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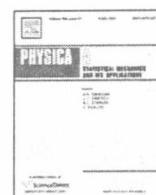
目 录 (博士)

序号	姓 名	职 称	单位	论 文 题 目	刊物、会议名称	年、卷、期	类别
1	林敏	博士	091	Scale-Free Network Provides an Optimal Pattern for Knowledge Transfer	Physica A : Statistical Mechanics and its Applications	2010 (389) : 473-480	SCI 源期刊
2	林敏	博士	091	Spatial evolutionary game with bond dilution	Physica A : Statistical Mechanics and its Applications	2010 (389) :1753-1758	SCI 源期刊
3	林敏	博士	091	研发团队知识交流网络的小世界特性分析与证明	情报学报	2010,29 (4)	CSSCI
4	彭浪	博士	091	经济危机下中国汽车市场的趋势研究	经济问题	2010.1	
5	万青	博士	093	国有企业知识密集型服务部门创新激励研究	科学学研究	2010年28 卷 8 期	
6	姚延婷	博士	093	农业温室气体排放现状及低碳农业发展模式研究	科技进步与对策	2010 年27 卷22 期	
7	张小雪	博士	093	规模报酬递增、要素供给结构与经济增长研究	数理统计与管理	2010 年29 卷 1期	
8	邹治	博士	093	关于构建高校毕业生就业预警系统理论模型的探讨	高教探索	2010 年2 期	
9	邹治	博士	093	非营利组织:政府应对高校毕业生就业困境的路径选择	中国行政管理	2010 年 3 期	
10	李宪印	博士	093	高等教育大众化下高校毕业生结构性失业及其对策	现代教育科学	2010 年 2 期	
11	张敏	博士	093	中小企业人才聚集效应的虚拟化实现	管理学报	2010 年 7 卷 3 期	
12	张敏	博士	093	抗震救灾过程中人才聚集效应探析——以汶川8.0级地震和玉树7.1级地震为例	华南地震	2010年30 卷 2 期	
13	吴中伦	博士	093	中国教育人力资本非均衡性的测度分析	现代教育管理	2010 年3 期	
14	吴中伦	博士	093	私营企业劳动关系信任治理的实证研究:心理契约视角	统计与决策	2010 年 7 期	
15	吴中伦	博士	093	组织间信任、渠道伙伴关系与企业绩效的实证研究	工业技术经济	2010 年29 卷 5 期	
16	沈春光	博士	093	区域R&D经费投入结构的灰色优化预测方法	工业技术经济	2010 年29 卷 8 期	
17	沈春光	博士	093	区域科技人才创新能力评价指标体系与方法研究	科学学与科学技术管理	2010 年 2 期	
18	黄绍益 周德群	博士	091	Robust decentralized adaptive output feedback fuzzy controller design and application to AHS	Journal of Systems Engineering and Electronics	2010	
19	黄益绍 周德群	博士	091	一类MIMO非线性大系统基于 H_{∞} 跟踪的分散自适应输出反馈模糊控制	控制与决策	2010, (11)	
20	陈小毅 周德群	博士	091	关于我国煤电产业布局重构的战略思考	科技管理研究	2010, (10)	
21	陈小毅 周德群	博士	091	中国煤炭产业市场集中度的实证研究	当代财经	2010, (02)	
22	陈小毅 周德群	博士	091	中国煤炭产业地理集中的变动趋势及影响因素	系统工程	2010, (07)	

23	鞠可一 周德群 王群伟	博士	091	中国能源消费结构与能源安全关联的实证分析	资源科学	2010, (09)	
24	鞠可一 周德群 吴君民	博士	091	混合概念格在案例相似性度量中的应用	控制与决策	2010, (07)	
25	邢小军 孙利娟 周德群	博士	091	安徽省参与泛长三角区域发展分工的产业选择	华东经济管理	2010, (06)	
26	邢小军 孙利娟 周德群	博士	091	基于REEES模型的区域发展目标研究	价格月刊	2010, (04)	
27	邢小军 孙利娟 周德群	博士	091	基于Malmquist-DEA模型的中外农业生产效率比较研究	商业经济与管理	2010, (09)	
28	侯建敏 周德群	博士	091	Agent-Based Modeling of Distributed Energy System	IEEE International Conference on Grey Systems and Intelligent Services	2010	
29	侯健敏 周德群	博士	091	分布式能源系统的复杂性特征分析	中国矿业	2010, (02)	
30	王向青 周德群	博士	091	中国能源需求影响因素的主成分分析	价格月刊	2010, (05)	
31	陈超 周德群	博士	091	Resource Emergency Dispatching Mathematical Model under Transport Capacity Constraints	Conference on Grey Systems and Intelligent Services	2010	
32	余德建 周德群	博士	091	A Weighted Grey Target Theory-based Strategy Model for Emergency Facility Location	IEEE International Conference on Grey Systems and Intelligent Services	2010	
33	王群伟 周德群	博士	091	我国全要素能源效率的测度与分析	管理评论	2010, (03)	
34	王群伟 周德群	博士	091	中国全要素二氧化碳排放绩效的区域差异——考虑非期望产出共同前沿函数的研究	财贸经济	2010, (09)	
35	王群伟 周鹏	博士	091	我国二氧化碳排放绩效的动态变化、区域差异及影响因素	中国工业经济	2010, (01)	
36	高岩 周德群	博士	091	基于指数型新弱化缓冲算子的能源需求预测	管理学报	2010, (08)	
37	高大伟 周德群	博士	091	国际贸易与中国全要素能源效率的实证研究	统计与决策	2010, (08)	
38	高大伟 周德群	博士	091	中国制造业全要素能源效率研究	价格月刊	2010, (01)	
39	高大伟 周德群	博士	091	国际贸易、R&D技术溢出及其对中国全要素能源效率的影响	管理评论	2010, (08)	
40	朱佩枫 周德群	博士	091	技术差距对西部制造业吸收企业跨区直接投资技术溢出的影响	科学学与科学技术管理	2010, (01)	
41	杨阳 周德群	博士	091	基于灰色聚类的高校上市公司动态竞争力评价	价值工程	2010, (29)	
42	刘浩	博士	091	无约束优化的二次三对角插值直接搜索法	数学物理学报	2010. 30. 4	
43	王海军	博士	091	A CONVEX APPROXIMATION METHOD FOR LARGE SCALE LINEAR INEQUALITY CONSTRAINED MINIMIZATION	Asia-Pacific Journal of Operation	2010. 27. 1	

