-objective Shape Design in Aerodynamics Using Game Strategy	Transactions of Nanjing University of Aeronautics & Astronautics	2005.22.03
48	唐智礼	中级	012	Multi Criteria Robust Design Using Adjoint Methods and Game Strategies For Solving Drag Optimization Problems with Uncertainties	East West High Speed Flow Field Conferernce	2005
49	张震宇 明　晓	中级 正高	012 012	七孔探针的神经网络校准与制造偏差分析	实验力学	2005.20.03
50	王成鹏 张堃元 杨建军	中级 正高 中级	012 021 外地	Analysis of Flows in Scramjet Isolator Combined with Hypersonic Inlet (AIAA2005-0024)	43rd AIAA Aerospace Sciences Meeting and Exhibit	2005
51	王成鹏 张堃元 徐惊雷 金志光	中级 正高 副高 博士	012 021 021 021	Geometric effedts on aerodynamic performance of isolator (ISABE2005-1278)	17th International Symposium on Airbreathing Engines	2005
52	张红英 童明波 王跃全	初级 正高 博士	012 0112 0112	基于ADAMA的伞绳断裂分析	系统仿真学报	2005.17.10
53	张红英 童明波 吴剑萍	初级 正高 教授	012 0112 014	降落伞充气理论的发展	航天返回与遥感	2005.26.03

航空宇航学院2005年学术论文清单（0131）

序号	姓名	职称	单位	论文题目	刊物、会议名称	年、卷、期
1	孙 伟 胡海岩	博士 正高	0131 0131	Semi-active Vibration Control for Wind Aileron Using Stepped Magneto-rheological Damper	International Journal of Nonlinear Sciences and Numerical Simulation	2005.06.01
2	孙 伟 胡海岩	博士 正高	0131 0131	Design, Testing, and Modeling of Magnetorheological damper with stepped restoring torque	Proceedings of the Ninth International Conference on Electrorheological Fluids and Magnetorheological Suspensions	2005
3	孙 伟 胡海岩	博士 正高	0131 0131	基于多级磁流变阻尼器的操纵面振动半主动抑制——数值仿真与风洞试验	振动工程学报	2005.18.01
4	孙 伟 胡海岩	博士 正高	0131 0131	基于多级磁流变阻尼器的操纵面振动半主动抑制——阻尼器设计与试验建模	振动工程学报	2005.18.01
5	王 昊 胡海岩	博士 正高	0131 0131	Optimal Fuzzy Control of a Semi-active Suspension of a Full-vehicle Model Using MR Dampers	International Journal of Modern Physics B	2005.19.7-9
6	王 昊 胡海岩	博士 正高	0131 0131	Optimal fuzzy control of a semi-active suspension of a full-vehicle model using MR dampers	Proceedings of the Ninth International Conference on Electrorheological Fluids and Magnetorheological Suspensions	2005
7	王 昊 胡海岩	博士 正高	0131 0131	整车悬架的最优模糊半主动控制	振动工程学报	2005.18.04
8	于明礼 胡海岩	博士 正高	0131 0131	基于超声电机作动器的翼段颤振主动抑制	振动工程学报	2005.18.05
9	郭达蕾 胡海岩	博士 正高	0131 0131	Nonlinear Stiffness of a Magneto-Rheological Damper	Nonlinear Dynamics	2005.40.03
10	王在华 胡海岩	正高 正高	0131 0131	Hopf Bifurcation Control of Delayed Systems with Weak Nonlinearity via Delayed State Feedback	International Journal of Bifurcation and Chaos	2005.15.05
11	王在华 胡海岩	正高 正高	0131 0131	An Energy Analysis Of Amplitude Death of a Pair of Oscillators with delayed coupling	Proceedings of IDETC' 2005, ASME International Design Engineering Technical Conferences and Computers and Information in Engineering Conference	2005
12	赵永辉 胡海岩	副高 正高	0131 0131	Structural Modeling and Aeroelastic Analysis of a High-Aspect-Ratio Composite Wing	Chinese Journal of Aeronautics	2005.18.01
13	王怀磊 胡海岩	中级 正高	0131 0131	Bifurcation Analysis of a Delayed Dynamic System via Method of Multiple Scales and Shooting Technique	International Journal of Bifurcation and Chaos	2005.15.02
14	史友进 张曾锠	博士 正高	0131 0131	大柔性飞机着陆响应弹性机体模型	东南大学学报（自然科学版）	2005.35.04
15	张令弥 王 彤 田村幸雄	正高 中级 正高	0131 0131 外校	A Frequency-Spatial Domain Decomposition (FSDD) Technique for Operational Modal Analysis	The First International Conference on Operational Modal Analysis(IOMAC)	2005
16	张令弥 Rune Brincker Palle Andersen	正高 正高	0131 外校 外校	An Overview of Operational Modal Analysis:Major Development and Issues	The First International Conference on Operational Modal Analysis(IOMAC)	2005
17	张令弥 费庆国 郭勤涛	正高 博士 中级	0131 0131 051	Dynamic Finite Element Model Updating Using Meta-model and Genetic Algorithm	The 23rd International Modal Analysis Conference	2005
18	张令弥	正高	0131	复杂结构动态分析/试验/设计与计算机仿真技术的进展与应用	强度与环境	2005.32.01
19	费庆国 张令弥 王 彤	博士正高 中级	0131 0131 0131	用于结构计算仿真的神经网络样本点选择方法研究	地震工程与工程振动	2005.25.01
20	费庆国 张令弥	博士正高	0131 0131	Samplling Technique for Structural Dynamics Computational Simulation Based on Neural Networks	The 23rd International Modal Analysis Conference	2005
21	王 彤 张令弥 田村幸雄	中级 正高 正高	0131 0131 外校	An Operational Modal Analysis Method in Frequency and Spatial Domain	. Earthquake Engineering and Engineering Vibration	2005.04.02

