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A MULTIPLE CRITERIA SEQUENTIAL SORTING PROCEDURE

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ABSTRACT. A novel procedure having strategic flexibility is designed to handle multiple criteria sorting problems such that a decision maker (DM) can adjust the group count and fine-tune group numbers to improve sorting efficiency. Its unique features include interactive control, so that the DM can adjust the number of groups and other sorting characteristics; the capacity to aggregate cardinal and ordinal criteria using concepts from data envelopment analysis; and the integration of approximate information about criterion weights, which may help to ensure that the sorting results more closely reflect the DM's intrinsic preferences. A case study in inventory classification is carried out to demonstrate the efficacy of the proposed method.

1. Introduction. The main task of multiple criteria decision analysis (MCDA) is to assist a decision maker (DM) to *choose*, *rank* or *sort* a finite set of alternatives according to two or more criteria [19]. Over the past forty years, many methods have been proposed to solve choice and ranking problems, such as Multiattribute Utility Theory (MAUT) [12], Analytic Hierarchy Process (AHP) [20] and Outranking techniques [18]. But sorting problems have not been systematically explored until recently.

With the evolution of MCDA and the appearance of powerful new classification tools, MCDA sorting has recently become an important research focus. In 2002, Doumpos and Zopounidis published the first book on sorting in MCDA [9], while Zopounidis and Doumpos [29] gave a comprehensive literature review. Chen et al. [4] recently proposed a case-based distance sorting method with an application of water-use analysis for Canadian municipalities, and later Chen et al. [5] studied the extension of traditional sorting to multiple criteria nominal classification problems.

Generally speaking, there are two approaches to sorting: direct judgement and case-based reasoning. Direct judgement methods, such as ELECTRE TRI [28],

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possess a general preference model, and require the DM to provide enough explicit information to evaluate all of its parameters. Case-based reasoning methods require the DM to furnish decisions on selected cases; these decisions are used to calibrate parameters that mimic the DM's preferences as consistently as possible. Case-based approaches include the dominated-based rough sets method [24], UTADIS (UTilités Additives DIScriminantes), MHDIS (Multi-group Hierarchical DIScrimination) [9] and a case-based distance sorting method [4].

Most MCDA sorting methods classify alternatives into a pre-defined number of groups which is usually subjectively set by a DM. In practice, however, a DM may wish to avoid fixing the group count in advance, preferring instead a flexible sorting method that, based upon the sorting results, permits adjustment of the group count to organize the alternatives as efficiently as possible. For example, ABC analysis is a frequently used approach to classifying stock-keeping units (SKUs) in which the most important SKUs are placed in group $\mathcal{A}$, the least important SKUs fall into group $\mathcal{C}$; other SKUs belong to the middle group $\mathcal{B}$. However, this is not the only way to classify SKUs. Due to uncertainty considerations, the DM may wish to compare the results of 2-group, 3-group and 4-group sorting of SKUs before making a final decision. Hence, a sorting procedure with strategic flexibility needs to be used to fulfill this purpose. Many sorting methods would not be suitable for flexible sorting: case-based approaches to sorting, such as the dominated-based rough set method, may not be easily adapted for flexible sorting because of a lack of training case sets; other direct sorting methods, such as ELECTRE TRI, do not have a solid procedure to determine group thresholds to handle different sorting arrangements.

The motivation of this paper is to provide a theoretically sound approach to multiple criteria flexible sorting which allows the DM to explore and compare different kinds of sorting results. The method utilizes the concepts from Data Envelopment Analysis (DEA) [3] considering preference uncertainty. DEA, as first put forward in [3], is a technique used to measure the relative efficiency of a number of similar units performing essentially the same task. Within the past few decades, various types of research have been conducted to apply the concept of DEA in MCDA such as in [7], [8] and [25]. A methodological connection between MCDA and DEA is that if all criteria in an MCDA problem can be classified as either positive criteria (benefits or output) or negative criteria (costs or inputs), then DEA is relevant to MCDA using additive linear value functions [25].

Specifically, in this paper preferences in MCDA have been interpreted as values (preferences over consequences, i.e. physical measurements) and weights (preferences over criteria, the relative importance of criteria). Next, alternative values on ordinal criteria and criterion weights, which we believe the DM may feel more difficult to measure precisely, are expressed in a few interval data-based constraints to reduce the work load of precise value specifications for the DM. Then, a DEA-like model is applied to measure the relative efficiency of an alternative and, accordingly, assigns it to an appropriate group.

The unique features of the proposed method are that (1) the DM controls the group count, and can adjust it interactively; (2) both cardinal and ordinal criteria can be included; (3) preference uncertainty in MCDA has been addressed by considering any available information on values of ordinal criteria and criterion weights as interval data-constraints, so that the generated sorting results more closely reflect the DM's intrinsic preferences; (4) a DEA-like model is proposed to aggregate preference, and hence, applied to measure the efficiency of different alternatives.

A case study in inventory classification is carried out to demonstrate the practical usefulness of the method.

