南京航空航天大学

论文集

（二〇〇二年）

第10册

四院

南京航空航天大学科技部编

二〇〇三年六月

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37	沈国华 黄志球 柳雪涛 陶剑青	初级 副高 硕士 硕士	042 042 外 042	面向试飞工程数据仓库系统的设计与实现	计算机工程与设计	2002.23.10	J
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The Journal of Symbolic Logic
Volume 67, Number 1, March 2002

VALUATION STRUCTURE

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Abstract. This paper introduces valuation structures associated with preferential models. Based on *KLM* valuation structures, we present a canonical approach to obtain injective preferential models for any preferential relation satisfying the property **INJ**, and give uniform proofs of representation theorems for injective preferential relations appeared in the literature. In particular, we show that, in any propositional language (finite or infinite), a preferential inference relation satisfies **INJ** if and only if it can be represented by a standard preferential model. This conclusion generalizes the result obtained by Freund. In addition, we prove that, when the language is finite, our framework is sufficient to establish a representation theorem for any injective relation.

§1. Introduction. Nonmonotonic reasoning is one of important research fields in AI. Researches in this filed can be broadly classified into two categories: those that proposed systems in which nonmonotonic inferences are performed and those that presented general framework in which nonmonotonic reasoning systems could be compared and classified. In the former category, the best known are probably: negation as failure, circumscription, the modal system of McDermott and Doyle, default logic and autoepistemic logic. In the latter category, Gabbay was probably first to suggest focusing the study of nonmonotonic logics on their inference relations [4]. Inspired by Gabbay's work, there has been interest in researching into nonmonotonic inference relation from various angles.

One of the main tools in exploring nonmonotonic inference relations is the representation of these inference relations in terms of preferential models. A number of representation theorems have been established in the literature. Among them, some results illustrate that there exists a large class of nonmonotonic inference relations that can be represented by injective preferential models. These relations are referred to as injective inference relations.

The contribution of this paper is to provide an uniform framework to obtain representation theorems for injective inference relations. The key idea is the notion of the valuation structure (defined in section 3) associated with *KLM* models, where *KLM* model is introduced by Kraus, et al., in [5]. These valuation structures provide an unified approach to construct injective preferential models for inference relations satisfying **INJ**. Formally, let $|\sim$ be a preferential relation and W be the *KLM* model associated with $|\sim$, we will show that, if the relation $|\sim$ satisfies the

Received July 26, 2000; revised September 13, 2000.

Key words and phrases. nonmonotonic logic, injective inference relation, valuation structure, representation theorem, **INJ** property.

LLE (Left Logical Equivalence)	**RW (Right Weakening)**
$\dfrac{\vdash \alpha \leftrightarrow \beta, \alpha \mid\sim \gamma}{\beta \mid\sim \gamma}$	$\dfrac{\vdash \alpha \to \beta, \gamma \mid\sim \alpha}{\gamma \mid\sim \beta}$
CM (Cautious Monotony)	**CUT**
$\dfrac{\alpha \mid\sim \beta, \alpha \mid\sim \gamma}{\beta \wedge \alpha \mid\sim \gamma}$	$\dfrac{\alpha \wedge \beta \mid\sim \gamma, \alpha \mid\sim \beta}{\alpha \mid\sim \gamma}$

TABLE 1

property **INJ** then the valuation structure $I(W)$ is an injective preferential model, moreover this injective model represents the relation $\mid\sim$. This result generalizes the conclusion obtained by Freund. What is more, we will illustrate that, if the relation $\mid\sim$ satisfies properties that are stronger than **INJ**, then the valuation structure $I(W)$ has further fine features, for instance, if the relation $\mid\sim$ is disjunctive (resp., rational, rationally transitive) then $I(W)$ is *filtered* (resp., *ranked*, *quasi-linear*), and so on. These results indicate that, the valuation structure (associated with *KLM* model) is a powerful tool in exploring the semantic characterizations of preferential relations that satisfy the property **INJ**.

The rest of the paper is organized as follows: In section 2, we recall some basic definitions and results related to this paper. In section 3, we introduce valuation structures. In section 4, we discuss the valuation structure associated with *KLM* model, and adopt it to establish the representation theorems for injective relations appeared in the literature. In finite framework, section 5 explores valuation structures associated with ordinary preferential models, and shows that there exists only one injective model for any injective inference. In section 6, we compare our work with related works that have appeared in the literature.

§2. **Preliminaries.** In this section, we will recall some basic definitions and results from [5] and [6], which will be used in this paper.

2.1. Nonmonotonic inference relation. We consider formulae of classical propositional calculus built over a set of atomic formulae denoted L plus two constants $\top$ and $\bot$ (the formulae *true* and *false* respectively). If L is finite we will say that the propositional language is finite. We denote the set of all well formed formulas by Form(L). A valuation is a function $v\colon L \cup \{\top, \bot\} \to \{0, 1\}$ such that $v(\top) = 1$ and $v(\bot) = 0$. We use lower case letters of the Greek alphabet to denote formulae, the letters $v, v_1, v_2, \ldots, n, m, \ldots$ to denote valuations, and U to denote the set of all valuations.

As usually, $\vdash \alpha$ means that the formula α is a tautology and $v \models \alpha$ means that the valuation v satisfies α where compound formulae are evaluated as usually. If Σ is a set of valuations, then $\Sigma \models \alpha$ means that $v \models \alpha$ for any valuation $v \in \Sigma$.

