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ON THE INVERSE MINIMUM SPANNING TREE PROBLEM WITH MINIMUM NUMBER OF PERTURBED EDGES

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Abstract

Let G=<V, E, L> be a network with the vertex set V, the edge set E and the length vector L, and let T^* be a prior determined spanning tree of G. The inverse minimum spanning tree problem with minimum number of perturbed edges is to perturb the length vector L to $L+\delta$, such that T^* is one of minimum spanning trees under the length vector $L+\delta$ and the number of perturbed edges is minimum. This paper establishes a mathematical model for this problem and transforms it into a minimum vertex covering problem in a bipartite graph G_0, a path-graph. Thus a strongly polynomial algorithm with time complexity $O(mn^2)$ can be designed by using Hungarian method.

Keywords: Inverse network optimization problem, minimum spanning tree, vertex covering set

1. Introduction

Perhaps the inverse network optimization problems were first introduced by Burton and Toint (1992). They studied the inverse shortest path problem under L_2 norm and provided some applications of this problem in traffic models and transportation networks. They also discussed the inverse shortest path problem under L_1 norm and showed that this problem is NP-complete if the perturbation of every edge is bounded (Burton and Toint 1994). Since then, more and more researchers have been interested in the inverse network optimization problems and the studied field has been extended to other inverse network optimization problems, for example, Xu and Zhang (1995) considered the inverse weighted shortest path problem; Sokkaling (1995) studied the inverse minimum cost flow problem under L_1, L_2 and L_∞ norms; Yang and Zhang (1998) investigated the inverse maximum capacity problem. Recently, Huang and Liu (1999) discussed the inverse linear programming problem and gave some applications to the minimum perfect k-matching problem in bipartite graph. This paper will introduce inverse minimum

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spanning tree problem with the objective of minimum number of perturbed edges and transform it into minimum vertex covering set problem in bipartite graph.

The minimum spanning tree problem is a well-studied problem in network optimization, hence its inverse problems are also attractive in inverse network optimization field. Let $G=<V, E, L>$ be a network with the vertex set V ($|V|=n$), the edge set E ($|E|=m$) and the length vector L, let T^* be a prior determined spanning tree of G. The general inverse minimum spanning tree problem is to perturb the length vector L to $L+\delta$, such that T^* is one of minimum spanning trees under $L+\delta$ and objective function $\|\delta\|_p$ is minimum.

Based on our knowledge, L_1 norm, L_2 norm and L_∞ norm are three popular choice for objective functions. Zhang and Liu (1996) presented the first paper about inverse minimum spanning tree problem. Sokkalingam (1999) studied the inverse minimum spanning tree problem under L_1 norm and L_∞ norm.

In this paper, we introduce the number of perturbed edges as a new objective function in inverse minimum spanning tree problem, i.e., the objective function is

$$\left|\left\{e_j : \delta_j \neq 0, e_j \in E\right\}\right|.$$

The aim of this problem is to perturb the length of edges to ensure that the prior determined spanning tree T^* is one of minimum spanning trees and the number of perturbed edges is minimum. Considering the number of perturbed edges as an objective function is very significant and this kind of problems has strongly practical background in network design problems.

We first establish mathematical model of inverse minimum spanning tree problem with minimum number of perturbed edges and then prove that this problem can be transformed into minimum vertex covering set problem in a bipartite graph $G_0=<E_1 \cup E_2, A_0>$ arisen from the original network. So Hungarian method can be exploited as a subroutine to design an algorithm for this problem with time complexity $O(mn^2)$.

2. Establishment Process of the Mathematical Model

In this section, we want to establish mathematical formulation of inverse minimum spanning tree problem with minimum number of perturbed edges and reveal some properties of the mathematical model which play an important role in our algorithm. Let us adopt the network terminology and notation given by Bondy and Murty (1978).