44	徐斌 李南	博士	091	求解灰色双层线性规划模型的交互式模糊算法	系统工程学报	2010, 25 (2)	
45	田刚	博士	091	江苏省物流货运量灰色预测及灰色关联	价格月刊	2010.4	
46	田刚	博士	091	基于效率改善的物流低碳化模式探讨	价格月刊	2010.1	
47	董锋	博士	093	中国能源强度的影响因素分析—基于协整检验和因素分解	系统工程	2010,28(10)	
48	董锋	博士	093	技术进步对能源效率的影响—基于考虑环境因素的全要素生产率指数和面板计量分析	科学学与科学技术管理	2010(6)	
49	董锋	博士	093	技术进步、产业结构和对外开放程度对中国能源消费的影响—基于灰色关联分析-协整检验	中国人口·资源与环境	2010,20(6)	
50	董锋	博士	093	资源型城市可持续发展水平评价—以黑龙江省大庆市为例	资源科学	2010,32(8)	
51	董锋	博士	093	“苏州模式”、“温州模式”的协整分析与比较	华东经济管理	2010,24(11)	
52	李婧	博士	093	中国区域创新生产的空间计量分析—基于静态与动态空间面板模型的实证研究	管理世界	2010(7)	
53	施杨	博士	091	研发团队知识交流网络中心性对知识扩散影响及其实证研究	情报理论与实践	2010.33.4	
54	施杨	博士	091	基于社会关系网络的团队知识扩散影响因素探析	科技进步与对策	2010.27.14	
55	施杨	博士	091	基于灰色多层次评价的研发团队绩效测度模型及应用	企业经济	2010.12	
56	郭研	博士	091	多项目多资源均衡问题及其基于Pareto的向量评价微粒群算法	控制与决策	2010年 25卷 5 期	
57	郭研	博士		基于因子分析的泛在网络实施条件评价方法研究	工业技术经济	2010 年 29 卷 4 期	
58	石风光	博士	096	中国区域经济差距收敛性的协整检验	管理评论	2010. 4	
59	周明	博士	096	古诺竞争的选址模型研究	数学的实践与认识	2010. 40. 20	
60	李遂江 江可申	博士	095	高技术产业科技创新传导能力研究——以江苏省为例	科技进步与对策	2010 年27 卷 7 期	
61	李遂江 江可申	博士	095	新兴产业与中国产业结构优化升级有序度研究	科学学与科学技术管理	2010 年 31 卷 12 期	
62	李遂江 江可申	博士	095	高技术产业研发创新效率与全要素生产率增长	科学学与科学技术管理	2010 年31 卷 11 期	
63	卢山	博士	095	连云港科技资源配置与行业R&D有效性研究	科技进步与对策	2010. 27. 16	
64	卢山 江可申	博士	095	农户金融服务认知、融资需求与借贷行为研究——基于连云港市农村金融的多维分析	农村金融研究	2010 年 卷 4 期	
65	田泽永 江可申	博士	095	FDI与江苏民营制造业全要素生产率的改进——基于Malmquist生产率指数	中国科技论坛	2010 年 3 期	
66	田泽永 江可申	博士	095	FDI溢出效应对民营经济的影响——基于省际面板数据的实证研究	研究与发展管理	2010 年22 卷 1 期	
67	汪本强 江可申	博士	095	中国航空工业企业融资机制的演变路径与资源易得性	软科学	2010 年 24 卷 4 期	
68	白俊红 江可申	博士	095	中国地区研发创新的技术效率与技术进步	科研管理	2010 年 31 卷 6 期	
69	曾小舟 江可申	博士	095	我国航空网络枢纽机场中心化水平比较分析	系统工程	2010 年 28 卷 9 期	

70	王正新	博士	091	Integration of Regional Science and Technology Resources Based on GM(1, 3)	JOURNAL OF GREY SYSTEM	2010 年22 卷3期	
71	王正新	博士	091	缓冲算子的光滑性	系统工程理论与实践	2010 年30 卷9期	
72	王正新	博士	091	基于灰熵优化的加权灰色关联度	系统工程与电子技术	2010 年32 卷4 期	
73	王正新	博士	091	基于累积前景理论的多指标灰关联决策方法	控制与决策	2010 年25 卷2 期	
74	熊萍萍	博士	091	The Optimization of Time Response Function in Non-Equidistant Verhulst Model	JOURNAL OF GREY SYSTEM	2010 年22 卷2期	
75	王育红	博士	091	An approach to increase prediction precision of GM(1,1) model based on optimization of the initial condition	Expert Systems with Applications	2010 年37期	
76	宋捷	博士	091	基于灰色聚类的群决策方法研究	控制与决策	2010 年25 卷10 期	
77	宋捷	博士	091	正负靶心灰靶决策模型	系统工程理论与实践	2010 年30 卷10 期	
78	宋捷	博士	091	基于强“奖优罚劣”算子的多指标灰靶决策模型	系统工程与电子技术	2010 年32 卷6 期	
79	王庆丰	博士	091	基于Moore值的中国就业结构滞后时间测算	管理评论	2010 年22 卷7期	
80	崔杰	博士	091	一类新的强化缓冲算子及其数值仿真	中国工程科学	2010 年12 卷2期	
81	崔杰	博士	091	基于矩阵条件数的NGM(1, 1, k)模型病态性研究	控制与决策	2010 年25 卷2 期	
82	刘成斌	博士	091	区间直觉模糊动态规划方法	控制与决策	2010 年25 卷1 期	
83	姚奕	博士	091	中国经济与环境系统协调发展的实证分析	统计与决策	2010. 1	
84	汤少梁	博士	091	基于电子商务的物流企业客户保持研究	工业技术经济	2010. 4	
85	王强	博士	095	不完全成本信息下差异产品厂商古诺竞争博弈分析	运筹与管理	2010. 8	
86	冯忠垒	博士	095	事前被许可企业自主创新投资决策研究-基于技术基础和资金机会成本的分析	科技进步与对策	2010. 1	
87	冯忠垒	博士	095	许可租金分配条件下事前被许可企业自主创新投资决策研究	科技进步与对策	2010. 4	
88	石盛林	博士	095	我国县域金融发展水平收敛性问题的实证研究	中央财经大学学报	2010. 12	
89	郑兵云	博士	095	转型期中国工业全要素生产率与效率	数理统计与管理	2010. 29. 3	



Scale-free network provides an optimal pattern for knowledge transfer

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ABSTRACT

We study numerically the knowledge innovation and diffusion process on four representative network models, such as regular networks, small-world networks, random networks and scale-free networks. The average knowledge stock level as a function of time is measured and the corresponding growth diffusion time, τ is defined and computed. On the four types of networks, the growth diffusion times all depend linearly on the network size N as $\tau \sim N$, while the slope for scale-free network is minimal indicating the fastest growth and diffusion of knowledge. The calculated variance and spatial distribution of knowledge stock illustrate that optimal knowledge transfer performance is obtained on scale-free networks. We also investigate the transient pattern of knowledge diffusion on the four networks, and a qualitative explanation of this finding is proposed.