A nonmonotonic inference relation is a binary relation over formulae which satisfies some Horn or non-Horn conditions defined in the style of Gentzen. Gabbay uses the relation symbol $\mid\sim$ to denote nonmonotonic consequence to distinguish it from monotonic logical consequence. If α, β are formulas, then the sequence $\alpha \mid\sim \beta$ is called a conditional assertion. Let $\mid\sim$ be an inference relation. As usual, the set

$\{\beta : \alpha \mid\sim \beta\}$ is denoted by $C_{\mid\sim}(\alpha)$. If there is no ambiguity we shall write $C(\alpha)$ instead of $C_{\mid\sim}(\alpha)$. If Γ is a set of formulas then $\mathrm{Cn}(\Gamma)$ will denote the set of classical consequence of Γ. If α is a formula, we shall write $\mathrm{Cn}(\alpha)$ instead of $\mathrm{Cn}(\{\alpha\})$.

A consequence relation $\mid\sim$ is said to be *cumulative* if and only if it contains all instances of the axiom **Reflexivity** (i.e., $\alpha \mid\sim \alpha$) and is closed under the inference rules described in Table 1.

A relation $\mid\sim$ is said to be *preferential* inference relation if and only if it is *cumulative* and satisfies the following rule **OR**

$$\frac{\beta \mid\sim \gamma, \alpha \mid\sim \gamma}{\beta \vee \alpha \mid\sim \gamma}.$$

2.2. Preferential Model. Following the definition in [5], a preferential model W is a triple $\langle S, l, \prec\rangle$, where S is a set, the elements of which will be called states, the interpretation function $l : S \longrightarrow U$ assigns a valuation to each state, where U is the set of all valuations, and $\prec$ is a strict partial order on S (i.e., transitive and irreflexive) satisfying the following smoothness condition: for any $\alpha \in \mathrm{Form}(L)$, the set $\|\alpha\|_w = \{s : s \in S \text{ and } l(s) \models \alpha\}$ is smooth.[1] If there is no ambiguity, we shall write $\|\alpha\|$ instead of $\|\alpha\|_w$. A preferential model $W = \langle S, l, \prec\rangle$ is said to be injective model if and only if l is injective.

A *ranked* model $W = \langle S, l, \prec\rangle$ is a preferential model for which the strict partial order $\prec$ is modular, i.e., for any $x, y, z \in S$, if $x \not\prec y$, $y \not\prec x$ and $z \prec x$, then $z \prec y$.

Let $W = \langle S, l, \prec\rangle$ be a preferential model, the inference relation generated by W will be denoted by $\mid\sim_w$ and is defined as follows: for any formulas α and β, $\alpha \mid\sim_w \beta$ if and only if for any s minimal in $\|\alpha\|$, $l(s) \models \beta$. We denote the set $\{\beta : \alpha \mid\sim_w \beta\}$ by $C_w(\alpha)$.

An inference relation $\mid\sim$ is said to be injective preferential relation if and only if there exists an injective preferential model W such that $\mid\sim = \mid\sim_w$.

§3. Valuation structure. In this section, we will introduce *valuation structure* which is the key concept in this paper. We first recall some basic definitions and notations in [9].

Let $W = \langle S, l, \prec\rangle$ be a preferential model. We adopt the following notations: the range of l will be denoted by $\mathrm{rang}(l)$ (i.e., $\mathrm{rang}(l) =_{\mathrm{def}} \{v \in U : \exists s\ (s \in S \text{ and } l(s) = v)\}$). If $X \subseteq S$, then $l(X) =_{\mathrm{def}} \{v \in U : \exists s\ (s \in X \text{ and } l(s) = v)\}$, and $\min(X)$ is the set of all minimal element of X with respect to $\prec$ (i.e., $\min(X) =_{\mathrm{def}} \{t \in X : \neg\exists s\ (s \in X \text{ and } s \prec t)\}$). If $\Sigma \subseteq \mathrm{rang}(l)$, then $l^{-1}(\Sigma) =_{\mathrm{def}} \{s \in S : \exists v\ (v \in \Sigma \text{ and } l(s) = v)\}$; we shall write $l^{-1}(v)$ instead of $l^{-1}(\{v\})$.

Previously, in order to establish representation theorems for some inference relations, we introduced the binary relation $\propto_w$ which is defined as follows.

Definition 3.1 ([9]). Let $W = \langle S, l, \prec\rangle$ be a preferential model, the relation $\propto_w$ is defined as follows, for any $X_1, X_2 \subseteq S$,

$$X_1 \propto_w X_2 \qquad \text{if and only if} \qquad \forall s\ (s \in X_2 \Longrightarrow \exists t\ (t \in X_1 \text{ and } t \prec s)).$$

[1] Let W be a set, $\prec$ be a strict partial order over W and $V \subseteq W$, we shall say that V is smooth if and only if for any $t \in V$, either t is itself minimal in V (i.e., there is no $w \in V$ such that $w \prec t$), or there exists $s \in V$ such that $s \prec t$ and s is minimal in V.

By the transitivity of the relation $\prec$, it is easy to show that the relation $\propto_w$ is transitive.