Assume that $G=<V, E, L>$ is an undirected network and that T^* is a prior determined spanning tree of G. Let $E(T^*)$ be the edge set of T^*. The task we face with now is to determine whether T^* is a minimum spanning tree of G. If the answer is "yes", then stop. Otherwise, we perturb the length vector L to a new length vector $L+\delta$, such that T^* is one of minimum spanning trees of perturbed network $G'=<V,E,L+\delta>$ and simultaneously, the number of perturbed edges from G to G' is minimum, i.e.,

$|\{e_j : \delta_j \neq 0, e_j \in E\}|$ is minimum.

Noticing that every spanning tree of G always has $n-1$ edges, without loss of the generality, we assume that $E(T^*) = \{e_1, e_2, \cdots, e_{n-1}\}$ and the length parameter l_i for every edge $e_i \in E$ satisfies $l_i > 0$.

Definition 1 Refer to the set of all the edges in T^* as tree-edge set denoted by E_1 and the set of all the edges in $E \backslash E(T^*)$ notree-edge set denoted by E_2, i.e.,

$$E_1 = \{e_i : e_i \in E(T^*)\} \text{ and } E_2 = E - E_1 .$$

Based on the Properties of spanning tree, we know that for an arbitrary notree –edge e_j, there exists a unique path in T^* that connects the two vertices of edge e_j. So we give the next definition which is the key step to establishing our model.

Definition 2 For any $e_j \in E_2$, let $P_j = \{ e_i : e_i$ is the edge in the unique path in T^* that connects the two vertices of edge $e_j \}$.

To establish our mathematical model, we need a rule to determine whether the given spanning tree T^* is a minimum spanning tree under perturbed length vector $L+\delta$.

Lemma 1 (Bondy, 1978) *The necessary and sufficient condition for T^* to be a minimum spanning tree in perturbed network $G' = < V, E, L+\delta >$ is that for any $e_j \in E_2$ and for any $e_i \in P_j$,*

$$l_i + \delta_i \leq l_j + \delta_j ,$$

where $\delta = < \delta_1, \delta_2, \cdots, \delta_m >$ is one perturbed value vector and δ_j is perturbed value of edge $e_j, 1 \leq j \leq m$.

From the original network G to the perturbed network G', the set of perturbed edges is clearly $\{e_j : \delta_j \neq 0, e_j \in E\}$.

From Definition 1, Definition 2 and Lemma 1, we can formulate the inverse minimum spanning tree problem with minimum number of perturbed edges as the following non-linear mathematical programming:

$$\text{Minimize } |\{e_j : \delta_j \neq 0, e_j \in E\}|$$

Subject to:

$$l_i + \delta_i \leq l_j + \delta_j, e_i \in P_j, e_j \in E_2 \qquad (1)$$

δ_j is unrestricted, $1 \leq j \leq m$.

For simplicity, we use $\|\delta\|_1$ to denote $|\{e_j : \delta_j \neq 0, e_j \in E\}|$ in what follows.

Proposition 1 *Assume that $\delta^* = < \delta_1, \delta_2, \cdots, \delta_m >$ is an optimal solution of formulation (1). Then $\delta_i \leq 0$ for all $e_i \in E_1$ and $\delta_j \geq 0$ for all $e_j \in E_2$.*

Proof. Assume on the contrary that there is a $e_i \in E_1$, such that $\delta_i > 0$ in the optimal solution δ^*. By optimal perturbed vector δ^*, we construct a new perturbed vector

$$\delta' = < \delta_1', \delta_2', \cdots, \delta_m' >$$

in the following way that $\delta_i' = 0$ and $\delta_j' = \delta_j$ if $j \neq i$. Note that δ' also satisfies all the constraints of formulation (1), but

$$\|\delta'\|_1 = \|\delta^*\|_1 - 1,$$

which contradicts the optimal property of δ^*. This proves the first claim.

A similar proof can be given for the second claim.