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1. Introduction

Knowledge transfer provides opportunities for interpersonal cooperation. It stimulates the creation of new knowledge and contributes to the innovation ability in organization. Knowledge system is open and self-organized that knowledge evolves and diffuses through innovation and communication. Knowledge transfer has received much attention from management scholars, and it has become increasingly important in organizations. A growing body of empirical evidence indicates that organizations that are able to transfer knowledge effectively from one unit to another are more productive and more likely to survive than those that are less adept at knowledge transfer [1]. The ability to transfer knowledge represents a distinct source of competitive advantage for organizations over other institutional arrangements such as markets [2]. This knowledge-based theory of the firm views organizations as social communities specializing in efficient knowledge creation and transfer [3]. Although organizations are able to realize remarkable increases in performance through knowledge transfer, successful knowledge transfer is difficult to achieve [4,5]. Researchers have concluded key elements that affect knowledge transfer, such as the stickiness of knowledge [6,7], the absorptive capacity of receivers [8], and intermediary and context for knowledge transfer [9,10]. Knowledge transfer in organizations occurs through a variety of mechanisms. These include personnel movement [11,12], training [13,14], communication [15–17], technology transfer [18], and other forms of inter-organizational relationships [19,20].

More recently, there has been a surge of research activity on various aspects of complex networks since some important features of real networks were successfully explained by simple model networks [21–32]. Complex networks, which can well mimic the interactions between individuals in real systems, provide a substrate for the researchers to study many interesting dynamical processes. Epidemic spreading [33–37] as well as classical and quantum diffusion [38,39] on complex

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networks have been extensively studied and served as a first step toward the complete understanding of the impact of network structure on the dynamical process. In light of this, close attentions are paid to the knowledge transfer on networks, which can be considered as a diffusion process. Ingram and Roberts [40] described how dense friendship networks affected the performance of Sydney hotels. One explanation for the observed effect is that friendship networks promote knowledge transfer, allowing managers facing similar market conditions to learn from each other's experience. In Reagans and Zuckerman's analysis of corporate research and development teams [41], they described how interactions among scientists with non-overlapping networks outside of their team improved productivity. Instead of examining how the structure of social relations affected performance, Tsai [42] found that the most innovative and profitable business units were central. Some researchers find that the performance of knowledge diffusion in organizations exhibits "small-world" properties [43,44]. In all these cases, knowledge transfer was assumed to be the causal mechanism linking network structure to performance.

Although network analysis method has been used in the study of knowledge transfer, we still cannot obtain a clear picture about how network structures influence the knowledge transfer performance, and whether there exists certain network structure that can facilitate and promote knowledge transfer more efficiently. Motivated by these considerations, in this article, we numerically study a simple model of knowledge innovation and diffusion process on different types of network models. The important topological properties of the real-life population structures are well depicted by these network models. Regular network, where every agent is directly connected to the same small number of his nearest neighbors, is the simplest model representing the geometry of the social system. Random networks, where any pairs of agents in the population are connected with the same probability, describe the existence of disorder in real population. On the other hand, many real networks also exhibit special topological characteristics: a small average distance as random networks, large clustering coefficient as regular networks (small-world property) and a power-law degree distribution (scale-free property). The models introduced by Barabási and Albert (BA) [26] and by Watts and Strogatz (WS) [25] can well mimic these two properties separately. The growth and diffusion of knowledge are examined in details on the four network models, and we found that the scale-free networks provide an optimal pattern for knowledge transfer process.

This paper is organized as follows. In Section 2, the models of networks and knowledge transfer process are introduced. In Section 3, We define the computed statistical quantities. The simulation results are given in Section 4. Finally in Section 5 we give a discussion and summarize the article.

2. The model

2.1. Knowledge evolution

Consider N agents existing on an undirected, connected graph $G(S, \Gamma)$, where $S = \{1, \dots, N\}$ is a finite set of nodes (agents) and $\Gamma = \{\Gamma_i, i \in S\}$ is the list of connections. Specially, $\Gamma_i = \{j \in S - \{i\} | d(i, j) = 1\}$, where $d(i, j)$ is the length of the shortest path from node i to node j on the graph. Only neighboring agents can interact. Each agent $i \in S$ is characterized by a knowledge stock which evolves over time as the agent innovates and receives knowledge broadcast by other agents. Formally, let $v_i(t)$ denote agent i 's knowledge stock at time t .

(1) Knowledge innovation

If agent i innovates at time t , his knowledge increases according to

$$v_i(t+1) = (1 + \beta_i)v_i(t) \quad (1)$$

with $\beta_i > 0$ the innovative ability of agent i . Innovative abilities are independently and identically distributed among agents.

(2) Knowledge absorption

Absorption involves an individual incorporating new knowledge and is assumed to be proportional to the difference in knowledge stock between broadcaster and recipient. The innovating agent i broadcasts to every $j \in \Gamma_i$. When a broadcast takes place, all the recipients absorb part of the knowledge that is transferred. Formally, when i broadcasts, j 's knowledge stock increases immediately according to

$$v_j(t+1) = \begin{cases} v_j(t) + \alpha_j\{v_i(t+1) - v_j(t)\}, & \text{if } v_i(t+1) > v_j(t), \\ v_j(t), & \text{otherwise} \end{cases} \quad (2)$$

with α_j the absorptive capability of agent j . There is no consequent loss of knowledge to agent i . Since recipient can only partially assimilate received knowledge, $\alpha_j < 1$ captures an important aspect of knowledge transfer [45].

We should note that, the value of knowledge stock is a scalar measure of the knowledge amount. When an agent innovates, which can be considered as self-study, his knowledge stock will increase according to his primary amount of knowledge (Eq. (1)). It means that, although an agent innovates (self-study), he cannot broadcast if his knowledge stock is less than that of his neighbor (Eq. (2)).

The value of β_i and α_i is obtained as

$$\alpha_i = \underline{\alpha} + a_i r, \quad \beta_i = b_i \bar{\beta}, \quad (3)$$

where $\underline{\alpha}$ and $\bar{\beta}$ are separately lower bound of absorptive capability and upper bound of innovative ability. a_i and b_i are random numbers uniformly distributed in $(0, 1]$, and r is a global constant. We set $r = 0.2$ in this paper.

2.2. Network model

The simulation of knowledge transfer is run on the four representative network models: regular network, random network, small-world network and scale-free network.

Regular network: The regular network used here is a one-dimensional lattice consisting of N nodes with periodic boundary condition and coordination number $2z$. It is easily found that every node has the same degree $2z$.

Random network: According to the Erdős and Rényi model [24], we start with N nodes and connect every pair of nodes with probability p , creating a graph with approximately $pN(N-1)/2$ edges distributed randomly. In order to keep the average degree the same with the other three networks, we choose $p = 2z/(N-1)$, and only the connected networks are considered.