DEFINITION 3.2. Let $W = \langle S, l, \prec \rangle$ be a preferential model. The relation $\sqsubset_w$ over the set rang(l) is defined by the following: for any valuations m, n in rang(l),

$$n \sqsubset_w m \qquad \text{if and only if} \qquad l^{-1}(n) \propto_w l^{-1}(m).$$

In other words, $n \sqsubset_w m$ if and only if for any state s such that $l(s) = m$, there exists a state t with $l(t) = n$ and $t \prec s$. If there is no ambiguity, we shall write $n \sqsubset m$ instead of $n \sqsubset_w m$. The valuation structure associated with the model W is the triple $\langle \text{rang}(l), \text{id}, \sqsubset \rangle$, where id is the identity function over the set rang(l). In the following, we denote this triple by $I(W)$. For any formula α, we denote the set $\{ v \in \text{rang}(l) : v \models \alpha \}$ by $\|\alpha\|_{I(w)}$.

DEFINITION 3.3. A preferential model $W = \langle S, l, \prec \rangle$ is said to be *valuation parsimonious* if and only if for any valuation $m \in \text{rang}(l)$, there is a formula α such that $m \in l(\min(\|\alpha\|))$.

In [1] the authors define *parsimonious* preferential model in the following way:

DEFINITION 3.4 ([1]). A preferential model $W = \langle S, l, \prec \rangle$ is said to be *parsimonious* if and only if for every state $s \in S$ there is a formula α such that $s \in \min(\|\alpha\|)$.

It is evident that, if a preferential model is *parsimonious* then it is *valuation parsimonious*, but not conversely.

LEMMA 3.1. *Suppose that $W = \langle S, l, \prec \rangle$ is a preferential model, then*

(i) *The relation $\sqsubset$ is transitive, and*
(ii) *If the model W is valuation parsimonious then the relation $\sqsubset$ is irreflexive.*

PROOF. (i) Immediate from the definition of $\sqsubset$ and the transitivity of the relation $\propto_w$.

(ii) Let m be any valuation in the set rang(l). We want to show $m \not\sqsubset m$. Suppose not. Since $m \in \text{rang}(l)$ and the model W is *valuation parsimonious*, there is a formula α such that $m \in l(\min(\|\alpha\|))$. Hence, there exists a state $s \in S$ such that $l(s) = m$ and $s \in \min(\|\alpha\|)$. Furthermore, by $m \sqsubset m$, there exists $t \in S$ such that $l(t) = m$ and $t \prec s$. This contradicts $s \in \min(\|\alpha\|)$. Thus, the relation $\sqsubset$ is irreflexive, as desired. ⊣

When the language is infinite and W just is an ordinary preferential model, the relation $\sqsubset$ is not always irreflexive. This can be seen by the following example. Let the language $L = \{p_0, p_1, \dots, p_n, \dots\}$, and n be a valuation. Consider the preferential model $W = \langle S, l, \prec \rangle$ where $S = \{ s_i : i \geq 1 \} \cup X$, the interpretation function l satisfies $l(s_i) = n$ for any $i \geq 1$, and $l(X) = U - \{n\}$, and the relation $\prec$ is the transitive closure of the relation $\{ \langle s_{i+1}, s_i \rangle : i \geq 1 \} \cup \{ \langle x, s_i \rangle : i \geq 1 \text{ and } x \in X \}$. Graphically, see Figure 1.

Clearly, $n \sqsubset_w n$. So, the relation $\sqsubset_w$ is not irreflexive. What is more, in part (ii) of the above lemma, *valuation parsimonious* is just a sufficient condition, not necessary one. This can be seen by the following example. Suppose that the language is infinite and W_1 is an injective *ranked* model whose states are valuations, with two levels: v_0 in the upper level and the rest of the valuations in the lower level, i.e., the order is

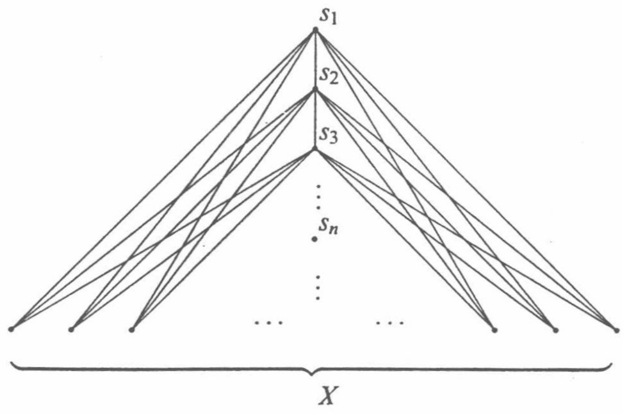

FIGURE 1. *Preferential* model W.

$v \prec v_0$ for all valuations $v \neq v_0$. It is clear that the relation $\sqsubset_{w_1}$ is irreflexive, but the model W_1 is not *valuation parsimonious*.

In addition, by the above lemma, for any *valuation parsimonious* preferential model W, if the relation $\sqsubset$ is smooth then the structure $I(W)$ is an injective preferential model. However, the relation $\sqsubset$ is not always smooth when the language is infinite. For instance, let the language $L = \{p_0, p_1, \dots, p_n, \dots\}$, consider the following preferential model $W = \langle S, l, \prec \rangle$, where

$$S = \{s, t\} \cup \{ s_i : i \geq 0 \} \cup \{ t_i : i \geq 0 \} \cup \{ k_i : i \geq 0 \},$$

$l(s) = \{p_1\}$,[2] $l(t) = \{p_1, p_2\}$, $l(s_0) = l(t_0) = \{p_0, p_1, p_2\}$, $l(s_i) = l(t_i) = \{p_1, p_{i+2}\}$ for any $i \geq 1$, $l(k_i) = \{p_1\} \cup \{ p_{i+n} : n \geq 3 \}$ for any $i \geq 0$, and the relation $\prec$ is the transitive closure of the relation

$$\{ \langle s, s_i \rangle : i \geq 0 \} \cup \{ \langle s_{i+1}, s_i \rangle : i \geq 0 \} \cup \{ \langle t, t_i \rangle : i \geq 0 \} \\ \cup \{ \langle t_{i+1}, t_i \rangle : i \geq 0 \} \cup \{ \langle k_i, t_i \rangle : i \geq 0 \}.$$

Graphically, see Figure 2.