■

From Proposition 1, we obtain a variant of formulation (1) and restate this variant in the following formulation :

$$\text{Minimize}\left|\{e_j : \delta_j \neq 0, 1 \le j \le m\}\right|$$

Subject to:

$$\begin{aligned} & l_i - \delta_i \le l_j + \delta_j, \quad e_i \in P_j, \quad e_j \in E_2 \\ & \delta_i \ge 0, \text{ for all } e_i \in E_1 \qquad\qquad (2) \\ & \delta_j \ge 0, \text{ for all } e_j \in E_2 \end{aligned}$$

Now we investigate the constraints in formulation (2). If there exists an $e_i \in P_j$ which satisfies $l_i \le l_j$, then the inequality $l_i - \delta_i \le l_j + \delta_j$ is trivial, since any nonnegative perturbed vector δ satisfies automatically inequality $l_i - \delta_i \le l_j + \delta_j$. So in the remainder of this paper, we always assume that $l_i > l_j$ for all $e_i \in P_j$ and $e_j \in E_2$, otherwise, delete e_i from set P_j.

Observing that every constraint in formulation (2) is a contrast relation between tree-edge length and notree-edge length, we then define a new graph G_0 to show this contrast relation.

Definition 3 Let $G_0 =< E_1 \cup E_2, A_0 >$, where

$$A_0 = \{< e_i, e_j >: e_i \in P_j, e_j \in E_2\}.$$

G_0 is termed as path-graph in this paper.

From Formulation (2) and Definition 3, we know that there is a one-to-one correspondence between the edges in G_0 and the constraints in formulation (2).

Denote neighbor set of vertex e_i in graph G_0 by $N_{G_0}(e_i)$ for all $e_i \in E$.

Proposition 2 *Assume that* $\delta^* = <\delta_1, \delta_2, \cdots, \delta_m>$ *is an optimal solution of formulation (2). If* $\delta_i > 0$, *then there exists an* $e_j \in N_{G_0}(e_i), \delta_j = 0$, *for some* $e_i \in E_1$. *Similarly, if* $\delta_j > 0$, *then there exists an* $e_i \in N_{G_0}(e_j), \delta_i = 0$, *for some* $e_j \in E_2$.

Proof. We proceed by contradiction.

Let $\delta^* =< \delta_1, \delta_2, \cdots, \delta_m >$ be an optimal solution to formulation (2) and assume that there exists an $e_i \in E_1$, such that $\delta_i > 0$ and $\delta_j > 0$ for all $e_j \in N_{G_0}(e_i)$. δ^* is a feasible solution of formulation (2) and automatically satisfies the following inequalities:

$$l_i - \delta_i \le l_j + \delta_j, \quad e_i \in P_j, \quad e_j \in E_2$$

$$\delta_i \ge 0, \; e_i \in E_1$$

$$\delta_j \ge 0, \; e_j \in E_2$$

Now utilizing δ^*, we construct a new vector $\bar{\delta} =< \bar{\delta}_1, \bar{\delta}_2, \cdots, \bar{\delta}_m >$ in the following way that:

$$\bar{\delta}_i = 0, \quad \bar{\delta}_k = \delta_k, e_k \in E_1 \setminus \{e_i\};$$

$$\bar{\delta}_j = \delta_j + \delta_i \text{ for all } e_j \in N_{G_0}(e_i),$$

and

$$\bar{\delta}_j = \delta_j \quad \text{for all} \quad e_j \in E_2 \setminus \{N_{G_0}(e_i)\}.$$

From the above construction progress, it is obvious that $\bar{\delta}$ also satisfies all the constraints in formulation (2), but

$$\|\bar{\delta}\|_1 = \|\delta^*\|_1 - 1.$$

This contradicts the optimal property of δ^*.

The second claim can be proved in the same way.