Small-world network: The algorithm of the WS model is the following [25]: (1) start with a ring lattice with N nodes in which every node is connected to its first $2z$ neighbors. In order to have a sparse but connected network at all times, consider $N \gg 2z \gg \ln(N) \gg 1$. (2) Randomly rewire each edge of the lattice with probability P such that self-connections and duplicate edges are excluded. This process introduces PNz long-range edges. We choose $P = 0.2$ throughout this article.

Scale-free network: The scale-free network is generated according to the BA's algorithm [26]: (1) *Growth*: Starting with a small number (m) of nodes, at every time step, we add a new node with $z (< m)$ edges that link the new node to z different nodes already present in the system. (2) *Preferential attachment*: When choosing the nodes to which the new node connects, we assume that the probability Π that a new node will be connected to node i depends on the degree k_i of node i , such that

$$\Pi(k_i) = \frac{k_i}{\sum_j k_j}. \quad (4)$$

After t time steps this procedure results in a scale-free network with $N = t + m$ nodes and zt edges, whose average degree is approximately $2z$.

3. Statistics

We measure the performance of knowledge transfer in three aspects: the growth and diffusion of knowledge, the adequacy of knowledge transfer, and the spatial distribution of knowledge.

3.1. The growth and diffusion of knowledge

The average knowledge stock level at time t is defined as

$$\bar{v}(t) = \frac{1}{N} \sum_{i \in S} v_i(t), \quad (5)$$

which is a major concern in the study of knowledge transfer. To characterize knowledge diffusion, we define the growth diffusion time τ associated with the average knowledge stock level by the condition

$$\bar{v}(t = \tau) = C, \quad (6)$$

where the constant C indicates certain average knowledge stock level.

On the other hand, the knowledge growth rate at time t is measured by

$$\rho(t) = \frac{\bar{v}(t)}{\bar{v}(t-1)} - 1, \quad (7)$$

which directly illustrates the speed of knowledge growth.

3.2. The variance coefficient of knowledge

The adequacy of knowledge transfer is judged by the variation of knowledge stocks. To some extent, one of the purposes of knowledge transfer is to narrow the gap of knowledge stocks among agents. The variance can be used to measure the degree of discrepancy in knowledge stocks,

$$\sigma^2(t) = \frac{1}{N} \sum_{i \in S} v_i^2(t) - \bar{v}^2(t). \quad (8)$$

However, since the value of variance will increase as the average knowledge stock level grows, we use the coefficient of variance instead of variance,

$$c(t) = \sigma(t)/\bar{v}(t). \quad (9)$$

The value of variance coefficient ranges in value from 0 to 1. Large value of variance coefficient corresponds to large gap of knowledge stocks among agents. On the contrary, small value of variance coefficient denotes similarity in knowledge stocks.

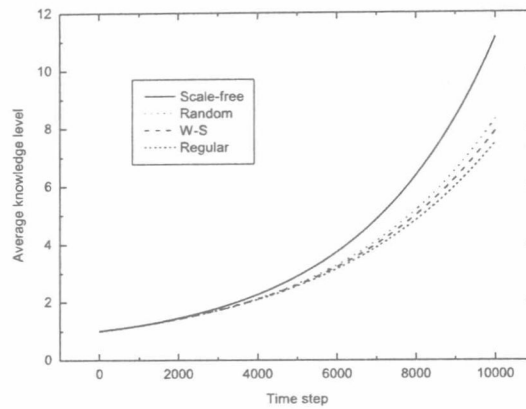


Fig. 1. The average knowledge stock as a function of time on four different networks. The system size is 10^3 .

3.3. The spatial distribution of knowledge

The last measurement is related to the degree of spatial order in a system of locally interconnected components. To check whether the broadcast mechanism on different network structures generates spatially auto-correlated knowledge allocations, we compute the Moran coefficient,

$$S(t) = \frac{1}{\sigma^2(t)} \sum_{i \in S} \sum_{j \neq i} w_{ij} [v_i(t) - \bar{v}(t)][v_j(t) - \bar{v}(t)], \quad (10)$$

where

$$w_{ij} = \frac{X(i, j)}{\sum_{i \in S} \sum_{j \neq i} X(i, j)}. \quad (11)$$

$X(i, j)$ indicates whether there is a direct connection between i and j .

The Moran coefficient usually ranges in value from -1 to 1 , with values close to zero indicating spatial randomness (i.e., no spatial dependence). A positive Moran coefficient indicates positive spatial dependence, that is, agents with similar values of knowledge stocks tend to be located close to each other. Negative values of the Moran coefficient indicates negative spatial dependence (i.e., agents with unlike values of knowledge stocks tend to be located close to each other).

4. Simulation results

Throughout the simulations, the average degree of each network is $2z = 6$ and the initial knowledge stock for each agent is unity. All of the results have been computed for one thousand independent runs with different configurations of the innovative and absorptive ability as well as different network realizations, over which averages have been taken.

4.1. The growth and diffusion of knowledge

At first, we investigate the behavior of average knowledge stock level $\bar{v}(t)$ in time evolution on the four different networks. Fig. 1 shows the dependence of $\bar{v}(t)$ on time. We can see that knowledge growth varies greatly on different networks. Obviously, scale-free network is more conducive to promote the growth of knowledge in the system. We can get a more clear picture from Fig. 2, which illustrates the knowledge growth rate as a function of time on the four types of networks. It is observed that the knowledge growth speed on the four different networks follows $\rho(\text{scale-free}) > \rho(\text{random}) > \rho(\text{small-world}) > \rho(\text{regular})$, which will be discussed in detail in Section 5.

On the four types of networks, the temporal behaviors of average knowledge stock level with different system size are computed (Fig. 3) and the growth diffusion time τ is measured from the condition in Eq. (6), where we choose the constant $C = 10$. It has been examined that another choice for the value of C does not make any qualitative difference in the scaling behavior of τ . In Fig. 4, we show the diffusion time τ depending on the system size N on the four different networks. It is observed that the growth diffusion times all depend linearly on the network size N as $\tau \sim N$, and the slope corresponding to scale-free network is minimal, which indicates the fastest growth and diffusion of knowledge.

This result that scale-free network provides an optimal framework for knowledge growth and diffusion can be qualitatively explained from the viewpoint of network structure. It is known that in scale-free network there are few large degree nodes who are connected with a finite fraction of the system. We call them hub agents. When small degree agent broadcasts,

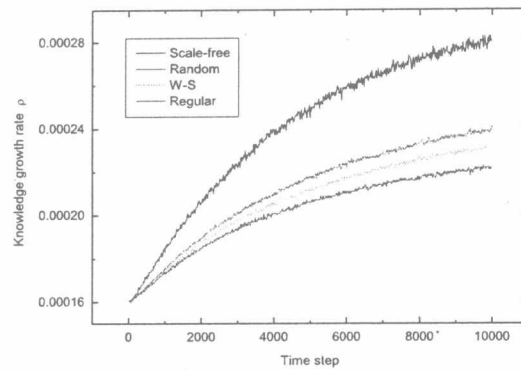


Fig. 2. The knowledge growth rate ρ as a function of time on four different networks. The system size is 10^3 .