It is easy to see that the above preferential model W is *parsimonious*. The valuation structure $I(W)$ is represented by the following figure.

Since the set $\|p_1\|_{I(w)}$ does not satisfy smoothness condition, the relation $\sqsubset$ is not smooth. Thus, it can be seen that *parsimonious* model is not always a guarantee of the smoothness of the relation $\sqsubset$ associated with it.

LEMMA 3.2. *If $W = \langle S, l, \prec \rangle$ is a preferential model, then for any formula α,*

$$l(\min(\|\alpha\|_w)) \subseteq \min(\|\alpha\|_{I(w)})$$

where the set $\{ v \in \text{rang}(l) : v \models \alpha$ and $\neg \exists m \in \text{rang}(l)$ $(m \sqsubset v$ and $m \models \alpha) \}$ is denoted by $\min(\|\alpha\|_{I(w)})$.

PROOF. Suppose $n \in l(\min(\|\alpha\|_w))$. We want to show $n \in \min(\|\alpha\|_{I(w)})$. Suppose not. Then there is a valuation m such that $m \in \|\alpha\|_{I(w)}$ and $m \sqsubset n$. Since $n \in l(\min(\|\alpha\|_w))$, there exists a state $s \in \min(\|\alpha\|_w)$ such that $l(s) = n$. By the

[2] We give the valuations as for a Herbrand model, that is identifying the subset of variables with its characteristic function.

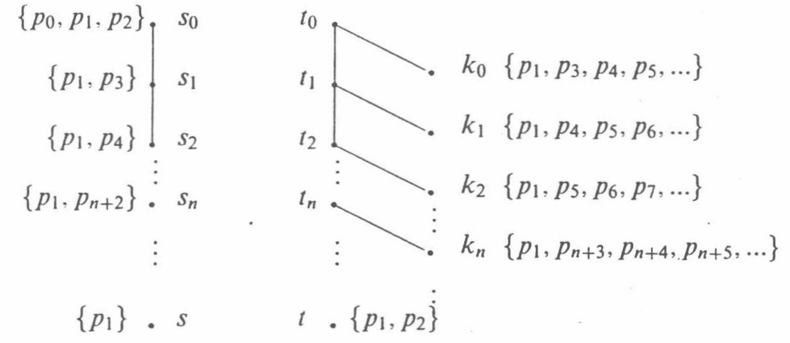

FIGURE 2. *Preferential* model W.

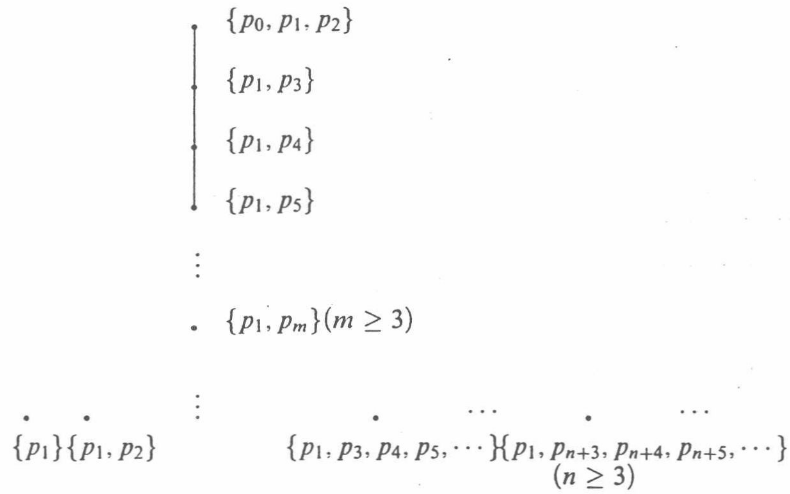

FIGURE 3. $I(W)$.

definition of $\sqsubset$ and $m \sqsubset n$, there is a state $t \in S$ such that $l(t) = m$ and $t \prec s$. We get a contradiction because $t \prec s$, $l(t) \models \alpha$ and $s \in \min(\|\alpha\|_w)$. ⊣

From the above lemma, the following holds.

COROLLARY 3.1. *Given a preferential model W, if the valuation structure $I(W)$ is a preferential model then* $|\!\sim_{I(w)} \subseteq |\!\sim_w$.

Note that the equation $|\!\sim_{I(w)} = |\!\sim_w$ does not always hold. Consider the following model $W = \langle S, l, \prec \rangle$ where $S = \{s_0, s_1, s_2, s_3\}$, $\prec = \{\langle s_0, s_1\rangle, \langle s_3, s_2\rangle\}$, $l(s_0) = \{p, q\}, l(s_3) = \{p\}$ and $l(s_1) = l(s_2) = \{q\}$. This model and its valuation structure $I(W)$ are represented by the following figure.

It is clear that $\top |\!\sim_w p$ and $\top |\!\not\sim_{I(w)} p$. Hence, $|\!\sim_{I(w)} \neq |\!\sim_w$. In section 5, we will show that, when the language is finite, the equation $|\!\sim_{I(w)} = |\!\sim_w$ holds for exactly those preferential models which represent the inference relations satisfying the property **INJ**.