■

Proposition 3 *Suppose that* $\delta^* = <\delta_1, \delta_2, \cdots, \delta_m>$ *is an optimal solution of formulation (2). Calculate* $\alpha = \max\{\delta_i : e_i \in E_1\}$ *and* $\beta = \max\{\delta_j : e_j \in E_2\}$ *from* δ^*. *Then construct a new vector*

$$\bar{\delta} = <\bar{\delta}_1, \bar{\delta}_2, \cdots, \bar{\delta}_m>$$

in following way:

$$\bar{\delta}_i = \begin{cases} \alpha & \text{if } \delta_i > 0 \\ 0 & \text{otherwise} \end{cases}, \textit{for all } e_i \in E_1;$$

$$\bar{\delta}_j = \begin{cases} \beta & \text{if } \delta_j > 0 \\ 0 & \text{otherwise} \end{cases}, \textit{for all } e_j \in E_2.$$

Then $\bar{\delta}$ *is also an optimal solution of formulation (2).*

Proof. In the inequalities $l_i - \delta_i \le l_j + \delta_j$, if δ_i>0, then substitute δ_i for α; if δ_j>0, then substitute δ_j for β. After the progress of substitution, the inequalities are all preserved, and

$$\left\|\bar{\delta}\right\|_1 = \left\|\delta^*\right\|_1.$$

So we can claim that $\bar{\delta}$ is also an optimal solution of formulation (2).

■

Proposition 3 implies that we can make the optimal solution uniformed by only two nonzero number α and β.

3. Main Result and Algorithm

In this section, we want to design a strongly polynomial algorithm for solving the formulation (2). Our algorithm depends on the concept of minimum covering set in bipartite graph.

Definition 4 Let G=<V, E> be a bipartite graph with the vertex partition $V = V_1 \cup V_2$. Let M be a subset of edge set E. If arbitrary two edges in M are not adjacent in G, then M is called a matching set. Let C be a subset of vertex set $V_1 \cup V_2$. If every edge in G has at least one vertex in C, then C is called a vertex covering set.

Lemma 2 (Bondy, 1978) *For any bipartite graph G,* $|M^*| = |C^*|$, *where* C^* *is a minimum cardinality vertex covering set and* M^* *is a maximum cardinality matching set.*

After the above preparing work, we now give the main result of this paper.

Theorem 1 $\left\|\delta^*\right\|_1 = \left|C^*\right|$ *for any optimal solution* δ^* *of formulation (2) and any minimum vertex covering set* C^* *of bipartite graph* G_0, *where* $|C^*|$ *is the cardinality of set* C^*.

Proof. Assume that

$$\delta^* = <\delta_1, \delta_2, \cdots, \delta_m>$$

is an optimal solution of formulation (2), which has been uniformed by two nonzero numbers α and β. Then construct two vertex subsets X and Y of bipartite graph G_0 as follows:

$$X = \{e_i : \delta_i = \alpha, \text{for all } e_i \in E_1\} \text{ and}$$
$$Y = \{e_j : \delta_j = \beta, \text{for all } e_j \in E_2\}.$$

We can conclude that $X \cup Y$ is a vertex covering set of G_0. From the definition of G_0, we know that $l_i > l_j$, for all $<e_i, e_j> \in A_0$. Since δ^* is a feasible solution of formulation (2), we also have that

$$l_i - \delta_i \le l_j + \delta_j \text{ for all} <e_i, e_j> \in A_0.$$

The two inequalities above mean that at least one of δ_i and δ_j is greater than 0, i.e., at least one vertex of every edge in G_0 is covered by $X \cup Y$. So $X \cup Y$ is a vertex covering set of bipartite graph G_0. This implies that

$$\left\|\delta^*\right\|_1 = |X| + |Y| \ge \left|C^*\right|,$$

where C^* is a minimum vertex covering set of bipartite graph G_0.