2. Sorting in multiple criteria decision analysis.

2.1. Multiple Criteria Decision Analysis (MCDA).

2.1.1. *The Structure of MCDA.* Any multiple criteria analysis must begin with the processes of defining objectives, arranging them into criteria, identifying all available alternatives, and then determining the consequence of each alternative on each criterion. A consequence is a direct measurement of the success of an alternative according to a criterion, such as cost in dollars.

Figure 1 shows the basic structure of an MCDA problem, with the following definitions and notation:

- $\mathbf{A} = \{A^1, A^2, \cdots, A^i, \cdots, A^n\}$ is the set of alternatives.
- $\mathbf{I} = \mathbf{I}^c \cup \mathbf{I}^o$ is the set of criteria, where $\mathbf{I}^c = \{I_1^c, I_2^c, \ldots, I_p^c\}$ is the set of cardinal criteria and $\mathbf{I}^o = \{I_1^o, I_2^o, \ldots, I_q^o\}$ is the set of ordinal criteria. Of course, $\mathbf{I}^c \cap \mathbf{I}^o = \emptyset$.
- $c_j^i \in \mathbb{R}$ is the consequence (measure) of alternative A^i on cardinal criterion I_j^c. (At this stage, there is no assumption about which values of a consequence are preferable.)
- $d_k^i \in \{1, 2, \ldots, l\}$ is the consequence of alternative A^i on ordinal criterion I_k^o. Formally, $\mathbf{L} = \{L_1, L_2, \ldots, L_l\}$ is the linguistic grade set, where L_1 represents the best grade, L_2 the next best, and so on down to the worst grade, L_l, and $d_k^i = r$ means that A^i has been assessed at grade L_r on criterion I_k^o, i.e. as r^{th} grade. For example, $d_1^3 = 2$ means that alternative A^3 is considered to be 2^{nd} grade on ordinal criterion I_1^o. (For simplicity, we assume that all ordinal criteria have the same linguistic grade set.)

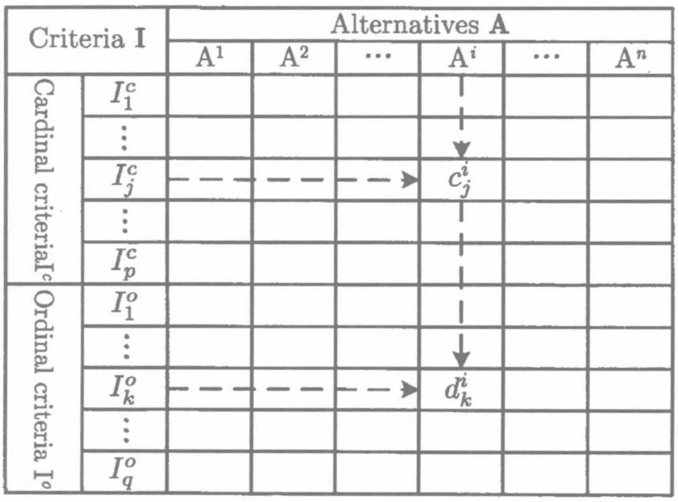

FIGURE 1. The structure of MCDA

Note that it is assumed that there is no uncertainty in the consequences. For cardinal criteria, the DM can measure the consequences of alternatives directly while for ordinal criteria, the values of d_k^i may summarize several estimates of the consequences, perhaps from different individuals.

Roy [19] proposed several MCDA **problématiques**, or problem approaches, with respect to alternative set $\mathbf{A}$:

(α): The **Choice problématique**: Choose the best alternative from $\mathbf{A}$.

(β): The **Sorting problématique**: Sort the alternatives of $\mathbf{A}$ into a predefined number of (relatively) homogeneous groups, arranged in preference order.

(γ): The **Ranking problématique**: Rank the alternatives of $\mathbf{A}$ from best to worst.

2.1.2. *Preference Expression in MCDA.* Obviously, the DM's preferences must be crucial to the solution of any MCDA problem. But different ways of expressing preferences can lead to different results. We distinguish two kinds of preferences, *values*, which are preferences on consequences, and *weights*, which are preferences on criteria.

The value of an alternative on a criterion is a function of the consequence of the alternative on that criterion; this function must reflect the DM's needs, understanding, and objectives. For cardinal criterion I_j^c, the value of alternative A^i is $u_j(A^i) = u_j^i \in \mathbb{R}$; for ordinal criterion I_k^o, it is $v_k(A^i) = v_k^i \in \mathbb{R}$. Thus,

$$u_j(A) = f_j\left(c_j(A)\right) \text{ or } v_k(A) = g_k\left(d_k(A)\right) \tag{1}$$

where $f_j(\cdot)$ and $g_k(\cdot)$ are mappings from consequences to values for the j^{th} cardinal and the k^{th} ordinal criterion, respectively. Better performance of A^i on these criteria is indicated by increases in u_j^i and v_k^i. For ease of comparison and aggregation, u_j^i and v_k^i are usually adjusted so that the same minimum and maximum apply to all criteria.