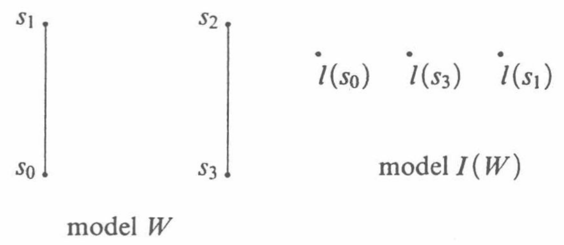

FIGURE 4. $|\!\sim_{I(w)} \neq |\!\sim_w$.

§4. Valuation structure associated with *KLM* Model and injective representation. One of the topics in the study of nonmonotonic inference relations is establishing the representation theorems for them. Suppose that Φ is a set of Horn or non-Horn conditions defined in the style of Gentzen, and suppose Ω is a set of preferential models. The representation theorem $\mathrm{TH}(\Phi, \Omega)$ is usually consisted of the following two statements:

(i) If an inference relation $|\!\sim$ satisfies the rules in Φ, then there exists a preferential model $W \in \Omega$ such that $|\!\sim\, = |\!\sim_w$, and
(ii) If a preferential model $W \in \Omega$ then the relation $|\!\sim_w$ satisfies the rules in Φ.

In the above two statements, the proof of the part (i) is usually harder than (ii). We call the part (i) the hard part of the representation theorem $\mathrm{TH}(\Phi, \Omega)$.

The aim of this section is to present uniform proofs of the hard part of representation theorems for injective inference relations. Valuation structures associated with *KLM* models play a key role in this section, where *KLM* model is introduced by Kraus, et al. In the following, we recall some basic definitions and results from [5] which will be used.

Kraus, et al., have investigated the semantic characterization of preferential relations in [5]. In particular, given a preferential relation $|\!\sim$, they introduce a method to construct a preferential model for the relation $|\!\sim$. We describe this method as follows.

Let $|\!\sim$ be a preferential inference relation, following the technic in [5], we say that the formula α is not less ordinary than β and write $\alpha \leq \beta$ if and only if $\alpha \vee \beta \,|\!\sim \alpha$. A valuation m is said to be normal for the formula α (or, *α-normal*) if and only if $m \models C(\alpha)$. In the following, for any valuation n, Σ_n will denote the set $\{\, \gamma \in \mathrm{Form}(L) : n \text{ is a normal valuation for } \gamma \,\}$.

Given a preferential relation $|\!\sim$, Kraus, et al., construct the preferential model $W = \langle S, l, \prec \rangle$ as follows:

(i) $S = \{\, \langle m, \alpha \rangle : \alpha \in \mathrm{Form}(L)$ and m is a *α-normal valuation* $\}$,
(ii) $l(\langle m, \alpha \rangle) = m$, and
(iii) $\langle m, \alpha \rangle \prec \langle n, \beta \rangle$ if and only if $\alpha \leq \beta$ and $m \models \neg\beta$.

For convenience, in the following, we call the above model $W = \langle S, l, \prec \rangle$ the *KLM* model associated with the relation $|\!\sim$.

LEMMA 4.1 ([5]). *Let $|\!\sim$ be a preferential inference relation and $W = \langle S, \ell, \prec \rangle$ be the KLM model associated with the relation $|\!\sim$, then*

(i) $|\!\sim\, = |\!\sim_w$.

(ii) *If* $\alpha \leq \beta$ *and* m *is a* α*-normal valuation that satisfies* β, *then* m *is* β*-normal.*
(iii) $\langle m, \beta\rangle \in \min(\|\alpha\|)$ *if and only if* $m \models \alpha$ *and* $\beta \leq \alpha$.

By the construction of *KLM* models and the part (iii) of the above lemma, it is clear that *KLM* models are *parsimonious*. Let $|\!\sim$ be a preferential relation and $W = \langle S, l, \prec\rangle$ be the *KLM* model associated with $|\!\sim$. By the definitions of valuation structure and *KLM* model, it is easy to see that, for any valuations m, n in $\mathrm{rang}(l)$, $m \sqsubset_w n$ if and only if for every formula α such that n satisfies $C(\alpha)$, m does not satisfy α and there exists a formula β such that $m \models C(\beta)$ and $\beta \leq \alpha$.

LEMMA 4.2. *Suppose that* $W = \langle S, l, \prec\rangle$ *is a KLM model. If the valuation* n *is* α*-normal then* $n \in \min(\|\alpha\|_{I(w)})$.

PROOF. Since the valuation n is α*-normal*, we get $\langle n, \alpha\rangle \in S$. By the part (iii) of Lemma 4.1, $\langle n, \alpha\rangle \in \min(\|\alpha\|_w)$. Furthermore, by Lemma 3.2, it is easy to show that $n \in \min(\|\alpha\|_{I(w)})$. ⊣

In the following, we will discuss the smoothness of valuation structures associated with *KLM* models. The following property is introduced in [3].

INJ $\quad C(\alpha \vee \beta) \subseteq \mathrm{Cn}(C(\alpha) \cup C(\beta))$, where α and β are formulas.

We will show that, if a model W is *KLM* model associated with an inference relation satisfying **INJ**, then the relation $\sqsubset$ is smooth.