On the other hand, we assume that $C^* = X^* \cup Y^*$ is a minimum vertex covering set of bipartite graph G_0, where

$$X^* \subseteq E_1, Y^* \subseteq E_2 .$$

Construct a perturbed vector $\delta = <\delta_1, \delta_2, \cdots, \delta_m>$ as follows:

$$\delta_i = \begin{cases} \alpha & e_i \in X^* \\ 0 & e_i \in E_1 \setminus X^* \end{cases} \quad \text{for all } e_i \in E_1 ;$$

$$\delta_j = \begin{cases} \beta & e_j \in Y^* \\ 0 & e_j \in E_2 \setminus Y^* \end{cases} \quad \text{for all } e_j \in E_2 ,$$

where

$$\alpha = \max\{l_i - l_j : e_i \in X^*, e_j \in E_2\} > 0,$$

$$\beta = \max\{l_i - l_j : e_i \in E_1, e_j \in Y^*\} > 0.$$

Since C^* is a covering set of G_0, it follows that for every edge $<e_i, e_j> \in A_0$, we have $e_i \in X^*$ or $e_j \in Y^*$. This means that at least one of δ_i and δ_j is greater than 0. From the definition of δ_i and δ_j, the inequality

$$l_i - \delta_i \le l_j + \delta_j$$

is obviously satisfied. This implies that the vector δ is a feasible solution of formulation (2). We then have that

$$\|\delta^*\|_1 \le \|\delta\|_1 = |X^*| + |Y^*| = |C^*| ,$$

where δ^* is an optimal solution of formulation (2).

Summarizing the above analysis, we claim that

$$\|\delta^*\|_1 = |C^*| ,$$

where δ^* is an optimal solution of formulation (2) and C^* is a minimum vertex covering set of bipartite graph G_0.

■

In fact, the proof of theorem 1 establishes a one-to-one correspondence between the optimal solution of the formulation (2) and the minimum covering set of bipartite graph G_0.

Up to now, the inverse minimum spanning tree problem with minimum number of perturbed edges has been transformed into minimum vertex covering set problem in bipartite graph G_0. Hungarian method is well known as a good algorithm to solve minimum vertex covering problem in bipartite graph. This implies that Hungarian method can be used as a subroutine to design our algorithm.

Combining Theorem 1 and Hungarian method, we now present our algorithm for solving the inverse minimum spanning tree problem with minimum number of perturbed edges. This algorithm is termed as (inverse-MST-1) algorithm.

(Inverse-MST-1) Algorithm:

Step 1 From the original network $G=<V, E, L>$ and the prior determined spanning tree T^*, construct the bipartite graph G_0.

Step 2 Take Hungarian method as a subroutine to solve the maximum matching problem in G_0. Let M^* denote the maximum matching set obtained by Hungarian method and $V(M^*)$ denote the vertex set labeled as Hungarian tree in G_0.

Step 3 Construct a minimum vertex covering set $C^* = X^* \cup Y^*$, where $X^* = E_1 \cap V(M^*)$ and $Y^* = E_2 \setminus V(M^*)$.

Step 4 Based on C^*, construct an optimal solution of formulation (2)

$$\delta^* = <\delta_1, \delta_2, \cdots, \delta_m>$$

in the following way:

$$\delta_i = \begin{cases} \alpha, & e_i \in X^* \\ 0, & e_i \in E_1 \setminus X^* \end{cases} \quad \text{and}$$

$$\delta_j = \begin{cases} \beta, & e_j \in Y^* \\ 0, & e_j \in E_2 \setminus Y^* \end{cases}, \text{ where}$$

$\alpha = \max\{l_i - l_j : e_i \in X^*, e_j \in E_2\}$ and

$\beta = \max\{l_i - l_j : e_i \in E_1, e_j \in Y^*\}$.

We now analyze the running time of (inverse-MST-1) algorithm. The running time of Step 1 is $O(mn)$. Step 2 and Step 3 is Hungarian method and its running time is $O(mn^2)$, for $|E_1| = O(n)$ and $|E_2| = O(m-n)$, where m is the edge number and n is the vertex number in the original network. The running time of Step 4 is $O(mn)$. So the total time complexity of (inverse-MST-1) algorithm is $O(mn^2)$.