22	王　彤 张令弥 郭勤涛	中级 正高 中级	0131 0131 051	基于奇异值分解的运行状态模态参数识别及其应用分析	江苏省博士后学术大会	2005
23	陈前 U Kruger	正高	0131 外校	Analysisi of Extended Partial Least Squares for Monitoring Large-Scale Processes	IEEE Transactions on Control Systems Technology	2005.13.05
24	陈前 K Worden J Rongong	正高	0131 外校 外校	Characterisation of Particle Dampers Using Restoring Force Surface Technique	The Proceedings of The Sixth Eurpean Conference on Structural Dynamics	2005
25	范颖峰 陈　前	硕士 正高	043 0131	基于案例推理分布式系统的研究与实现	计算机与现代化	2005.00.07
26	童国强 陈　前	硕士 正高	033 0131	基于数据融合技术的多模型状态监测与故障预报	工业控制与计算机	2005.00.06
27	顾宝栋 陈怀海 申　凡 许　锋	硕士 正高 副高 副高	0131 0131 0131 0112	力传感器动态标定及其精度分析研究	振动与冲击	2005.24.02
28	贺旭东 陈怀海	博士 正高	0131 0131	多点随机振动控制中的互谱矩阵研究	南京航空航天大学学报	2005.36.06
29	陆勇星 陈怀海 贺旭东	硕士 正高 博士	0131 0131 0131	电动车后避振疲劳损坏概率估计	振动、测试与诊断	2005.25.04
30	刘先斌 Liew.K.W	正高	0131 外校	Maximal Lyapunov Exponent of a Co-dimension Two Bifurcation System Excited by a white noise	Internation Journal of Non-linear Mechanics	2005.40
31	刘先斌 Liew.K.W	正高	0131 0131	On the Stability Properties of a Van der Pol-Duffing Oscillator That is driven by a Real Noise	Journal of Sound &Vibration	2005.285
32	纪国宜	副高	0131	Adaptive Walelet Analysisi of Non-Stationary Vibration Signal in Rotor Dynamics	International Journal of Precision Engineering and Manufacturing	2005.06.04
33	王　轲 张福祥	副高	0131 外校	钢丝网垫减振器结构动响应计算	兵工学报	2004.25.03

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A complete boundary integral formulation for compressible Navier–Stokes equations

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SUMMARY

A complete boundary integral formulation for compressible Navier–Stokes equations with time discretization by operator splitting is developed using the fundamental solutions of the Helmholtz operator equation with different order. The numerical results for wall pressure and wall skin friction of two-dimensional compressible laminar viscous flow around airfoils are in good agreement with field numerical methods. Copyright © 2004 John Wiley & Sons, Ltd.

KEY WORDS: boundary integral formulation; boundary element method; compressible Navier–Stokes equations

INTRODUCTION

The boundary integral equation method is closely related to classical Green's function method. In the classical Green's function method, one applies the definition of the adjoint operator to a special function, which satisfies certain suitable boundary conditions to get an explicit expression for the solution. however, for non-linear problems as well as for linear problems with geometry of practical interest, obtaining an expression for the Green's function may be hard. It would be desirable to develop computational models of handling complexity, but based on cause-and-effect concepts accessible to the applications engineer. Such a project is offered by the new generation of boundary integral methods now starting to emerge. For non-linear problems, as in the case under consideration, the non-linear terms are formally treated as non-homogeneous terms. This yield the presence of domains integrals. In this paper the methods for transformation of domain integrals into boundary integrals presented in References [1, 2] are extended further and a complete boundary integral formulation for compressible Navier–Stokes equations with time discretization by operator splitting is developed. The advantages

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of complete boundary integral formulation are impressive: no mesh is needed external to the body boundary; very complex geometries can be treated; computation time are vastly smaller; conventional computers can be employed. The numerical results for the surface pressure and skin friction of airfoil given by present method show good agreement with field numerical methods.

THEORETICAL BASIS

The non-dimensional compressible Navier–Stokes equations are as follows:

$$\begin{aligned} &\partial\rho/\partial t + \nabla\cdot(\rho\mathbf{u}) = 0 \\ &\rho\partial\mathbf{u}/\partial t + \rho(\mathbf{u}\cdot\nabla)\mathbf{u} + (\gamma-1)T\nabla\rho = (1/Re)\{\nabla^2\mathbf{u} + (1/3)\nabla(\nabla\cdot\mathbf{u})\} \\ &\rho\partial T/\partial t + \rho\mathbf{u}\cdot\nabla T + (\gamma-1)\rho T\nabla\cdot\mathbf{u} = (1/Re)\{(\gamma/Pr)\nabla^2 T + F(\nabla u)\} \end{aligned} \tag{1}$$

where pressure p, density ρ, velocity $\mathbf{u}=\{u_i\}$, temperature T are non-dimensionalized by the free stream values $\rho_\infty|\mathbf{u}_\infty|^2$, ρ_∞, $\mathbf{u}_\infty$, and $|\mathbf{u}_\infty|^2/c_v$, respectively. Re, M_∞, Pr, c_v and γ are the Reynolds number, the free stream mach number, the Prandtl number, the specific heat at constant volume and the ratio of specific heat, respectively.

For two-dimensional flow:

$$F(\nabla\mathbf{u}) = (4/3)\{(\partial u_1/\partial x)^2 + (\partial u_2/\partial y)^2 - (\partial u_1/\partial x)(\partial u_2/\partial y)\} + (\partial u_1/\partial x + \partial u_2/\partial y)^2$$

where u_1 and u_2 are the velocity components along x and y directions. For simplicity, only Direchlet boundary conditions are considered.