The DM's evaluations of alternative $A \in \mathbf{A}$ on all cardinal criteria are collected in the *cardinal value vector*, $\mathbf{u}(A) = (u_1(A), u_2(A), \ldots, u_p(A))$. Analogously, the *ordinal value vector* associated with A is $\mathbf{v}(A) = (v_1(A), v_2(A), \ldots, v_q(A))$.

Weights are positive real numbers that indicate the relative importance of criteria. The weight of cardinal criterion I_j^c is $w_j^c \in \mathbb{R}^+$; the weight of ordinal criterion I_k^o is $w_k^o \in \mathbb{R}^+$. The cardinal weight vector is $\mathbf{w}^c = (w_1^c, w_2^c, ..., w_j^c, ..., w_p^c)$, and the ordinal weight vector is $\mathbf{w}^o = (w_1^o, w_2^o, ..., w_j^o, ..., w_q^o)$.

After an MCDA problem has been structured (as in Figure 1), and after the DM's preferences have been acquired, an MCDA problématique can be solved using a global model to aggregate preferences. This model can be represented using an overall evaluation function, $V(\cdot)$, which we write as

$$V(A) = F_c\left(\mathbf{u}(A), \mathbf{w}^c\right) + F_o\left(\mathbf{v}(A), \mathbf{w}^o\right),$$

where $V(A) \in \mathbb{R}$ is the overall evaluation of alternative A, $F_c(\cdot)$ represents the input of the cardinal value vector $\mathbf{u}(A)$ and weight vector $\mathbf{w}^c$ to the overall evaluation, and $F_o(\cdot)$ represents the input of the ordinal value vector $\mathbf{v}(A)$ and the ordinal weight vector $\mathbf{w}^o$ to the overall evaluation. A simple example is the linear additive value function, which has the form

$$V(A^i) = \sum_{j=1}^{p} w_j^c \cdot u_j^i + \sum_{k=1}^{q} w_k^o \cdot v_k^i \tag{2}$$

2.1.3. *Preference Uncertainty.* In some situations, DMs may not easily express their preferences precisely. For example, DMs may not feel comfortable to specify the values of alternatives or criterion weights using real numbers. Instead, they prefer interval data for preference expressions. Much research has been carried out for preference uncertainty in MCDA. For instance, Rios Insua [17] introduced a general framework for sensitivity analysis that expanded results of the traditional Bayesian approach to decision making. Stochastic multicriteria acceptability analysis (SMAA) comprises a family of techniques to handle MCDA problems including incomplete, imprecise, and uncertain information [13]. Vetschera [26] presented a recursive algorithm for volume-based sensitivity analysis of linear decision models and the efficiency of his approach is analyzed both analytically and via computational experiments.

In this paper, preference uncertainty is taken into account by maximizing the possibility of obtaining the best possible evaluation score for alternatives. Such a procedure can provide a fair overall assessment. More specifically, two kinds of preference expressions, the values of alternatives for ordinal criteria and criterion weights, which are usually hard to be measured precisely, are allowed to be set as interval data-based upon DMs' estimates. Then, an optimization model is constructed for each alternative to find the best possible outcome within the predefined interval constraints. Although a similar idea is put forward by Cook and Kress [7, 8], our paper focuses upon the extension of this idea to handle flexible sorting problems.

2.2. Multiple criteria sorting.

Definition 2.1. An *m-sorting* of the alternative set $\mathbf{A}$ is a partition of $\mathbf{A}$ into $m > 1$ non-empty subsets, denoted as $\mathbf{S} = (\mathbf{S}_1, \mathbf{S}_2, \ldots, \mathbf{S}_m)$, satisfying the following conditions:

- $\forall g, h = 1, 2, \ldots, m$, and $g \neq h$, $\mathbf{S}_g \cap \mathbf{S}_h = \varnothing$,
- $\bigcup_{g=1}^{m} \mathbf{S}_g = \mathbf{A}$.

Note that m is the *count* of $\mathbf{S}$, and $\mathbf{S}_g$ is the g^{th} *element* of the sorting $\mathbf{S}$, or the g^{th} *group* of $\mathbf{S}$.

In a sorting, order matters; $\mathbf{S}$ is often written $\mathbf{S}_1 \succ \mathbf{S}_2 \succ \ldots \succ \mathbf{S}_m$, where $\succ$ is pronounced "is preferred to." The idea is that earlier groups contain "better" alternatives, and alternatives in the same group are "about equally good." Figure 2 suggests this interpretation of $\mathbf{A}$, $\mathbf{S}$, and $\mathbf{S}_g$.