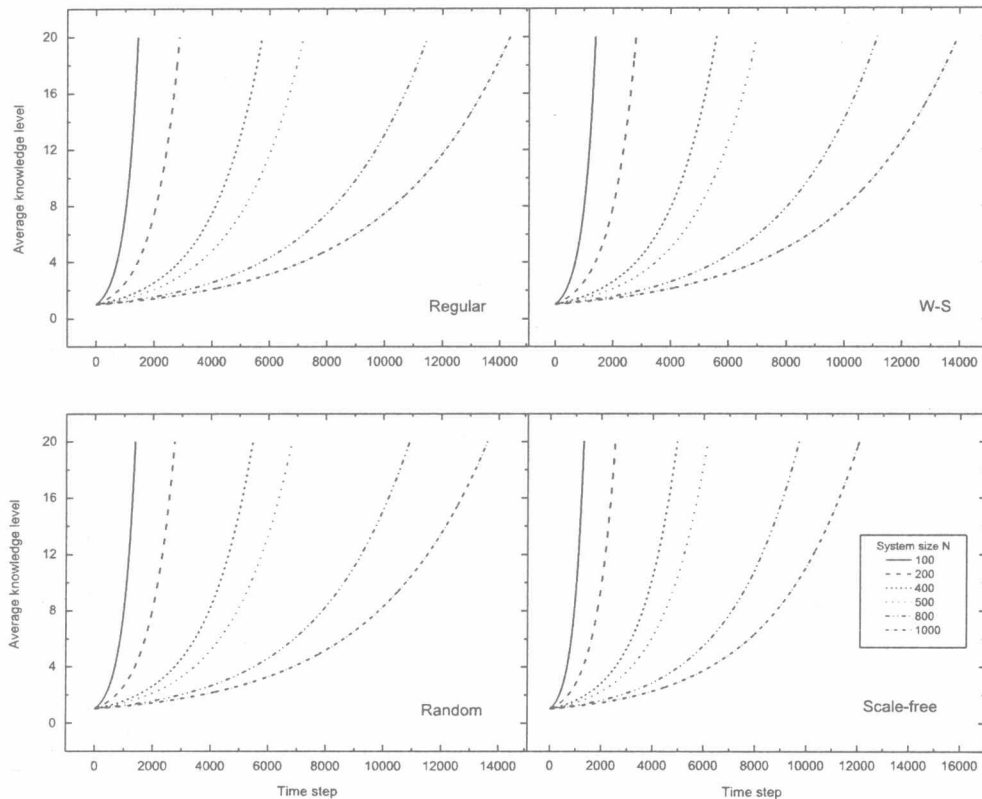


Fig. 3. The dependence of average knowledge stock level $\bar{v}(t)$ on the system size N for regular network, small-world network, random network, and scale-free network.

hub agent may probably learn because he is very likely to be connected with the broadcaster. This provides them with more chance to absorb knowledge from others and results in higher knowledge stock. On the other hand, when hub agent broadcasts, a finite fraction of the system agents connected with him will learn immediately, and thus the average knowledge stock of the system is largely promoted. On scale-free networks, the hub agent play a very important role. The newly innovated knowledge is quickly transferred to the hub agents from a few innovating agents, and then broadcasted to the rest of the systems. This mechanism provides fast knowledge growth and diffusion on scale-free network.

4.2. Variance coefficient of knowledge

The variance coefficient of knowledge stock is a measure of the level of overall difference in knowledge stock. The smaller of the value the more adequate knowledge transfer is. Table 1 shows the variance coefficient as a function of both lower bound of absorptive ability and network structure. We can see that regular network shows a large variation of knowledge stock among agents which means that regular network structure is not conducive to the balanced distribution of knowledge.

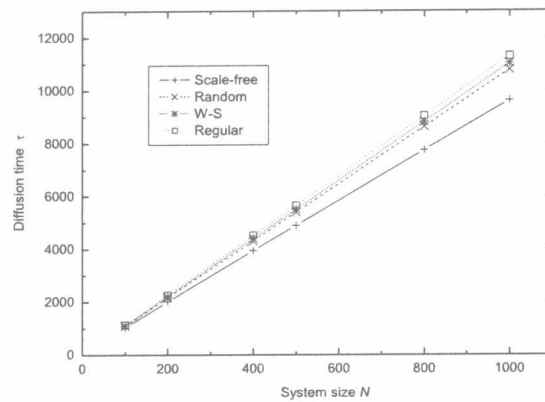


Fig. 4. Growth diffusion time τ versus system size N on four types of networks. Linear behavior $\tau \sim N$ is observed. The slope corresponding to scale-free network is minimal indicating the fastest growth and diffusion of knowledge.

Table 1

Variance coefficient as a function of the network structure and the lower bound of absorptive capacity α . The data is obtained by averaging over 10,000 generations after a transient time of 1000 generations for each network. The system size is 10^3 .

α	Regular	Small-world	Random	Scale-free
0.0	0.67407	0.28427	0.12426	0.10163
0.2	0.47731	0.09185	0.04291	0.04214
0.4	0.62761	0.10593	0.04448	0.05057
0.6	0.63178	0.11035	0.05556	0.06670
0.8	0.35916	0.12547	0.07908	0.09084

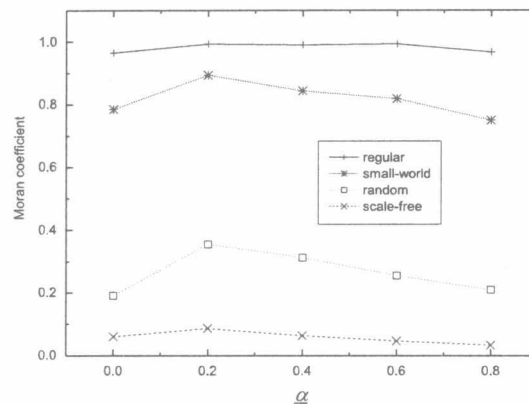


Fig. 5. Illustration of Moran coefficient as a function of the network structure and the lower bound of absorptive capacity. The data is obtained by averaging over 10,000 generations after a transient time of 1000 generations for each network. The system size is 10^3 .