On far field boundaries:

$$\begin{aligned} &\rho = 1 \\ &T = T\infty = 1/\gamma(\gamma-1)M_\infty^2 \\ &\mathbf{u} = \mathbf{u}_\infty \end{aligned} \tag{2}$$

On the rigid boundaries of body:

$$\begin{aligned} &|\mathbf{u}| = 0 \\ &T = T_B = T\infty\{1 + ((\gamma-1)/2)M_\infty^2\} \quad \text{(the free stream total temperature)} \end{aligned} \tag{3}$$

Since we consider time dependent equations, the initial conditions have also to be added:

$$\rho(\mathbf{r},0) = \rho_0(\mathbf{r}), \quad \mathbf{u}(\mathbf{r},0) = \mathbf{u}_0(\mathbf{r}), \quad T(\mathbf{r},0) = T_0(\mathbf{r})$$

In order to establish the complete boundary integral formulation for compressible Navier–Stokes equations, a new variable $\sigma = \ln\rho$ is introduced. With this variable, the compressible Navier–Stokes equations become,

$$\begin{aligned} &\partial\sigma/\partial t + \nabla\cdot\mathbf{u} + \mathbf{u}\cdot\nabla\sigma = 0 \\ &\partial\mathbf{u}/\partial t - \mu\nabla^2\mathbf{u} + \beta\nabla\sigma = \psi(\sigma,\mathbf{u},T) \\ &\partial T/\partial t - \pi\nabla^2 T = \chi(\sigma,\mathbf{u},T) \end{aligned} \tag{4}$$

where $\mu = 1/Re$, $\pi = \gamma\mu/(RePr)$, $\beta = (\gamma - 1)T_B = (1/\gamma)[(\gamma - 1)/2 + 1/M_\infty^2]$

$$\psi(\sigma, \mathbf{u}, T) = -(\gamma - 1)[\nabla T + (T - T_B)\nabla\sigma] - (\mathbf{u} \cdot \nabla)\mathbf{u}$$
$$+ (1/Re)\{e^{-\sigma}(\nabla^2\mathbf{u} + 1/3\nabla(\nabla \cdot \mathbf{u})) - \nabla^2\mathbf{u}\}$$
$$\chi(\sigma, \mathbf{u}, T,) = -(\gamma - 1)T\nabla \cdot \mathbf{u} - \mathbf{u} \cdot \nabla T + \gamma/(RePr)(e^{-\sigma} - 1)\nabla^2 T + (1/Re)e^{-\sigma}F(\nabla\mathbf{u})$$

Using time discretization by operator splitting methods, we should obtain the following θ scheme [3] from Equation (4). In this paper we take $\theta = \frac{1}{4}$. For $n \geqslant 0$, starting from $\sigma^n, \mathbf{u}^n, T^n$ we solve

$$(\sigma^{n+1/4} - \sigma^n)/(\Delta t/4) + \nabla \cdot \mathbf{u}^{n+1/4} = -\mathbf{u}^n \cdot \nabla\sigma^n \quad (5a)$$

$$(\mathbf{u}^{n+1/4} - \mathbf{u}^n)/(\Delta t/4) - a\mu\nabla^2\mathbf{u}^{n+1/4} + \beta\nabla\sigma^{n+1/4} = \psi(\sigma^n, \mathbf{u}^n, T^n) + b\mu\nabla^2\mathbf{u}^n \quad (5b)$$

$$(T^{n+1/4} - T^n)/(\Delta t/4) - a\pi\nabla^2 T^{n+1/4} = \chi(\sigma^n, \mathbf{u}^n, T^n) \quad (5c)$$

$$(\sigma^{n+3/4} - \sigma^{n+1/4})/(\Delta t/2) + \mathbf{u}^{n+3/4}\nabla\sigma^{n+3/4} = -\nabla \cdot \mathbf{u}^{n+1/4} \quad (6a)$$

$$(\mathbf{u}^{n+3/4} - \mathbf{u}^{n+1/4})/(\Delta t/2) - b\mu\nabla^2\mathbf{u}^{n+3/4} - \psi(\sigma^{n+3/4}, \mathbf{u}^{n+3/4}, T^{n+3/4}) = a\mu\nabla^2\mathbf{u}^{n+1/4} - \beta\nabla\sigma^{n+1/4} \quad (6b)$$

$$(T^{n+3/4} - T^{n+1/4})/(\Delta t/2) - b\pi\nabla^2 T^{n+3/4} - \chi(\sigma^{n+3/4}, \mathbf{u}^{n+3/4}, T^{n+3/4}) = a\pi\nabla^2 T^{n+1/4} \quad (6c)$$

$$(\sigma^{n+1} - \sigma^{n+3/4})/(\Delta t/4) + \nabla \cdot \mathbf{u}^{n+1} = -\mathbf{u}^{n+3/4} \cdot \nabla\sigma^{n+3/4} \quad (7a)$$

$$(\mathbf{u}^{n+1} - \mathbf{u}^{n+3/4})/(\Delta t/4) - a\mu\nabla^2\mathbf{u}^{n+1} + \beta\nabla\sigma^{n+1} = \psi(\sigma^{n+3/4}, \mathbf{u}^{n+3/4}, T^{n+3/4}) + b\mu\nabla^2\mathbf{u}^{n+3/4} \quad (7b)$$

$$(T^{n+1} - T^{n+3/4})/(\Delta t/4) - a\pi\nabla^2 T^{n+1} = \chi(\sigma^{n+3/4}, \mathbf{u}^{n+3/4}, T^{n+3/4}) + b\pi\nabla^2 T^{n+3/4} \quad (7c)$$

with $0 < a,\ b < 1,\ a + b = 1$ for $\theta = \frac{1}{4}$:

$$a = (1 - 2\theta)/(1 - \theta) = 2/3, \quad b = \theta/(1 - \theta) = 1/3 \quad (8)$$