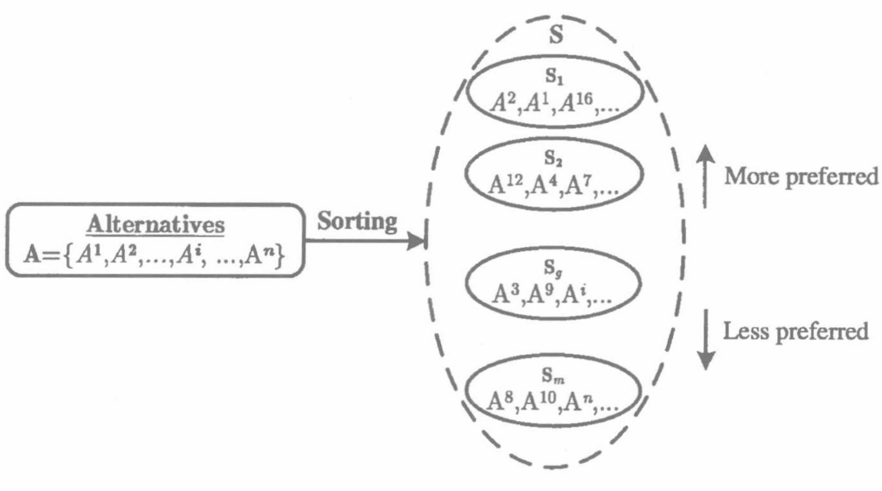

FIGURE 2. Relationship of **A** and **S**

In most sorting procedures, the count m is predetermined. Once the preferences are given, the value of m determines the sizes of the elements of the sorting. In the flexible procedure described next, the DM has the freedom to adjust the element sizes directly, indirectly affecting the count, m. Such flexible adjustments should enable the DM to organize the alternatives more efficiently.

3. Flexible multiple criteria sorting.

3.1. Value acquisition.

3.1.1. *Values Acquisition for Cardinal Criteria.* Many methods are available to obtain the DM's values, such as Multiattribute Utility Theory (MAUT) [12] and the Analytic Hierarchy Process (AHP) [20]. The following two transformations are commonly used and are simple:

- If a larger value of c_j^i represents better performance,

$$u_j^i = \frac{c_j^i - c_j^{\min}}{c_j^{\max} - c_j^{\min}}; \tag{3}$$

- If a smaller value of c_j^i represents better performance,

$$u_j^i = \frac{c_j^{\max} - c_j^i}{c_j^{\max} - c_j^{\min}}, \tag{4}$$

where $c_j^{\min}$ and $c_j^{\max}$ are the minimum and maximum consequences on I_j^c. Note that $0 \leq u_j^i \leq 1$. In practice, the DM may revise the transformations (3) and (4) to express his or her preference more accurately, provided the bounds $0 \leq u_j^i \leq 1$ are maintained.

3.1.2. *Values Acquisition for Ordinal Criteria.* Define an indifference threshold, $\alpha > 0$, with the interpretation that on any ordinal criterion, I_k^o, differences in v_k less than α can be ignored. (For simplicity, we assume that indifference thresholds of all ordinal criteria are the same.) Because of limitations in cognitive ability, non-negligible indifference thresholds are common in practice. For example, in grading a course, an instructor may ignore any differences between scores falling in the interval from 76.5% to 79.5%, assigning them all a grade of B+. Similarly, in thinking of the price of a car, people typically ignore differences of less than \$100, or perhaps \$10 or \$1.

Adapting ideas from DEA [3], our method assesses an alternative $A \in \mathbf{A}$ on the ordinal criteria in $\mathbf{I}^o$ by finding the maximum possible values of $\mathbf{v}(A)$ consistent with the grades of A on $I_1^o, I_2^o, \ldots, I_q^o$. Therefore, to evaluate A^i we find the maximum value of v_k^i that is consistent with the following conditions [8]:

- For alternatives A^m and A^l, and for $k = 1, 2, ..., q$,

$$\text{if } d_k^m > d_k^l,\ v_k^l - v_k^m \geq (d_k^m - d_k^l)\alpha; \tag{5}$$

- For any alternative, A^i, and for $k = 1, 2, ..., q$,

$$\alpha \leq v_k^i \leq 1. \tag{6}$$

Here, (5) provides lower bounds for value differences between alternatives at different grades, and (6) normalizes the alternative values on each ordinal criterion. For example, if $d_k^1 = 2$ and $d_k^2 = 5$, then the constraints read $v_k^2 - v_k^1 \geq (5-2)\alpha = 3\alpha$, $\alpha \leq v_k^1 \leq 1$ and $\alpha \leq v_k^2 \leq 1$. Note that the DM could provide information to represent more precisely his or her preferences over linguistic grades; for example, he could specify that the difference in value between L_1 and L_2 exceeds that between L_2 and L_3. For some useful suggestions, see [8].

3.2. **Sorting model construction.** To aggregate the values of A^i over different criteria, a linear additive function, (2), is selected, and re-written as follows:

$$V(A^i) = \sum_{j=1}^{p} w_j^c \cdot u_j^i + \sum_{k=1}^{q} w_k^o \cdot v_k^i. \tag{7}$$

To apply (7), we need only find the weights w_j^c for $j = 1, 2, \ldots, p$ and w_k^o for $k = 1, 2, \ldots, q$. First, select an indifference threshold for weights, $\beta > 0$. Again following the principle that an evaluation should be the maximum possible, consistent with all "natural" constraints, we assume the weights are chosen to maximize $V(A^i)$, subject to the following conditions:

- To make comparisons easy, $V(A^i)$ must lie between 0 and 1, i.e.
$$0 \leq V(A^i) \leq 1 \text{ for } i = 1, 2, \ldots, n. \tag{8}$$
- Because differences in w_j^c and w_k^o less than β are not meaningful, we require that
$$w_j^c \geq \beta \text{ for } j = 1, \ldots, p \text{ and } w_k^o \geq \beta \text{ for } k = 1, \ldots, q. \tag{9}$$
- Any preference information that the DM can provide about weights, however imprecise, must be satisfied, so that the sorting more closely reflects the DM's intrinsic preferences. The imprecise preference expressions listed by [21] can be adapted for this purpose, as suggested next.
  - **Strict ranking** If the DM can state that the j^{th} criterion is more important than the k^{th}, where $j \neq k$ and both I_j^c and I_k^c are cardinal criteria or both I_j^o and I_k^o are ordinal criteria, then
  $$w_j^c - w_k^c \geq \beta \text{ and } w_j^o - w_k^o \geq \beta, \tag{10}$$
  as appropriate. Also, for all $j = 1, \ldots, p$ and $k = 1, \ldots, q$, the DM may agree that cardinal criterion I_j^c is not equal in importance to ordinal criterion I_k^o, so that
  $$|w_j^c - w_k^o| \geq \beta. \tag{11}$$
  - **Fixed bounds** The DM may wish to provide lower and upper bounds on the importance of criteria. If L_j and U_j are lower and upper bounds for the weight w_j^c of cardinal criterion I_j^c, and L_k and U_k are the lower and upper bounds for the weight w_k^o of ordinal criterion I_k^o, then,
  $$\beta \leq L_j \leq w_j^c \leq U_j \text{ and } \beta \leq L_k \leq w_k^o \leq U_k. \tag{12}$$

The parameters α and β both represent indifference thresholds, for which α applies to the values of ordinal criteria and β to the weights of all criteria. Since values and weights are normalized between 0 and 1, it is useful to adjust these thresholds simultaneously, which we facilitate by setting

$$\alpha = J\beta.$$

We call $J \in \mathbb{R}^+$ the *adjustment ratio* for thresholds, and suggest that the DM fix J such that all thresholds can be set by adjusting β. Usually, larger cardinality of criteria sets and the larger linguistic grade set (associated with ordinal criteria), require smaller values of these thresholds to distinguish among alternatives. Hence, a reasonable starting value for J is $\dfrac{p+q}{l}$, where $l = |\mathbf{L}|$ and $p + q = |\mathbf{I}|$. Note that when there are no ordinal criteria, l and α are set as 0, and the proposed method is carried out simply by adjusting β.

The aggregate score for a particular alternative, $A^{i'}$, is the solution of the optimal program

$\mathbf{P}(A^{i'}, \mathbf{A}, \beta)$:

Maximize $V^*(A^{i'}) = \left[\sum_{j=1}^{p} w_j^c \cdot u_j^{i'} + \sum_{k=1}^{q} w_k^o \cdot v_k^{i'}\right]$

Subject to:

$0 \leq \sum_{j=1}^{p} w_j^c \cdot u_j^i + \sum_{k=1}^{q} w_k^o \cdot v_k^i \leq 1$, for $i = 1, 2, ..., n$;

$v_k^\ell - v_k^i \geq (d_k^i - d_k^\ell)\alpha$ whenever $d_k^i > d_k^\ell$, where $k = 1, 2, \ldots, q$ and $i, \ell \in \{1, 2, \ldots, n\}, i \neq \ell$;

$\alpha \leq v_k^i \leq 1$, for $k = 1, 2, ..., q$, and $i = 1, 2, ..., n$;

$w_j^c, w_k^o \geq \beta$, for $j = 1, 2, ..., p$, and $k = 1, 2, ..., q$;

Constraints (10)-(12) as applicable;

$\alpha = J\beta$.

The inputs to $\mathbf{P}(A^{i'}, \mathbf{A}, \beta)$ are u_j^i and d_k^i (for $i = 1, 2, ..., n$, $j = 1, 2, ..., p$, and $k = 1, 2, ..., q$), and β. (The parameter J can also be adjusted if the DM so desires.) The output of $\mathbf{P}(A^{i'}, \mathbf{A}, \beta)$, $V^*(A^{i'})$, is the highest aggregate score that $A^{i'}$ can achieve, consistent with all the specified conditions.

For $i = 1, 2, \ldots, n$, the aggregate score of alternative A^i must be calculated using $\mathbf{P}(A^i, \mathbf{A}, \beta)$. If $V^*(A^i) = 1$, then A^i can achieve the best possible performance, and we call it *efficient*. Of course, whether an alternative is efficient can change if β changes. We denote the set of efficient alternatives at threshold β as $\mathit{Eff}\,(\mathbf{A}, \beta) = \{A^i \in \mathbf{A} : V^*(A^i) = 1\}$; the alternatives of $\mathbf{A} - \mathit{Eff}\,(\mathbf{A}, \beta)$ are then *inefficient* at threshold β.