LEMMA 4.3. *Let* $|\!\sim$ *be a preferential relation satisfying* **INJ**. *Suppose that* $W = \langle S, l, \prec\rangle$ *is the KLM model associated with the relation* $|\!\sim$. *For any formula* α *and valuation* $n \in \mathrm{rang}(l)$, *if* $n \models \alpha$ *and* n *is not* α*-normal then there exists a* α*-normal valuation* m_0 *such that* $m_0 \sqsubset n$.

PROOF. Put $\Gamma = \bigcup\{ C(\alpha \vee \gamma) : \gamma \in \Sigma_n \} \cup \{\alpha\} \cup \{\neg\gamma : \gamma \in \Sigma_n\}$. We want to show that the set Γ is consistent. Suppose not. Then, by the compactness of classical consequence, there must be $\gamma_1, \gamma_2, \ldots, \gamma_k, \delta_1, \delta_2, \ldots, \delta_h \in \Sigma_n$ such that $C(\alpha \vee \gamma_1) \cup C(\alpha \vee \gamma_2) \cup \cdots \cup C(\alpha \vee \gamma_k) \vdash \alpha \to \beta$, where $\beta = \delta_1 \vee \delta_2 \vee \cdots \vee \delta_h$. By **INJ**, $C(\beta) \subseteq \mathrm{Cn}(\bigcup\{ C(\delta_i) : i \leq h \})$. So, by $\delta_1, \delta_2, \ldots, \delta_h \in \Sigma_n$, $\beta \in \Sigma_n$. Similarly, $\gamma_1 \vee \gamma_2 \vee \cdots \vee \gamma_k \vee \beta \in \Sigma_n$. Thus, $\langle n, \gamma_1 \vee \gamma_2 \vee \cdots \vee \gamma_k \vee \beta\rangle \in S$. Since $n \models \alpha$ and n is not a normal valuation for α, $\langle n, \gamma_1 \vee \gamma_2 \vee \cdots \vee \gamma_k \vee \beta\rangle \in \|\alpha\|_w$ and $\langle n, \gamma_1 \vee \gamma_2 \vee \cdots \vee \gamma_k \vee \beta\rangle \notin \min(\|\alpha\|_w)$. Furthermore, by the smoothness of the relation $\prec$, there exists a state $\langle m, \delta\rangle \in \min(\|\alpha\|_w)$ such that $\langle m, \delta\rangle \prec \langle n, \gamma_1 \vee \gamma_2 \vee \cdots \vee \gamma_k \vee \beta\rangle$. So, by the construction of *KLM* model and the part (iii) of Lemma 4.1, we obtain $m \models \neg(\gamma_1 \vee \gamma_2 \vee \cdots \vee \gamma_k \vee \beta)$, $\delta \leq \alpha$ and $\delta \leq \gamma_1 \vee \gamma_2 \vee \cdots \vee \gamma_k \vee \beta$. From $\delta \leq \alpha$ and $\delta \leq \gamma_1 \vee \gamma_2 \vee \cdots \vee \gamma_k \vee \beta$, we conclude $\alpha \vee \delta \mathrel{|\!\sim} \delta$ and $\gamma_1 \vee \gamma_2 \vee \cdots \vee \gamma_k \vee \beta \vee \delta \mathrel{|\!\sim} \delta$. Thus, by **OR**, $\alpha \vee \gamma_1 \vee \gamma_2 \vee \cdots \vee \gamma_k \vee \beta \vee \delta \mathrel{|\!\sim} \delta$. So, $\delta \leq \gamma_1 \vee \gamma_2 \vee \cdots \vee \gamma_k \vee \beta \vee \alpha$. For any $1 \leq i \leq k$, by $\delta \leq \gamma_1 \vee \gamma_2 \vee \cdots \vee \gamma_k \vee \beta \vee \alpha$ and $\gamma_1 \vee \gamma_2 \vee \cdots \vee \gamma_k \vee \beta \vee \alpha \leq \alpha \vee \gamma_i$, we get $\delta \leq \alpha \vee \gamma_i$.[3] Since $m \models \alpha \vee \gamma_i$, $\delta \leq \alpha \vee \gamma_i$ and m is δ-normal, by part (ii) of Lemma 4.1, m is a normal valuation for $\alpha \vee \gamma_i$. Hence, $m \models C(\alpha \vee \gamma_1) \cup C(\alpha \vee \gamma_2) \cup \cdots \cup C(\alpha \vee \gamma_k)$. Furthermore, by $m \models \neg(\gamma_1 \vee \gamma_2 \vee \cdots \vee \gamma_k \vee \beta)$ and $C(\alpha \vee \gamma_1) \cup C(\alpha \vee \gamma_2) \cup \cdots \cup C(\alpha \vee \gamma_k) \vdash \alpha \to \beta$, $m \models \neg\alpha$. So, we get a contradiction because $m \models \neg\alpha$ and $\langle m, \delta\rangle \in \min(\|\alpha\|_w)$. So, the set Γ is consistent, as desired.

[3]Note that if the relation $|\!\sim$ is preferential then the relation $\leq$ is transitive. (See [5].)