It can be seen that at both time step $n + 1/4$ and $n + 1$ all require the solution of two same systems of couple Equations (5a), (5b) and (7a), (7b). They can be written as:

$$\lambda\sigma + \nabla \cdot \mathbf{u} = g \quad (9)$$

$$\lambda\mathbf{u} - 2/3\mu\nabla^2\mathbf{u} + \beta\nabla\sigma = f \quad (10)$$

where $\lambda = 1/(\Delta t/4)$, g and f are known functions of σ, $\mathbf{u}$ and T at previous time step.

$$g = -\mathbf{u}^n \cdot \nabla\sigma^n + \lambda\sigma^n \quad \text{(for Equation (5a))}$$

$$= -\mathbf{u}^{n+3/4} \cdot \nabla\sigma^{n+3/4} + \lambda\sigma^{n+3/4} \quad \text{(for Equation (7a))} \tag{11}$$

$$f = \psi(\sigma^n, \mathbf{u}^n, T^n) + (1/3)\mu\nabla^2\mathbf{u}^n + \lambda\mathbf{u}^n \quad \text{(for Equation (5b))}$$

$$= \psi(\sigma^{n+3/4}, \mathbf{u}^{n+3/4}, T^{n+3/4}) + 1/3\mu\nabla^2\mathbf{u}^{n+3/4} + \lambda\mathbf{u}^{n+3/4} \quad \text{(for Equation (7b))} \tag{12}$$

Taking the divergence of both sides in Equation (10), we have

$$\lambda\nabla \cdot \mathbf{u} - 2/3\mu\nabla^2(\nabla \cdot \mathbf{u}) + \beta\nabla^2\sigma = \nabla \cdot f \tag{13}$$

On the other hand, Equation (9) yields

$$\nabla \cdot \mathbf{u} = g - \lambda\sigma \tag{14}$$

Combining Equations (13) and (14), we obtain

$$\lambda_1\sigma - \nabla^2\sigma = f_1 \tag{15}$$

with $\lambda_1 = \lambda^2/(\beta + (2/3)\lambda\mu)$, $f_1 = (\lambda g - \nabla \cdot f - (2/3)\lambda\mu\nabla^2 g)/(\beta + (2/3)\lambda\mu)$ in order to have a well posed problem in σ, it is necessary to have an additional boundary condition of type: $\sigma = k$ on body. After computing σ from Equation (15), $\mathbf{u}$ may be solved from Equation (10) which is now reduced to the same type as Equation (15) with the boundary condition (2)–(3) and then the value of k has to be calculated in order that Equation (9) is satisfied. Equations (5c) and (7c) already take the type as Equation (15). A linear variant of Equations (6b) and (6c) are obtained by substituting $\psi(\sigma^{n+1/4}, \mathbf{u}^{n+1/4}, T^{n+1/4})$ for $\psi(\sigma^{n+3/4}, \mathbf{u}^{n+3/4}, T^{n+3/4})$ in Equation (6b) and $\chi(\sigma^{n+1/4}, \mathbf{u}^{n+1/4}, T^{n+1/4})$ for $\chi(\sigma^{n+3/4}, \mathbf{u}^{n+3/4}, T^{n+3/4})$ in Equation (6c). After these substitutions, Equations (6b) and (6c) are also reduced to the type as Equation (15). Hence, the problem for the solution of compressible Navier–Stokes equations are now really reduced to the problems for the solution of a series of equation with the type of Equation (15). Equation (15) can be solved by following fundamental solution method. Multiplying Equation (15) with the fundamental solution H_0 of Helmholtz operator equation with order zero and integrate it with respect to domain Ω, we have

$$\int_\Omega (\lambda_1\sigma - \nabla^2\sigma) H_0 \, d\Omega = \int_\Omega f_1 H_0 \, d\Omega \tag{16}$$

where H_0 satisfies equation:

$$(\lambda_1 - \nabla^2) H_0 = \delta(\mathbf{r}) \tag{17}$$

Here δ is the impulse function, $\mathbf{r}$ is the position vector.
According to the Green theorem,

$$\int_\Omega H_0 \nabla^2\sigma \, d\Omega = \int_B (H_0 \partial\sigma/\partial n - \sigma\partial H_0/\partial n) \, dB + \int_\Omega \sigma\nabla^2 H_0 \, d\Omega \tag{18}$$

Substituting Equation (18) into Equation (16), we have

$$\int_\Omega \sigma(\lambda_1 - \nabla^2)H_0\,\mathrm{d}\Omega = \int_B (H_0\partial\sigma/\partial n - \sigma\partial H_0/\partial n)\,\mathrm{d}B + \int_\Omega f_1 H_0\,\mathrm{d}\Omega \tag{19}$$

where B are the boundaries of domain Ω. Substituting Equation (17) into the left-hand side of Equation (19) and considering the integrating properties of impulse function δ, we have

$$c\sigma(\mathbf{r}) = \int_B (H_0\partial\sigma/\partial n - \sigma\partial H_0/\partial n)\,\mathrm{d}B + \int_\Omega f_1 H_0\,\mathrm{d}\Omega \tag{20}$$

c is a coefficient, for smooth boundary $c = \frac{1}{2}$. In order to transform the domain integral $\int_\Omega f_1 H_0\,\mathrm{d}\Omega$ in Equation (20) into a series of boundary integrals, two new functions A_0 and H_1 are first introduced. $A_0 = f_1$, $H_0 = (\lambda_1 - \nabla^2)H_1$. Thus,

$$\int_\Omega f_1 H_0\,\mathrm{d}\Omega = \int_\Omega A_0(\lambda_1 - \nabla^2)H_1\,\mathrm{d}\Omega \tag{21}$$