3.3. **Setting the threshold.** Just as in [8], the following theorem holds:

Theorem 3.1. *For any $i = 1, 2, \ldots, n$, the aggregate score, $V^*(A^i)$, is non-increasing in β.*

Proof. The score $V^*(A^i)$ is the solution of $\mathbf{P}(A^i, \mathbf{A}, \beta)$. As β increases, the constraints of $\mathbf{P}(A^i, \mathbf{A}, \beta)$ cause the feasible region to shrink. □

A reasonable upper bound of β would therefore be the largest value consistent with efficiency, i.e. the largest value such that there is at least one efficient alternative. The following program finds this value:

$\mathbf{P}(\mathbf{A})$

$\beta^* = \max\ \beta$

Subject to:

$\mathit{Eff}\,(\mathbf{A}, \beta) \neq \emptyset$

All constraints of $\mathbf{P}(A^i, \mathbf{A}, \beta)$, for $i = 1, ..., n$.

It is immediate from Theorem 3.1 that, if A^i is efficient when $\beta = \beta^*$, then A^i is efficient whenever $\beta < \beta^*$. Also, $\mathit{Eff}\,(\mathbf{A}, \beta) \neq \emptyset$ whenever $\beta \leq \beta^*$.

A reasonable lower bound for β, say β_*, would have the property that

$$0 < \beta < \beta_* \Rightarrow \mathit{Eff}\,(\mathbf{A}, \beta) = \mathit{Eff}\,(\mathbf{A}, \beta_*) \tag{13}$$

In particular, reducing β below β_* would not increase the number of efficient alternatives. Note that β_* is analogous to ϵ in DEA [3]. The maximum value of β_* satisfying (13) depends on the specific problem, of course. The starting point suggested by the DEA software, Frontier Analyst [1], namely $\beta_* = 1 \times 10^{-6}$, is usually small enough to satisfy (13). Note that when the set of criteria or alternatives is small, there may has no efficient alternative no matter how small β_* it is. Under such extreme decision situation, the MCDA problem may not be so difficult to handle considering the relatively small number of criteria or alternatives. The DM can manually set the thresholds to sort alternatives into different groups based upon the values generated from the model.

In summary, Theorem 3.1 demonstrates that greater values of β increase the discrimination of the aggregate scores of the alternatives in $\mathbf{A}$, in that fewer alternatives are efficient. The program $\mathbf{P}(\mathbf{A})$ provides a good upper bound for β; at or below β^*, there is always at least one efficient alternative. The lower bound for β, ideally satisfying (13), generates the largest number of efficient alternatives. As β is decreased from β^* to β_*, more and more alternatives become efficient; once an alternative becomes efficient, it remains so as β decreases.

3.4. **Sorting strategies.** We now suggest sorting methods that can be applied to the alternatives of $\mathbf{A}$ to produce an m-sorting $(\mathbf{S}_1, \mathbf{S}_2, \ldots, \mathbf{S}_m)$. Each method assigns alternatives to groups in sequence, beginning with the best group, $\mathbf{S}_1$, continuing with $\mathbf{S}_2$, and so on down to the bottom group, $\mathbf{S}_m$. In each method, an initial value for β is determined, and then *Eff* $(\mathbf{A}, \beta)$ is assigned to $\mathbf{S}_1$. Then, perhaps after the value of β is reset, *Eff* $(\mathbf{A} - \mathbf{S}_1, \beta)$ is assigned to $\mathbf{S}_2$. In each step, the efficient unassigned alternatives (with respect to the current value of β) are assigned to the next sorting element.

If the value of m is predetermined, then the procedure described above should be used to determine sorting elements $\mathbf{S}_1, \mathbf{S}_2, \ldots, \mathbf{S}_{m-1}$, and then all unassigned elements should be assigned to $\mathbf{S}_m$. In fact, the process can be stopped at any stage (thereby determining m) by assigning all unassigned alternatives to the next group and declaring it to be the bottom group. Note, however, that the process must stop when all unassigned alternatives are efficient, in which case they must constitute the bottom group, which determines m. For this reason, it may not be possible to achieve a particular value of m. This problem may be remedied by selection of smaller values of β in the initial steps, which will reduce the size of the earlier sorting elements and thereby increase the count, as follows from Theorem 3.1. As illustrated below, however, this problem is not common.

In the initial step, the value of β must be selected in the interval $[\beta_*, \beta^*]$. The next lemma shows that selecting a value of β from this same interval in subsequent steps is always feasible, in the sense that there is at least one efficient alternative, so that the next group is non-empty.

Lemma 3.2. *Let $\mathbf{A}' \subseteq \mathbf{A}$, where $\mathbf{A}' \neq \emptyset$, and let β^* and $\beta^{*\prime}$ be the values of programs $\mathbf{P}(\mathbf{A})$ and $\mathbf{P}(\mathbf{A}')$, respectively. Then $\beta^* \leq \beta^{*\prime}$.*

Proof. Analogous to the proof of Theorem 3.1. □

It is clear that the indifference threshold, β, is a crucial input to each step: changing the value of β may drastically change the number of efficient alternatives. The sorting methods we suggest differ in indifference thresholds, which constitute a convenient source of sorting flexibility. We denote by β_1 the value of β used to determine the first group, $\mathbf{S}_1$, β_2 the value of β used to determine the second group, $\mathbf{S}_2$. etc.