Interestingly, the variance coefficients corresponding to scale-free network and random network are very small, which suggest high adequacy of knowledge transfer. In regular network, each agent can only exchange knowledge with few close neighbors, and thus the knowledge transfer is localized. According to the knowledge transfer mechanism (Eq. (1)), the agent with more knowledge stock will innovate more knowledge, while the agent with less knowledge stock will stay at lower knowledge level, which results in large variation of knowledge stock in the system. The situation is changed in networks with long-range edges, such as small-world network, random network, and scale-free networks. The newly innovated knowledge can be quickly transferred to the others agent, which reduce the overall gap in knowledge stocks.

4.3. The Moran coefficient

Moran coefficient is used to measure the spatial distribution of knowledge. The Moran coefficient as a function of both lower bound of absorptive ability and network structure is displayed in Fig. 5. For regular network, the value of Moran coefficient is in the vicinity of 1, which indicates that agents with similar knowledge stock level form exclusive clusters. This is because the knowledge transfer is localized in regular network, which destroys the equity of knowledge stocks distribution. On the contrary, scale-free network exhibits a random spatial pattern and its Moran coefficient is in the vicinity

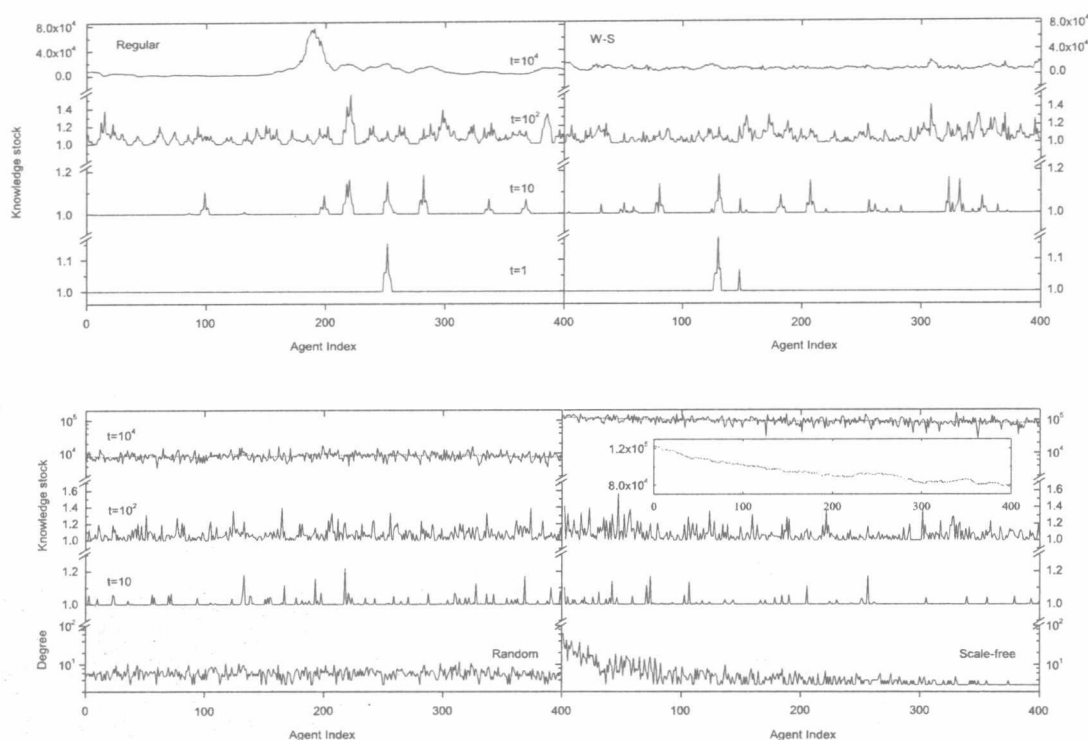


Fig. 6. The transient patterns of knowledge diffusion on regular network and small-world network (upper panel), and on random network and scale-free network (lower panel). The degree distributions according to the node index are also plotted for random network and scale-free network. The red lines are the adjacent averages of diffusion pattern at $t = 10^4$ for random network and scale-free network separately. The system size is 400.

of 0. This result suggests that knowledge transfer is non-discriminatory among agents, that is to say, agents with either high knowledge stock or low knowledge stock have the equal opportunity to communicate and transfer knowledge with others in harmony.

5. Discussion and conclusion

In order to get a deeper insight into the knowledge transfer process on the four types of networks, we investigate the transient pattern of the knowledge diffusion. Fig. 6 displays the knowledge stocks pattern according to the node index at different time step on regular network, small-world network, random network, and scale-free network. Firstly, we analyze the diffusion pattern of regular network. Due to the topological property of regular network where exist only short-range edges, the knowledge transfer is localized, which means the newly innovated knowledge can hardly be transferred to distant agents. At the early time step, every agents possess similar knowledge stock due to the same degree. However, few dominating agents with very high level of knowledge stock will emerge as time evolves, which can be seen in the diffusion pattern at time 10^4 on regular network in Fig. 6. Since each agent can only exchange knowledge with few close neighbors, the agents at higher knowledge level will innovate more and more knowledge, while the agents with less knowledge stock innovate less knowledge and will stay at lower knowledge level. In small-world network, there exist long-range edges, which make the knowledge transfer become easier. We can see from the diffusion pattern at the first time step on small-world network, the newly innovated knowledge can be transferred to faraway agent in only one time step. This property can largely reduce the variance of the knowledge stocks in the system. However, since the long-range edges are very few, the localization of knowledge transfer still occurs in some areas. Therefore, we can find several peaks in the growth and diffusion pattern on small-world network, which is shown in Fig. 6.

The Euclidean structure of the system is totally destroyed in random network where any pairs of agents in the population are connected with the same probability. The newly innovated knowledge can be transferred to any of the agents in the system through only several time steps, which largely improves the diffusion of knowledge and reduces the variance of the knowledge stocks of the system. Since in random network the degree of each agent is nearly the same (lower panel of Fig. 6), we expect a homogeneous diffusion pattern of knowledge on random network, which is confirmed by simulation (Fig. 6). On scale-free network, the behavior of the diffusion pattern of knowledge at the earlier time steps is almost the same as that on random network. As the time evolves, the growth of knowledge becomes much fast. It is known that in scale-free network there exist few hub agents who are connected with a considerable number of agents in the system. When the agent with small degree broadcasts, hub agent may probably receive the newly innovated knowledge because he is very likely to be

connected with the broadcaster. This provides the hub agents with higher knowledge stock. In Fig. 6, we display the adjacent average of knowledge diffusion pattern at time 10^4 on scale-free network as well as its degree distribution according to the agent index. It is found that the knowledge stock of an agent is proportional to its degree. On the other hand, when hub agent broadcasts, a large fraction of the agents connected with him will learn immediately, and thus the average knowledge stock of the system is efficiently improved. On scale-free networks, the hub agent plays a very important role that he gathers the newly innovated knowledge from several innovating agents, and then broadcast to the rest of the systems. This mechanism provides the fastest knowledge growth and diffusion on scale-free network.