According to the Green theorem

$$\int_\Omega A_0\nabla^2 H_1\,\mathrm{d}\Omega = \int_B (A_0\partial H_1/\partial n - H_1\partial A_0/\partial n)\,\mathrm{d}B + \int_\Omega H_1\nabla^2 A_0\,\mathrm{d}\Omega$$

Hence,

$$\int_\Omega f_1 H_0\,\mathrm{d}\Omega = \int_\Omega A_0(\lambda_1 - \nabla^2)H_1\,\mathrm{d}\Omega = \int_\Omega H_1(\lambda_1 - \nabla^2)A_0\,\mathrm{d}\Omega - \int_B (A_0\partial H_1/\partial n - H_1\partial A_0/\partial n)\,\mathrm{d}B \tag{22}$$

Similarly, if we set $A_1 = (\lambda_1 - \nabla^2)A_0$, $H_1 = (\lambda_1 - \nabla^2)H_2$, then the domain integral on the right-hand side of Equation (22) can also be rewritten as

$$\int_\Omega H_1(\lambda_1 - \nabla^2)A_0\,\mathrm{d}\Omega = \int_B (A_1\partial H_2/\partial n - H_2\partial A_1/\partial n)\,\mathrm{d}B + \int_\Omega H_2(\lambda_1 - \nabla^2)A_1\,\mathrm{d}\Omega \tag{23}$$

The procedure can be generalized by introducing two sequence of functions defined by the following recurrence formulae

$$A_{j+1} = (\lambda_1 - \nabla^2)A_j, \quad H_j = (\lambda_1 - \nabla^2)H_{j+1}, \quad j = 0, 1, 2, \ldots \tag{24}$$

Thus the domain integral $\int_\Omega f_1 H_0\,\mathrm{d}\Omega$ in Equation (20) can be expressed as the summations of infinite boundary integrals

$$\int_\Omega f_1 H_0\,\mathrm{d}\Omega = \sum_{j=0}^{\infty} \int_B (A_j\partial H_{j+1}/\partial n - H_{j+1}\partial A_j/\partial n)\,\mathrm{d}B \tag{25}$$

More generally, the jth order fundamental solution of Helmholtz equation Hj satisfies

$$(\lambda_1 - \nabla^2)H_j = H_{j-1}, \quad j = 1, 2, \ldots$$

and can be expressed as [4]

$$\begin{aligned} B_0 &= 1/(2\pi) \\ H_0 &= B_0 K_0(\lambda_1^{1/2} r) \\ B_j &= B_{j-1}/(2j\lambda_1) = B_0/((2\lambda_1)^j j!) \\ H_j &= B_j(\lambda_1^{1/2} r)^j K_j(\lambda_1^{1/2} r) = B_0 r^j K_j(\lambda_1^{1/2} r)/((2\lambda_1^{1/2})^j j!), \quad j = 1, 2, \ldots \end{aligned} \tag{26}$$

where $K_j(x)$ represents the second kind modified Bessel function of jth order. Substituting Equation (25) into Equation (20), a complete boundary integral formulation for Equation (15) is finally obtained and can be solved by well known boundary element method.

$$c\sigma(r) = \int_B (H_0 \partial\sigma/\partial n - \sigma \partial H_0/\partial n)\, \mathrm{d}B + \sum_{j=0}^{\infty} \int_B (A_j \partial H_{j+1}/\partial n - H_{j+1} \partial A_j/\partial n)\, \mathrm{d}B \tag{27}$$

Notice that the introduction of factor $(2\lambda_1^{1/2})^j j!$ into the denominator of expression H_j guarantees the rapid convergence of Equation (27) as j increase, especially for small Δt and the flow with higher Reynold's number because the smaller the Δt and the higher the Reynold's number, the larger the λ_1 will be.

NUMERICAL RESULTS AND CONCLUDING REMARKS

In 1987, a GAMM-workshop was organized to bring a small number of scientists highly concerned with the numerical solution of the compressible Navier–Stokes equations to calculate the assigned test problems [5] and to compare the results presented by the contributors each other. One of the assigned test problem was external 2D flow around a NACA0012 airfoil with Direchlet body boundary condition at $M_\infty = 0.8$, $Re = 73$ and 500, respectively, angles of attack $\alpha = 10°$. All the methods used by the contributors in Reference [5] were field method (finite differences, finite elements and finite volumes). In order to compare the results given by present complete boundary integral method with the results [6] given in Reference [5] the same test problems are calculated in this paper. One of the contributors [6] in Reference [5] solved the problems by using a new explicit Navier–Stokes code based on a combination of central finite differencing and rational Rung–Kutta time stepping. It is a more accurate field method. So it's results is used for comparison. Figures 1 and 2 show the laminar viscous wall pressure coefficient and skin friction coefficient on NACA 0012 airfoil calculated by present method and the results of field method given in Reference [6]. No field values (such as streamlines around airfoil, etc.) are compared because the results of the solution of boundary integral formulation are the values of variables on the wall boundary.