- Lower bound sorting strategy ($M1$)

  Let $\beta_1 = \beta_2 = \ldots = \beta_*$. As noted above, the lower bound indifference threshold, β_*, generates the largest possible efficient set of alternatives, so sorting method $M1$ tends to produce larger sorting elements, and (if the sorting is allowed to continue until all unassigned alternatives are efficient) minimizes m, the count of the sorting. Usually, the approximation given in (13) is sufficient for β_*.

- Maximum threshold sorting strategy ($M2$)
  Set $\beta_1 = \beta^*$, where β^* is determined by $\mathbf{P}(\mathbf{A})$, and let $\mathbf{S}_1 = \mathit{Eff}\,(\mathbf{A}, \beta_1)$. For $g = 2, 3, \ldots$, let β_g be the value of the program $\mathbf{P}(\mathbf{A} - \bigcup_{h=1}^{g-1} \mathbf{S}_h)$, and let $\mathbf{S}_g = \mathit{Eff}\,(\mathbf{A} - \bigcup_{h=1}^{g-1} \mathbf{S}_h, \beta_g)$. This procedure terminates when $\mathit{Eff}\,(\mathbf{A} - \bigcup_{h=1}^{g-1} \mathbf{S}_h, \beta_g) = \mathbf{A} - \bigcup_{h=1}^{g-1} \mathbf{S}_h$, in which case $m = g$ and $\mathbf{S}_m = \mathbf{A} - \bigcup_{h=1}^{g-1} \mathbf{S}_h$. It follows from Lemma 3.2 that $\beta_1 \leq \beta_2 \leq \ldots \leq \beta_m$. It also follows from the discussion above that sorting method $M2$ produces the smallest possible sorting elements, and therefore maximizes the count, m. Of course, the DM can reduce m by choosing to end the procedure whenever appropriate.
- Flexible threshold sorting strategy ($M3$)
  First set $g = 1$ and $\mathbf{A}_g = \mathbf{A}$. Choose β_g satisfying $\beta_* < \beta_g < \beta^*$, where β^* is determined by $\mathbf{P}(\mathbf{A}_g)$ and assign $\mathit{Eff}\,(\mathbf{A}_g, \beta_g)$ to $\mathbf{S}_g$. The DM may wish to adjust β_g upward (respectively, downward) to reduce (increase) the size of $\mathbf{S}_g$. Then $\mathbf{A}_{g+1} = \mathbf{A}_g - \mathbf{S}_g$. If $\mathbf{A}_{g+1} = \emptyset$, then $g = m$ and the procedure terminates. Otherwise, the DM has the choice of assigning $\mathbf{A}_{g+1}$ to $\mathbf{S}_{g+1}$ and terminating the procedure at $m = g + 1$, or iteratively repeating the above procedure with g increased by 1. Note that $M3$ produces smaller groups than $M1$ and larger groups than $M2$; of course, the DM can minimize or maximize the size of a sorting element by switching to $M1$ or $M2$. Also, unless the DM terminates the procedure by assigning all unassigned alternatives to the next group, the group count in $M3$ is intermediate between the group counts of $M1$ and $M2$. The discussion above, including Lemma 3.2, shows that the value of β^* increases at each step, though of course β_g may remain constant, or decrease, if the DM wishes to maximize the size of the next sorting element.

In summary, the composition and size of the sorting elements, as well as the group count, can be influenced by adjusting the indifference threshold input, β_g, in the sequence of sorting steps suggested above. The extremes are $M1$ and $M2$; we recommend $M3$ for the flexibility it can achieve. For example, it is usually possible to achieve (approximately) particular sizes of sorting elements, as may be required for budgetary or other practical reasons. Next, this flexibility is demonstrated using a case study in inventory classification.

4. **A case study in inventory classification.** As the globalization of business accelerates, firms increasingly need efficient and effective inventory management to maintain competitive advantage [22, 23]. Many research initiatives investigated different scenarios and designed various optimal inventory management solutions [10, 15, 27]. It is widely accepted that the basis of a sound inventory control scheme is a sorting of stock-keeping units (SKUs) into meaningful and manageable groups, so that different inventory policies can be designed for each group according to its importance to the firm and other relevant characteristics [2].

4.1. **Background.** This case study uses data provided by Flores et al. [11] for a hospital inventory management problem which has been investigated by several researchers using different approaches. For example, Chen et al. [6] proposed a case-based distance approach to sort SKUs into three groups; Ramanathan [16] examined the same problem using a DEA model but his method is fixed for a three-group sorting and only considers the preference uncertainty of criterion weights. All previous methods applying to this problem are limited to a three-group sorting and do not provide a flexible sorting procedure.