In this article, we study how the growth and diffusion of knowledge is affected by the network structure. Simulation results indicate that the steady-state level of average knowledge is maximal in scale-free network which holds adequacy and equity in knowledge transfer as well. We explain these results by carefully investigating the dynamics of knowledge transmission on networks with different architectures of connections among agents. Since the growth and diffusion of knowledge on the scale-free structure is very efficient, it may be a good target for reformation in company and government.

Finally, we point out that our study just offers a starting point for understanding the interplay between network structure and knowledge transfer. More profound conclusions need further investigations.

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Spatial evolutionary game with bond dilution

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ABSTRACT

In this paper, we study numerically the prisoner's dilemma game (PDG) and snowdrift game (SG) on a two-dimensional square lattice with both quenched and annealed bond dilution. For quenched bond dilution, the system undergoes a dynamical transition at the critical occupation probability q^* , which is higher than the bond percolation transition point for a square lattice. In the critical region, the defined order parameter has a scaling form as $P_c \sim (q - q^*)^\beta$ for $q < q^*$ with the critical exponents $\beta = 1.42$ for PDG and $\beta = 1.52$ for SG, which differ from those with quenched site dilution. For annealed bond dilution, the system exhibits a distinct cooperative behavior. We find that the cooperation is much enhanced in the range of small payoff parameters on a lattice with slightly annealed bond dilution.

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1. Introduction

The evolutionary game theory has been considered as an important approach to characterizing and understanding the emergence of cooperative behavior in systems consisting of selfish individuals [1]. Such systems are ubiquitous in nature, ranging from biological to economic and social systems. Since the groundwork on repeated games by Axelrod [2,3], the evolutionary prisoner's dilemma game (PDG) as a general metaphor for studying the cooperative behavior has drawn much attention from scientific communities. Due to the difficulties in assessing proper payoffs, the PDG has some restriction in the discussion of the emergence of cooperative behavior. Thus, the proposal of the snowdrift game (SG) was generated to be an alternative to PDG [4,5]. The SG, equivalent to the hawk–dove game, is also of biological interest. Both of the games are versions of matrix games describing the interaction between two players. The payoff of the players depend on their simultaneous decisions to cooperate or defect. For mutual cooperation both players receive the rewards R , but only the punishment P for mutual defection. A defector exploiting a cooperator gets an amount T (temptation to defect) and the exploited cooperator receives S (suckers payoff). These elements satisfy the condition that $T > R > P > S$ for PDG, and $T > R > S > P$ for SG. The values of these payoffs create an unresolvable dilemma for intelligent players who wish to maximize their own income; namely, defection brings higher individual income independently of the other players' decisions, but for mutual defection they receive the second worst result. Thus, the undesired outcome of mutual defection emerges in well-mixed populations [6], which is contrary to the empirical evidence. In order to solve these social dilemmas, a variety of suitable extensions on these basic models have been investigated. It is found that several mechanisms, such as kin selection [7], “tit-for-tat” [3,8] strategy, and voluntary participation [9–11], facilitate the emergence and persistence of cooperation in the populations.

The spatial game, introduced by Nowak and May [12,13], is a typical extension, which can result in emergence and persistence of cooperation. The dynamics are governed by a deterministic rule: in each subsequent round, players adopt

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the strategy of one who has gained the highest payoff among its neighbors (including also themselves) in the previous round. It has been shown that the spatial effects promote substantially the survival of cooperators [14–17]. The reason is that individuals usually occupy well-defined territorial regions. For a certain range of values of the payoff parameters, complex spatial patterns show up with cooperators and defectors coexisting (spatial chaos). In these structured populations, cooperative strategies can build clusters in which the benefits of mutual cooperation outweigh the losses against defectors, keeping the population of cooperators stable. In light of this, many attentions have been paid to the effects of spatial structures, such as lattices [18–21] and networks [22–29], on cooperative behavior. Recently, Santos and Pacheco studied the evolution of cooperation on more realistic network of contacts (NOCs) with scale-free degree distribution [30–33]. They demonstrated that, for all dilemmas, heterogeneous NOCs favor the emergence of cooperation, such that long-term cooperative behavior easily resists short-term noncooperative behavior.

Among the previous studies on evolutionary game in spatially-structured populations, the NOCs between those individuals is always deterministic; namely, all pairs of individuals, directly connected, must engage in a single round of a given game. In real populations, however, both the topological and temporal aspects of the environment affect the interactions between players. In light of this, Vainstein et al. study the robustness of cooperation in spatial evolutionary games by introducing quenched site diluted lattices [34,35]. However, two shortcomings prevent Vainstein's model from well characterizing the features of the social and biological systems. On one hand, the players on the diluted sites are permanently eliminated, and the total number of players are not fixed for different values of dilution probability. On the other hand, the quenched site diluted lattice keeps unchanged during the evolution. These two disadvantages can be overcome by using lattice with bond dilution, in which all of the players are active. Moreover, we can also introduce annealed bond dilution to mimic the dynamical NOCs in real systems. According to these considerations, in this paper, we push the research to more extensive situations by investigating spatial evolutionary PDG and SG on a two-dimensional square lattice with both quenched and annealed bond dilution. It is found that, in quenched bond dilution case, the system presents a pinning–depinning transition. In the subcritical region, the boundary of the clusters are pinned due to the presence of defects in diluted lattice, and the clusters are localized and stable. In the supercritical region, however, the large clusters with unconstrained boundaries are formed. This transition separates a region with spatial chaos from one with stable groups of cooperators [34]. We determine the critical bond occupation probability and the critical exponent for this transition, which are different from those with quenched site dilution. For annealed bond dilution, on the other hand, there is no such crossover behavior, and the cooperation is much enhanced in certain parameter region.

The remainder of the paper is organized as follows. The model and the measurements are presented in Section 2. Simulation results and discussions are provided in Section 3. Finally we draw the conclusion remarks in Section 4.

2. The model

We consider a $N = 100 \times 100$ two-dimensional square lattice with periodic boundary conditions. The bond dilution is introduced, which means the bonds in the square lattice are chosen randomly to be active with probability q , and the others are set to be inactive. This disorder can be either quenched or annealed. For quenched bond dilution, the diluted lattice is fixed throughout the evolution. While, for annealed bond dilution, the diluted lattice evolves with time and the population structure is dynamical.

The interacting players are placed on the sites of the diluted lattice. Each player follows one of the two simple strategies: cooperation (C) or defection (D), represented by the variable $S_i = \pm 1$ respectively. In each generation, the players combat with each other through the *active bond* and self-interactions are excluded. The players collect their payoffs depending on the chosen strategies, according to the rescaled payoff tables with single parameter for each game: $T = b$, $R = 1$, and $P = S = 0$ for PDG and $T = 1 + r$, $R = 1$, $S = 1 - r$, and $P = 0$ for SG, where $1 < b < 2$ and $0 < r < 1$. At the end of each generation, the players update their strategies according to the deterministic rule [12,22], i.e. each individual will adopt the strategy of the player with the greatest payoff among its *connected neighbors* and itself in the next round.