Corollary 1 *The inverse minimum spanning tree problem with minimum number of perturbed edges can be solved in $O(mn^2)$ times.*

Theorem 1 also gives the following result as a by-product.

Corollary 2 *In optimal solution δ^* of formulation (2), δ_i δ_j $\neq 0$ for edge $<e_i, e_j> \in A_0$ if and only if there exists a minimum covering set C^* of G_0, such that two vertices of edge $<e_i, e_j>$ are both covered by C^*.*

Proof. From Theorem 1, we know that there is a one-to-one correspondence between the optimal solution δ^* and the minimum covering set C^* of bipartite graph G_0.

In optimal solution δ^*, δ_i δ_j $\neq 0$ for edge

$$<e_i, e_j> \in A_0 \Leftrightarrow \delta_i \neq 0,$$

$$\delta_j \neq 0 \Leftrightarrow e_i \in X^*, e_j \in Y^*,$$

where $C^* = X^* \cup Y^*$ is a minimum covering set based on optimal solution δ^* $\Leftrightarrow$ two vertices of edge $<e_i, e_j>$ are both covered by C^*.

■

We now use the following numerical example given in Figure 1 to illustrate the construction process of the bipartite graph G_0 and the computational process of (inverse -MST-1) algorithm. Note that, in figure 1, every edge has been labeled and its original length is given in the blank. The edges in T^* are drawn in thick and black line. It is easy to test and to verify that the spanning tree T^* is not a minimum spanning tree of G with respect to the given length. To make T^* a minimum spanning tree with minimum number of perturbed edges, we must put the original length l_j of every edge e_j a small perturbation $\delta_j, 1 \leq j \leq 10$.

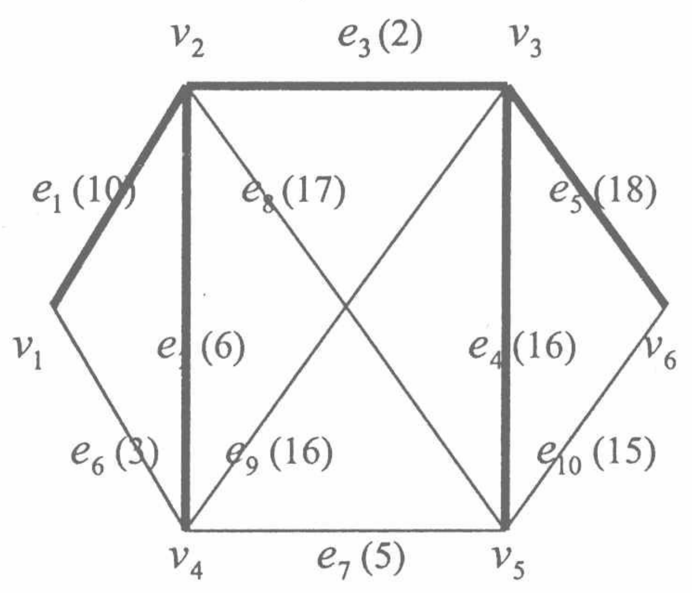

Figure 1 G and T^*

The process of (inverse-MST-1) algorithm is showed as follows.

Step 1 From the original network G in figure 1, we know that $E_1=\{e_1,e_2,e_3,e_4,e_5\}$, $E_2=\{e_6,e_7,e_8,e_9,e_{10}\}$. From the special structure of T^*, we know that $P_6=\{e_1,e_2\}$, $P_7=\{e_2,e_4\}$, $P_8=P_9=\varnothing$ and $P_{10}=\{e_4,e_5\}$.

By $e_i(e_i \in E_1)$ and P_j ($e_j \in E_2$), $G_0=<E_1 \cup E_2, A_0>$ can be constructed as shown in Figure 2.