For the flow with $M_\infty = 0.8$ and $Re = 73$, if we take time step $\Delta t = 0.1$ then we have $\mu = 0.014$, $\beta = 1.26$, $\lambda = 40$, $\lambda_1 = 979.792$. The relationships between j and $H_j/(r^j K_j)$ are as follows:

j	$H_j/(r^j K_j)$
0	0.159

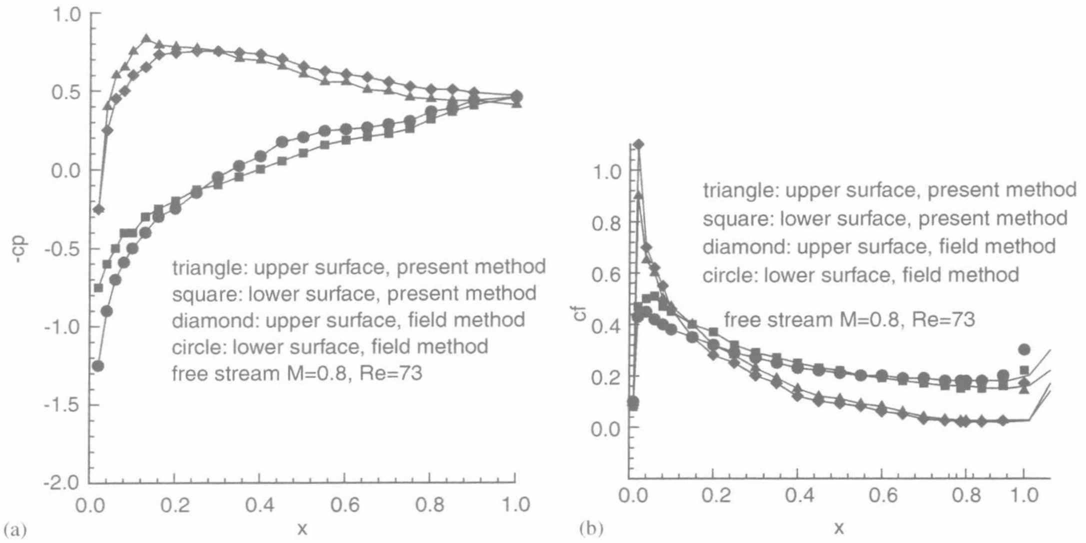

Figure 1. (a) Surface pressure coefficient cp; and (b) skin friction coefficient cf.

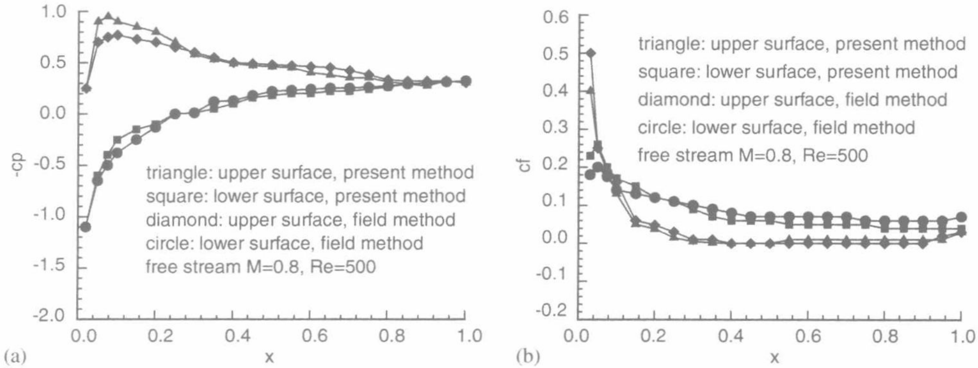

Figure 2. (a) Surface pressure coefficient cp; and (b) skin friction coefficient cf.

1	2.540×10^{-3}
2	2.029×10^{-5}
3	1.080×10^{-7}
4	4.313×10^{-10}
5	1.378×10^{-12}

The solutions are convergent at $j = 4$ and the maximum value of the relative difference of pressure coefficient between j and $j-1$ $(cp_j - cp_{j-1})/cp_j$ is less than 10^{-7}.

For the flow with $M_\infty = 0.8$ and $Re = 500$, if we take time step $\Delta t = 0.1$ then we have $\mu = 0.002$, $\beta = 1.26$, $\lambda = 40$, $\lambda_1 = 1218.58$. The relationships between j and $H_j/(r^j K_j)$ are as follows:

j	$H_j/(r^j K_j)$
0	0.159
1	2.277×10^{-3}
2	1.631×10^{-5}
3	7.787×10^{-8}
4	2.788×10^{-10}
5	7.988×10^{-13}

The solutions are convergent at $j=3$ and the maximum value of the relative difference of pressure coefficient between j and $j-1$ $(cp_j - cp_{j-1})/cp_j$ is less than 10^{-7}. The computing results show good agreement with the field method [6]. It can also be seen that even for low Reynold's number, the solution can still be converged at a small number of j. Obviously, the number of j for convergence will be reduced as the time step is further reduced.

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NUMERICAL SIMULATION OF TURBULENT SPOTS IN INCLINED OPEN-CHANNEL FLOW *

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ABSTRACT: The generation and evolution of turbulent spots in the open-channel flow are simulated numerically by using the Navier-Stokes equations. An effective numerical method with high accuracy and high resolution is developed. The fourth-order time splitting methods with high accuracy is proposed. Three-dimensional coupling difference methods are presented for the spatial discretization of the Poisson equation of pressure and Hemholtz equations of velocity, therefore, the fourth-order three-dimensional coupling central difference schemes are constituted. The fourth-order explicit upwind-biased compact difference schemes are designed to overcome the difficulty for the general higher-order central difference scheme which is inadaptable in the boundary neighborhood. The iterative algorithm and overall time marching is used to enhance efficiency. The method is applied in the numerical simulation of turbulent spots at various complex boundary conditions and flow domains. The generation and the developing process of turbulent spots are given, and the basic characteristics of turbulent spots are shown by simulating the evolution of the wall pulse in inclined open-channel flow.