To characterize the macroscopic behavior of the system, we introduce three order parameters [34].

(1) Cooperator density

Let $\rho_c(t)$ represents the cooperator density at a given time,

$$\rho_c(t) = \frac{1}{2N} \sum_{i=1}^N (S_i + 1). \quad (1)$$

Since we are interested in the long time regime, and the results depend on the choice of the diluted bonds both for quenched and annealed cases, we define the order parameter as the average over time ($\langle \cdot \rangle$) and over the realizations of the disorder ($\overline{(\cdot)}$) of the asymptotic cooperator density $\rho_c = \overline{(\rho_c(\infty))}$. Thus $\rho_c = 0$ means that the population is invaded by defectors, and $\rho_c = 1$ that the population is full of cooperators. In an intermediate case $0 < \rho_c < 1$, both strategists coexist.

(2) Active player density

The fraction of active players, i.e. the fraction of players who change strategy at each time, is

$$\rho_a(t) = \frac{1}{2N} \sum_{i=1}^N (1 - S_i^t S_i^{t-1}), \quad (2)$$

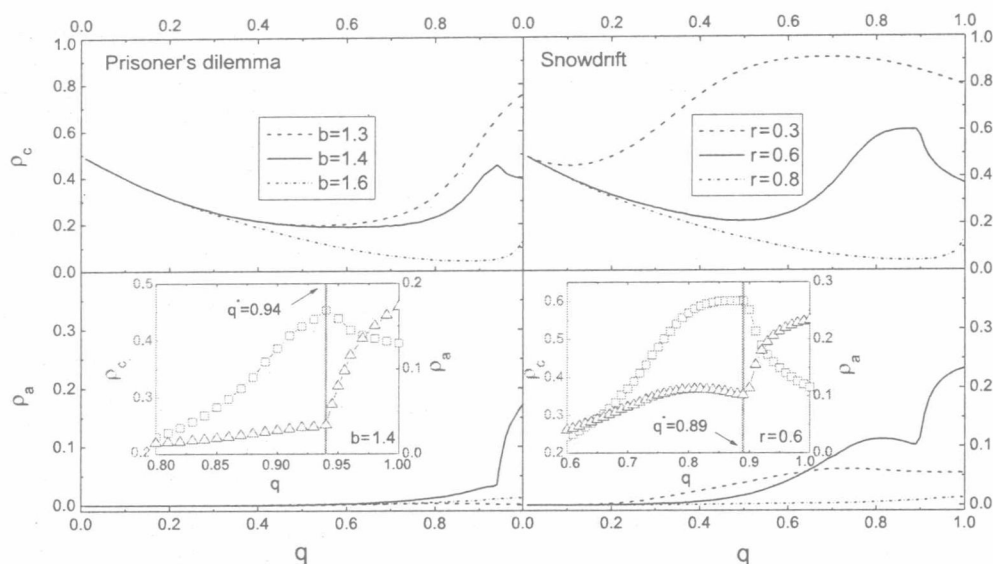


Fig. 1. The cooperator density ρ_c (upper panels) and the active player density ρ_a (lower panels) as functions of the bond occupation q for PDG (left panels) and SG (right panels) with different values of b and r . The critical values q^* of bond occupation are illustrated for $b = 1.4$ in PDG (left inset) and $r = 0.6$ in SG (right inset).

where S_i^t denotes the strategy of player i at time step t . Our second order parameter is defined as $\rho_a = \overline{\langle \rho_a(\infty) \rangle}$. Thus $\rho_a = 0$ means that all players are frozen, and $\rho_a = 1$ means that all individuals are changing strategy.

(3) Persistence

The fraction of players that do not change strategy before time step t is represented by persistence $P(t)$, which defines the third order parameter $P_e = \overline{\langle P(\infty) \rangle}$.

3. Simulation results and discussions

3.1. Quenched bond dilution

Through Monte Carlo (MC) simulations, we study the three quantities defined above for different values of bond occupation fraction q . The simulation results are obtained by averaging over 5000 MC time steps after a transient time of 10,000. We confirmed that averaging over larger periods or using different transient times did not change the conclusion. Furthermore, final data have been computed for 128 independent runs with different realizations of bond dilution, over which averages have been taken. All simulations start with an equal percentage of strategies (cooperators and defectors) randomly distributed among the individuals. In this way, all sites are initially populated with a strategy, and no initial advantage is given to cooperators or defectors. We have confirmed that other choices of the initial condition do not bring qualitative difference.

In Fig. 1, the asymptotic cooperator density ρ_c for PDG and SG are plotted against the bond occupation probability q , in which the behavior of ρ_c is similar to that in site dilution case [34]. It is found that, for small occupation fraction, the curves for different values of b and r merge in PDG and SG separately. This is because that, when q is near zero, almost all players are isolated and do not change strategy since there is no combat at all. The asymptotic cooperator density is the same as the initial one $\rho_c = 1/2$. As the occupation density increases, the clusters with small numbers of sites form, in which, irrespective of the values of b and r , the defectors are dominant. It can be seen as a decreasing curve of ρ_c from the origin. When the occupation fraction reaches certain value, the cooperative behavior will be determined by the dynamics, and the curves for different parameter values will depart from each other: the higher the b and r are, the more favorable the defector is; and vice versa. For PDG, the maximum cooperator density occurs at $q = 1$ in the region of $1 < b < 1.33$; on the other hand, in the region of $1.5 < b < 2$, the optimal ρ_c is located at $q = 0$. For SG, in the region of $0 < r < 0.34$, the maximal value of ρ_c occur at large q ; for $0.65 < r < 1$, the trend of the curve is opposite to that for $0 < r < 0.34$, and the optimal value of ρ_c is located at $q = 0$. In these cases cooperators and defectors, respectively, are at an advantage when interacting, and the fraction of active players stays at very small value (see lower panels of Fig. 1). The most interesting case occurs in the intermediate region of $1.33 < b < 1.5$ for PDG and $0.34 < r < 0.65$ for SG, where a sharp decrease in cooperator density ρ_c and a sharp increase in active players density ρ_a simultaneously appear at $q^* = 0.94$ for PDG and at $q^* = 0.89$ for SG. We should notice that this point is much higher than the bond percolation transition $q_{\text{bond}} = 1/2$ on a square lattice [36]. For $q < q^*$, the boundary of the clusters are pinned due to the presence of defects in diluted lattice, and the clusters are localized and stable. For $q > q^*$, however, the large clusters with unconstrained boundaries are formed, and