Therefore, there is a valuation m_0 such that $m_0 \models \Gamma$. In the following, we want to show $m_0 \sqsubset n$. Let γ be any formula in Σ_n. Since $m_0 \models C(\alpha \vee \gamma)$, m_0 is a normal valuation for $\alpha \vee \gamma$. Furthermore, from $m_0 \models \{\neg\gamma : \gamma \in \Sigma_n\}$ and $\alpha \vee \gamma \leq \gamma$, we get $\langle m_0, \alpha \vee \gamma \rangle \prec \langle n, \gamma \rangle$. Hence, by $l^{-1}(n) = \{\langle n, \gamma \rangle : \gamma \in \Sigma_n\}$ and the definition of the relation $\sqsubset$, we conclude $m_0 \sqsubset n$. Moreover, since $m_0 \models \alpha$, $\alpha \vee \gamma \leq \alpha$ and m_0 is a normal valuation for $\alpha \vee \gamma$, by the part (ii) of Lemma 4.1, the valuation m_0 is *α-normal.* ⊣

LEMMA 4.4. *Let $|\!\sim$ be a preferential relation satisfying* **INJ**. *Suppose that $W = \langle S, l, \prec \rangle$ is the KLM model associated with the relation $|\!\sim$. Then, the relation $\sqsubset$ is smooth.*

PROOF. Let α be any formula and n be any valuation. Suppose that $n \in \|\alpha\|_{I(w)}$ and $n \notin \min(\|\alpha\|_{I(w)})$. By Lemma 4.2, the valuation n is not *α-normal.* Furthermore, by Lemma 4.3, there exists a *α-normal* valuation m_0 such that $m_0 \sqsubset n$. Since the valuation m_0 is *α-normal*, by Lemma 4.2, $m_0 \in \min(\|\alpha\|_{I(w)})$. Hence, the relation $\sqsubset$ is smooth. ⊣

The following theorem is due to Freund:

THEOREM 4.1 ([3]). *Let L be a logically finite language and $|\!\sim$ a preferential inference relation on L. Then*

(i) *If the relation $|\!\sim$ satisfies* **INJ** *then there exists an injective preferential model W such that $|\!\sim \, = \, |\!\sim_w$.*
(ii) *If W is an injective preferential model then the relation $|\!\sim_w$ satisfies* **INJ**.

The above theorem characterizes injective preferential relations in the finite language. However, when the language is infinite, how to characterize the preferential relations that can be represented by an injective preferential model remains open. Recently, R. Pino Pérez and Carlos Uzcategui show that, if the preferential relation $|\!\sim$ satisfies **INJ** then it can be represented by an *essential pre-structure* $\langle S, \mathrm{id}, \prec_e \rangle$.[4] Unfortunately they do not prove that the relation $\prec_e$ is transitive in this case, and so far it is unknown whether this structure is a preferential model. However this result suggests that the property **INJ** could be considered a postulate that characterizes injective relations even when the language is infinite.

In the following, based on the valuation structure associated with *KLM* model, we will show that, in any propositional language (finite or infinite), if a preferential inference relation satisfies **INJ** then it indeed can be represented by an injective preferential model.

By Lemma 4.2 and Lemma 4.3 we get the following.

LEMMA 4.5. *Let $|\!\sim$ be a preferential relation satisfying* **INJ**. *Suppose that $W = \langle S, l, \prec \rangle$ is the KLM model associated with the relation $|\!\sim$. Then, for any formula α and valuation n, $n \in \min(\|\alpha\|_{I(w)})$ if and only if the valuation n is α-normal.*

Recently, two particular kinds of injective model appear in literatures; one is the *standard* model introduced in [3], the other is the *collapsed* model presented in [1]. They are defined as follows, respectively.

[4]Technical details may be found in [8].

DEFINITION 4.1 ([3]). An injective preferential model $W = \langle S, l, \prec \rangle$ is said to be a *standard* model if and only if for any $s \in S$, $s \in \min(\|\alpha\|)$ if and only if $l(s) \models C_w(\alpha)$.

DEFINITION 4.2 ([1]). A preferential model is said to be a *collapsed* model if and only if it is injective and *parsimonious*.

The following theorem reveals that the valuation structures associated with *KLM* models provide a canonical approach to obtain *standard* and *collapsed* preferential model for any preferential relation satisfying **INJ**.

THEOREM 4.2. *Let* $\mid\sim$ *be a preferential relation satisfying* **INJ**. *Suppose that* $W = \langle S, l, \prec \rangle$ *is the KLM model associated with the relation* $\mid\sim$. *Then*

(i) *The structure* $I(W)$ *is an injective preferential model,*
(ii) *The structure* $I(W)$ *is standard and collapsed, and*
(iii) $\mid\sim_w = \mid\sim_{I(w)}$.

PROOF. (i) By Lemma 3.1, the relation $\sqsubset$ is transitive. It is clear that *KLM* models are *valuation parsimonious*, so, by Lemma 3.1, $\sqsubset$ is irreflexive. Moreover, by Lemma 4.4, the relation $\sqsubset$ is smooth. Hence, the structure $I(W)$ is an injective preferential model.

(ii) Immediate from the part (i), the construction of *KLM* models and Lemma 4.5.

(iii) By Lemma 4.5, and parts (ii) and (iii) of Lemma 4.1, it is easy to show that $\min(\|\alpha\|_{I(w)}) = l(\min(\|\alpha\|_w))$ for any formula α. Thus, $\mid\sim_w = \mid\sim_{I(w)}$, as desired. ⊣

The following theorem provide a characterization of all the preferential inference relations that may be represented by a *standard* model, which generalize a result established in finite case in [3].

THEOREM 4.3. *A preferential inference relation* $\mid\sim$ *satisfies* **INJ** *if and only if there exists a standard preferential model* W *such that* $\mid\sim_w = \mid\sim$.