KEY WORDS: turbulent spot, Navier-Stokes equation, numerical simulation, three-dimensional coupling difference scheme, wall pulse

1. INTRODUCTION

The local turbulent regions in the laminar are observed often in the natural transition process from laminar flow to turbulence in wall-bounded shear flows, and have been termed a turbulent spot. These spots randomly appear in different positions of the wall region, and grow and upraise gradually as they travel downstream, then mix each other and form full turbulence. Turbulent spots were observed first by Emmons on a free-surface water table, and then it is thought to be an important role in the transition process[1], which lead researchers to give attention widely, and made many research works by using the experiments[2,3] and the numerical methods[4]. Up to now, the some significant results on turbulent spots in boundary layer[5] such as Poiseuille[6] and Couette[7] flows have been obtained, whereas in complex geometric shape, such as the open-channel etc., only few investigation have been made, which remain to be further researched.

The behavior of the forming and developing of turbulent spots are investigated in a inclined open-channel flows. A group of high-order compact difference schemes for incompressible flow have been proposed to simulate effectively turbulent spots. The accuracy and resolution of this method are high, and don't require harshly boundary conditions[8] which appear in the general spectral method. It will be applied to boundary neighborhood in which it retains also high accuracy[9]. The method is very effective to the research deeply the multiple-scale turbulent spots in complex flow-field, and is used to simulate the stable flow and the evolutions process from wall impulse induced by ridged wall to turbulent spots in the channel. Finally, the basic feature and properties of turbulent spots are shown in the different results.

2. GOVERNING EQUATIONS

An inclined open-channel flow will be studied as shown in Fig. 1 where, normal direction (y), span-wise (z) and stream-wise (x) length are Ly, Lz and Lx , respectively, θ is oblique angle of the

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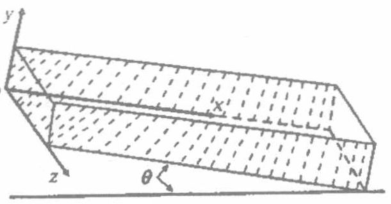

Fig. 1 Governing domain and coordinate

open-channel to the horizontal plane.

There is the gravitational fluid flow towards the right along the open-channel, which are governed by the non-dimensional Navier-Stokes equations written in the form

$$\nabla \cdot \boldsymbol{U} = 0 \qquad (1)$$

$$\frac{\partial \boldsymbol{U}}{\partial t} + (\boldsymbol{U} \cdot \nabla) = - \nabla p + \boldsymbol{F} + \frac{1}{Re} \nabla^2 \boldsymbol{U} \qquad (2)$$

where, $\boldsymbol{U} = (u, v, w)^{\mathrm{T}}$ is the velocity vector, $\boldsymbol{F}$ is gravity, p is the static pressure, $Re = \bar{u}h/\nu$, is Reynolds number, where h is the half-width of the open-channel, $\bar{u}$ is the streamwise velocity on the center of upper part of the open-channel, and ν is the kinematic viscosity.

For simplifying Eq. (2), we introduce symbols

$$\boldsymbol{F} = - \nabla g(- x\sin\theta + z\mathrm{con}\theta),$$

$$\Pi = p + g(- x\sin\theta + z\mathrm{con}\theta)$$

and Eq. (2) becomes

$$\frac{\partial \boldsymbol{U}}{\partial t} + (\boldsymbol{U} \cdot \nabla)\boldsymbol{U} = - \nabla \Pi + \frac{1}{Re} \nabla^2 \boldsymbol{U} \qquad (3)$$

3. NUMERICAL METHOD

In the section, the fourth-order compact difference methods would be presented for solving Eqs. (1) and (3).

On the base of the third-order time splitting methods[10], we have developed the fourth-order time splitting scheme, which is used in the temporal discretization of the Eq. (3), and the semi-discrete system are given by

$$\boldsymbol{U}' - 8\boldsymbol{U}^n + 6\boldsymbol{U}^{n-1} - 8\boldsymbol{U}^{n-2}/3 + \boldsymbol{U}^{n-3}/2 =$$
$$- \Delta t[4\boldsymbol{U}^n \cdot \nabla \boldsymbol{U}^n - 6\boldsymbol{U}^{n-1} \cdot \nabla \boldsymbol{U}^{n-1} +$$
$$4\boldsymbol{U}^{n-2} \cdot \nabla \boldsymbol{U}^{n-2} - \boldsymbol{U}^{n-3} \cdot \nabla \boldsymbol{U}^{n-3}] \qquad (4)$$

$$\boldsymbol{U}'' - \boldsymbol{U}' = - \Delta t \nabla p^{n+1} \qquad (5)$$

$$25\boldsymbol{U}^{n+1}/6 - \boldsymbol{U}'' = \Delta t \nabla^2 \boldsymbol{U}^{n+1}/Re \qquad (6)$$

where $\boldsymbol{U}'$, $\boldsymbol{U}''$ are the intermediate velocity fields.

Furthermore, we conduct spatial discretization to Eqs. (4)-(6). Corresponding to the fourth-order temporal splitting schemes, the fourth-order difference scheme will be used in the spatial discretization, and consistency of accuracy still remain.

The fourth-order upwind compact difference scheme is used first in the discretization of nonlinear terms on the right of the Eq. (4). In order to illustrate discretization scheme, we take $u\partial u/\partial x$ as an example.