The SKUs in this case study are the 47 disposable supply items required by a hospital-based respiratory therapy unit, which in the original study were classified using AHP [20]. The four criteria were defined as follows:

- Average unit cost (AUC) (\$), which ranges from \$5.12 to \$210.00;
- Annual dollar usage (ADU) (\$), which ranges from \$25.38 to \$5840.64;
- Lead time (LD) (weeks), the time for replenishment of an SKU after ordering, which ranges from from 1 to 7 weeks.
- Criticality factor (CF), 1, 0.50, or 0.01: a value of 1 indicates that an item is very critical, a value of 0.50 that it is moderately critical, and a value of 0.01 that it is non-critical;

Based on this information, AUC, ADU and LD are treated as cardinal criteria and identified as I_1^c, I_2^c and I_3^c, respectively. However, CF is treated as an ordinal criterion, and identified as I_1^o, since the values seemed easier to interpret as ordinal information only. Thus, SKUs with a CF value of 1 are assigned the best grade, L_1, on I_1^o, SKUs with a CF value of 0.50 are assigned the second grade, L_2, and SKUs with value of 0.01 are assigned the worst grade, L_3. The consequences of SKUs A^1 through A^{47} on these four criteria, with L_1, L_2 and L_3 represented using 1, 2, and 3, respectively, on criterion I_1^o, are shown in Table 4.1.

4.2. **Values acquisition and processing.** The consequence information in Table 4.1 is processed according to the definitions and constraints (3)—(6). The value information is shown in Table 4.2. Since the value of α has not yet been set, the value of alternative A^i on ordinal criterion I_1^o is simply denoted v_1^i; note that this value must satisfy constraints (5) and (6).

Also, it is assumed that the DM requires that criterion ADU receive more importance than any other criterion. Hence, conditions (10) apply, which imposes the constraints $w_2^c - w_1^c \geq \beta$, $w_2^c - w_3^c \geq \beta$ and $w_2^c - w_1^o \geq \beta$ on any sorting.

4.3. **Lower bound sorting.** The software Lingo [14], a comprehensive tool designed to solve linear, nonlinear and integer optimization models, is used to conduct the calculations in the case study. Firstly, sorting method $M1$ in Section 3.4, based on the lower-bound threshold, is applied first. It is a good idea to adopt this method first, as it establishes the minimum group count—if fewer groups are required, it will be necessary to curtail a procedure. Following the advice above, β_* is set at 1×10^{-6} and $J = \dfrac{p+q}{l} = \dfrac{4}{3}$. The detailed computation is omitted. The results are shown in Figure 3. But the DM may not be satisfied with this sorting, since $\mathbf{S}_1$, the group of SKUs requiring the greatest management attention, contains over half of all SKUs, and is by far the largest element of the sorting.

4.4. **Flexible threshold sorting.** Method $M3$ is now applied twice. The objectives are to achieve a three-group sorting and a four-group sorting with specified target sizes for the sorting elements, and then to compare the sortings to ensure consistency.

	Criteria			
SKU	I_1^c (AUC, \$)	I_2^c (ADU, \$)	I_3^c(LT, weeks)	I_1^o (CF)
A^1	49.92	5840.64	2	1
A^2	210.00	5670.00	5	1
A^3	23.76	5037.12	4	1
A^4	27.73	4769.56	1	3
A^5	57.98	3478.80	3	2
A^6	31.24	2936.67	3	2
A^7	28.20	2820.00	3	2
A^8	55.00	2640.00	4	3
A^9	73.44	2423.52	6	1
A^{10}	160.50	2407.50	4	2
A^{11}	5.12	1075.20	2	1
A^{12}	20.87	1043.50	5	2
A^{13}	86.50	1038.00	7	1
A^{14}	110.40	883.20	5	2
A^{15}	71.20	854.40	3	1
A^{16}	45.00	810.00	3	2
A^{17}	14.66	703.68	4	2
A^{18}	49.50	594.00	6	2
A^{19}	47.50	570.00	5	2
A^{20}	58.45	467.60	4	2
A^{21}	24.40	463.60	4	1
A^{22}	65.00	455.00	4	2
A^{23}	86.50	432.50	4	1
A^{24}	33.20	398.40	3	1
A^{25}	37.05	370.50	1	3
A^{26}	33.84	338.40	3	3
A^{27}	84.03	336.12	1	3
A^{28}	78.40	313.60	6	3
A^{29}	134.34	268.68	7	3
A^{30}	56.00	224.00	1	3
A^{31}	72.00	216.00	5	2
A^{32}	53.02	212.08	2	1
A^{33}	49.48	197.92	5	3
A^{34}	7.07	190.89	7	3
A^{35}	60.60	181.80	3	3
A^{36}	40.82	163.28	3	1
A^{37}	30.00	150.00	5	3
A^{38}	67.40	134.80	3	2
A^{39}	59.60	119.20	5	3
A^{40}	51.68	103.36	6	3
A^{41}	19.80	79.20	2	3
A^{42}	37.70	75.40	2	3
A^{43}	29.89	59.78	5	3
A^{44}	48.30	48.30	3	3
A^{45}	34.40	34.40	7	3
A^{46}	28.80	28.80	3	3
A^{47}	8.46	25.38	5	3

TABLE 1. SKUs Information, adapted from Flores et al. [11]