Step 2 Using Hungarian method as a subroutine to solve the maximum matching problem of G_0. The computational result shows that $M^*=<e_1,e_6>, <e_2,e_7>, <e_4,e_{10}>\}$ (drawn in black and thick line in Figure 2) and V(M^*)=$\{e_1,e_2,e_4,e_5,e_6,e_7,e_{10}\}$ (V(M^*) is obtained from the last labels of Hungarian spanning tree showed in Figure 2).

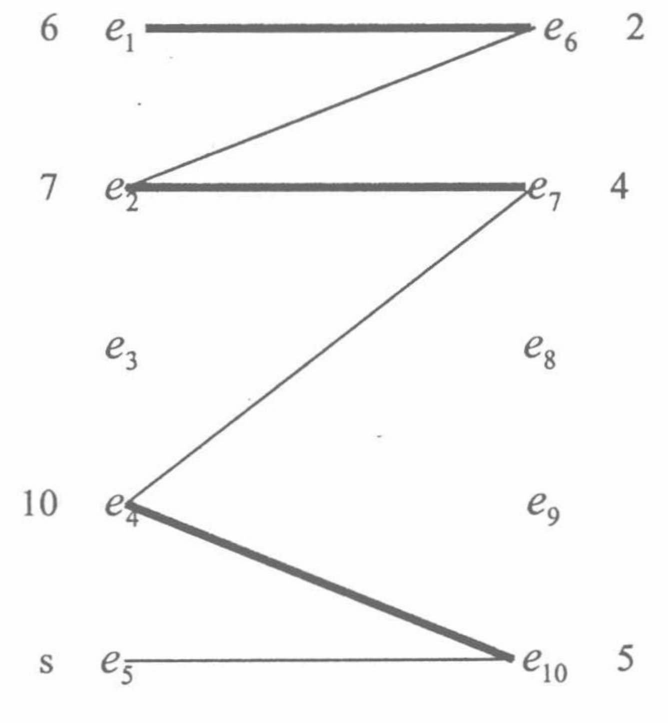

Figure 2 The Path-graph G_0

Step 2 Using Hungarian method as a subroutine to solve the maximum matching problem of G_0. The computational result shows that $M^*=<e_1,e_6>, <e_2,e_7>, <e_4,e_{10}>\}$ (drawn in black and thick line in Figure 2) and $V(M^*)=\{e_1,e_2,e_4,e_5,e_6,e_7,e_{10}\}$ ($V(M^*)$ is obtained from the last labels of Hungarian spanning tree showed in Figure 2).

Step 3 Using $V(M^*)$ construct the minimum vertex covering set $C^*=X^* \cup Y^*$. The computational result is $C^*=\{e_6,e_7,e_{10}\}$ for

$$X^*=E_1 \cap V(M^*)=\phi,$$

$$Y^*=E_2 \setminus V(M^*) = \{e_6,e_7,e_{10}\}.$$

Step 4 Using C^*, construct the optimal solution δ^* of formulation (2).

The computational result is that $\alpha=0, \beta=11$, and δ^*=<0, 0, 0, 0, 0, 11, 11, 0, 0, 11>.

From the perturbed vector δ^*, we present the perturbed network $G'=<V,E,L+\delta^*>$ in Figure 3. It is easy to verify that the prior spanning tree T^* is a minimum spanning tree of G' and the number of perturbed edges from G to G' is only 3.