PROOF. Suppose that the preferential inference relation $\mid\sim$ satisfies **INJ**, and $W_{klm} = \langle S, l, \prec \rangle$ is the *KLM* model associated with the relation $\mid\sim$. By part (i) of Lemma 4.1 and Theorem 4.2, we get $\mid\sim_{I(w_{klm})} = \mid\sim$. Furthermore, by Lemma 4.5 and part (i) of Theorem 4.2, the structure $I(W_{klm})$ is a *standard* model.

Suppose that $W = \langle S, l, \prec \rangle$ is a *standard* preferential model. Let α, β and γ be any formulas and $\gamma \in C_w(\alpha \vee \beta)$. We want to show $\gamma \in \mathrm{Cn}(C_w(\alpha) \cup C_w(\beta))$. Suppose not. Thus, $C_w(\alpha) \cup C_w(\beta) \cup \{\neg\gamma\}$ is consistent. So, there is a valuation m such that $m \models C_w(\alpha) \cup C_w(\beta) \cup \{\neg\gamma\}$. Let Γ be a set of formulas, in the following, we denote the set $\{n : n \models \Gamma\}$ by $\mathrm{Mod}(\Gamma)$. It is obvious that $\min(\|\alpha\|) \cap \min(\|\beta\|) \subseteq \min(\|\alpha \vee \beta\|)$. Since the model W is *standard*, we get $\mathrm{Mod}(C_w(\alpha)) \cap \mathrm{Mod}(C_w(\beta)) \subseteq \mathrm{Mod}(C_w(\alpha \vee \beta))$. Hence, $m \in \mathrm{Mod}(C_w(\alpha \vee \beta))$. This contradicts $\gamma \in C_w(\alpha \vee \beta)$. So, the relation $\mid\sim_w$ satisfies **INJ**. ⊣

In the literature, a number of rules that are stronger than **INJ** have been presented and explored. Certain especially interesting rules are described as follows, the intuition behind those rules may be found in [7], [2] and [1].

DR (Disjunctive Rationality)

$$\frac{\alpha \not\mid\!\sim \beta, \gamma \not\mid\!\sim \beta}{\alpha \vee \gamma \not\mid\!\sim \beta}$$

DP (Determinacy Preservation)

$$\frac{\alpha \wedge \gamma \not\mid\!\sim \neg\beta, \alpha \mid\!\sim \beta}{\alpha \wedge \gamma \mid\!\sim \beta}$$

RT (Rational Transitivity)

$$\frac{\alpha \mid\!\sim \beta, \beta \mid\!\sim \gamma, \alpha \not\mid\!\sim \neg\gamma}{\alpha \mid\!\sim \gamma}$$

CEM (Completely Determinated)

$$\frac{\alpha \not\mid\!\sim \beta}{\alpha \mid\!\sim \neg\beta}$$

FD (Fragmented Disjunction)

$$\frac{\alpha \mid\!\sim \beta \vee \gamma, \alpha \not\mid\!\sim \beta, \alpha \not\mid\!\sim \gamma}{\neg\beta \mid\!\sim \gamma}$$

FC (Fragmented Conjunction)

$$\frac{\alpha \wedge \beta \mid\!\sim \gamma, \beta \not\mid\!\sim \gamma, \alpha \not\mid\!\sim \gamma}{\alpha \mid\!\sim \neg\beta}$$

RM (Rational Monotony)

$$\frac{\alpha \wedge \gamma \not\mid\!\sim \beta, \alpha \not\mid\!\sim \neg\gamma}{\alpha \not\mid\!\sim \beta}$$

The representation theorems for the above rules have been established, respectively. Among them, Lehmann and Magidor study the non-Horn condition **RM** and demonstrate that rational inference relations[5] are exactly those that may be represented by a *ranked* model [6]. Freund provide a semantic characterization of preferential inference relations satisfying **DR** in terms of *filtered* model [3]. In [1], Bezzazi, et al., study non-Horn conditions **RT**, **DP**, **FD**, **FC**and **CEM** systematically, establish interrelations and provide semantic characterizations for them.

In [1], Bezzazi, et al., compare the strength of the rules **DP**, **RT**, **CEM**, **FD**, **FC** and **RM**, together with the results obtained in [7] and [3], the interrelations of these rules may be represented by the following diagram, where one condition implies another, given a preferential inference relation, if and only if one can follow arrows from the former to the latter.

Hence, if the preferential relation $\mid\!\sim$ satisfies the properties above mentioned, then the valuation structure associated with its *KLM* model represents the relation. Moreover, in the following, we will show that those valuation structures have further fine features. Firstly, we recall some basic definitions appeared in the literature.

Definition 4.3 ([3]). A preferential model $W = \langle S, l, \prec\rangle$ is said to be *filtered* if whenever two states s and t of S satisfy a formula α without being minimal in $\|\alpha\|_w$, there exists a state r, $r \prec s$ and $r \prec t$, such that $l(r) \models \alpha$.

Definition 4.4 ([1]). A preferential model $W = \langle S, l, \prec\rangle$ is said to be *quasi-linear* if and only if it is *ranked* and it has at most one state at any level above the lowest. In other words *quasi-linear* means *ranked* and whenever $r \prec s$, $r \prec t$ then either $s = t$ or $s \prec t$ or $t \prec s$.

Definition 4.5 ([1]). A preferential model $W = \langle S, l, \prec\rangle$ is said to be *linear* if and only if it is *ranked* and it has at most one state at each level.

[5] $\mid\!\sim$ is called a rational relation if and only if it is a preferential relation satisfying **RM**.