Let

$$a_1 = (u + | u |)/2, \ a_2 = (u - | u |)/2$$

$$\delta_x^{\pm} u_i = \pm (u_{i\pm1} - u_i),$$

$$u_x^{\pm} = \frac{3u_{i\mp1} + 13u_i - 5u_{i+1} + u_{i\pm2}}{12h}$$

so

$$u \frac{\partial u}{\partial x} = a_1 \delta_x^- u_x^- + a_2 \delta_x^+ u^+ \ o(h^4)$$

where h is a step along x direction. Other nonlinear items can be obtained with the same method. Finally, let $\boldsymbol{a}_1$, $\boldsymbol{a}_2$ and $\boldsymbol{\delta}^{\pm}$ be three-dimensional vectors, and the fourth-order explicit upwind finite difference schemes on $\boldsymbol{U}'$ can be written as

$$\boldsymbol{U}' = 8\boldsymbol{U}^n - 6\boldsymbol{U}^{n-1} + 8\boldsymbol{U}^{n-2}/3 - \boldsymbol{U}^{n-3}/2 -$$
$$\Delta t[4\boldsymbol{a}_1 \cdot \boldsymbol{\delta}^- \boldsymbol{U}^- + \boldsymbol{a}_2 \cdot \boldsymbol{\delta}^+ \boldsymbol{U}^+)^n -$$
$$6(\boldsymbol{a}_1 \cdot \boldsymbol{\delta}^- \boldsymbol{U}^- + \boldsymbol{a}_2 \cdot \boldsymbol{\delta}^+ \boldsymbol{U}^+)^{n-1} +$$

$$4(a_1 \cdot \delta^- U^- + a_2 \cdot \delta^+ U^+)^{n-2} -$$

$$(a_1 \cdot \delta^- U^- + a_2 \cdot \delta^+ U^{+n-3}] \quad (7)$$

In order to make intermediate velocity field satisfy incompressible constraint, we take the divergence of Eq. (5), and obtain

$$\nabla \cdot U'' - \nabla \cdot U' = -\Delta t \nabla^2 \Pi^{n+1}$$

Let

$$\nabla \cdot U'' = 0$$

then

$$\nabla^2 \Pi^{n+1} = \frac{\nabla \cdot U'}{\Delta t} \quad (8)$$

For the discretization of Eq. (8), we propose the three-dimensional coupling difference method, and the difference scheme is given by

$$(\frac{12\delta_x^2 + \delta_x^2\delta_y^2 + \delta_x^2\delta_z^2}{h_x^2})\Pi^{n+1} +$$

$$(\frac{12\delta_y^2 + \delta_x^2\delta_y^2 + \delta_z^2\delta_y^2}{h_y^2})\Pi^{n+1} +$$

$$(\frac{12\delta_z^2 + \delta_z^2\delta_x^2 + \delta_z^2\delta_y^2}{h_z^2})\Pi^{n+1} =$$

$$12f(x,y,z) + (\delta_x^2 + \delta_y^2 + \delta_z^2)f(x,y,z) \quad (9)$$

where h_x, h_y and h_z are the step sizes in x, y and z, respectively, $f(x, y, z) = \nabla \cdot U/\Delta t$, δ^2 is difference operator defined by

$$\delta^2 \Pi_{i-1}^{n+1} = \Pi_{i-1}^{n+1} - 2\Pi_i^{n+1} + \Pi_{i+1}^{n+1}$$

This is a fourth-order three-dimensional coupling central difference scheme, which not only has high-order accuracy, but also has higher resolution and is applied to nearer boundary points. Finally, let

$$f = -\frac{ReU''}{\Delta t},\ k = \frac{Re\gamma_0}{\Delta t},$$

$$f' = kU^{n+1} - f$$

and Eq. (6) becomes

$$\nabla^2 U^{n+1} = f' \quad (10)$$

Eq. (10) is similar to Eq. (8), and is studied using the same method. Then it can be written as following discrete form

$$(\frac{12\delta_x^2 + \delta_x^2\delta_y^2 + \delta_x^2\delta_z^2}{h_x^2})U^{n+1} +$$

$$(\frac{12\delta_y^2 + \delta_x^2\delta_y^2 + \delta_z^2\delta_y^2}{h_y^2})U^{n+1} +$$

$$(\frac{12\delta_z^2 + \delta_z^2\delta_x^2 + \delta_z^2\delta_y^2}{h_z^2})U^{n+1} -$$

$$12kU^{n+1} + k(\delta_x^2 + \delta_y^2\delta_z^2)U^{n+1} =$$

$$12f(x,y,z) + (\delta_x^2 + \delta_y^2 + \delta_z^2)f(x,y,z) \quad (11)$$

This is also a fourth-order three-dimensional coupling difference scheme.

Equations (7), (9), and (11) have composed a equation systems for solving the Eqs. (1) and (3). The method not only have the high-order accuracy, but also have the higher resolution and stability. It could be used to simulate various steady and unsteady flows. The boundary conditions and initial value will be given according to actual subjects.

4. BASIC FLOW FIELD (STEADY SOLUTION)

We will simulate the steady flow in the open-channel as the basic flow field. The boundary conditions and initial value are given later.

4.1 *Boundary conditions on the wall*

Velocity boundary conditions

$$U_{y=0} = U_{z=0} = U_{z=z_l} = 0 \quad (12a)$$

Pressure boundary conditions

$$(\frac{\partial \Pi^{n+1}}{\partial y})_{y=0} = -(\frac{1}{Re}[4(\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 w}{\partial x \partial y})_{y=0}^{n} -$$

$$6(\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 w}{\partial z \partial y})_{y=0}^{n-1} +$$