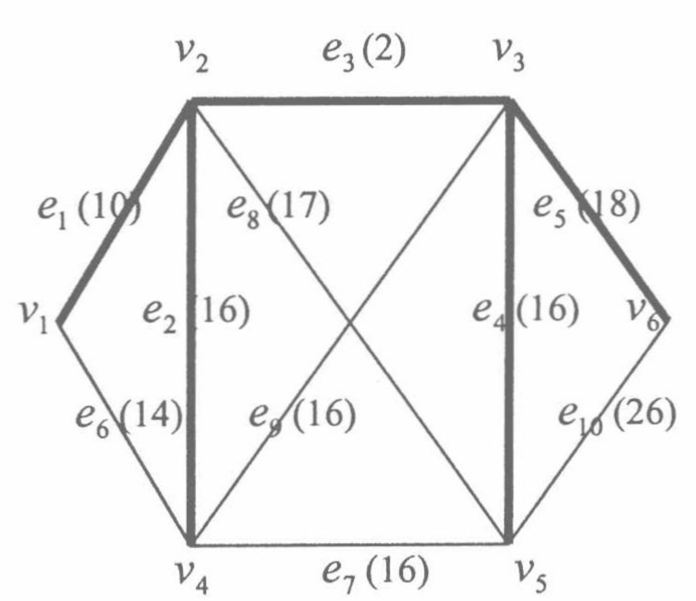

Figure 3 $G'=<V,E,L+\delta^*>$

4. Conclusions and Further Research Work

In this paper, we have discussed the inverse minimum spanning tree problem with minimum number of perturbed edges. Using the concept of path-graph, we prove that this problem can be transformed into minimum vertex covering set problem in bipartite graph G_0. This is the key idea to designing strongly polynomial time algorithm, since Hungarian method can be used as a subroutine to solve this problem. The main result of this paper is a strongly polynomial algorithm, i.e., inverse-MST-1 algorithm, whose time complexity is $O(mn^2)$.

The authors in (Sokkalingam, Ahuja and Orlin, 1999) studied the inverse minimum spanning tree problem with L_1 norm, i.e., the objective function is

$$\|\delta\|_{L_1} = \sum_{e_i \in E_1} \delta_i + \sum_{e_j \in E_2} \delta_j .$$

Combining L_1 norm and the number of perturbed edges, we then propose a biobjective inverse minimum spanning tree problem as follows:

$$\text{Minimize} \begin{cases} \left|\{e_j : \delta_j \neq 0, 1 \le j \le m\}\right| \\ \sum_{e_i \in E_1} \delta_i + \sum_{e_j \in E_2} \delta_j \end{cases}$$

Subject to:

$$\begin{aligned} & l_i - \delta_i \le l_j + \delta_j, \quad e_i \in P_j, \quad e_j \in E_2 \\ & \delta_i \ge 0, \text{ for all } e_i \in E_1 \\ & \delta_j \ge 0, \text{ for all } e_j \in E_2 \end{aligned} \tag{3}$$

Up to now, the complexity status of the above biobjective inverse minimum spanning tree problem is still an open problem. Is this problem polynomial time solvable, or NP-complete? This problem is worthy of further researching and we will pay more attention to it.

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目标函数为∑和 max 的双目标最短路问题：算法和复杂性

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摘 要:本文研究了一个双目标最短路问题。在该问题中,一个目标函数是∑形式,另一个目标函数是 max 形式。首先给出了一个时间复杂性为 $O(m^2\log n)$ 的算法产生代表有效解集合。然后研究了∑和 max 的组合目标函数最短路问题,对动态问题和静态问题,分别给出了一个时间复杂性都为 $O(m^2\log n)$ 的算法。最后在字典序最优解的意义下,本文给出了两个时间复杂性都为 $O(m\log n)$ 的算法。

关键词:双目标;最短路;有效解;算法
中图分类号:O1224;(1991MR)05O12 **文献标识码**:A

1 引言

随着信息科学和优化技术的发展,经典的最短路问题出现了一系列新变形问题,如[1]中分析了依赖于时间的最短路问题,即其中边的长度依赖于到达的时间,证明了问题的 NP-C 复杂性;[2]中研究了 robust 最短路问题,其中,每条边的长度是个 k 维向量,自然按照 k 个参数,一条路可以计算出 k 个长度,robust 最短路问题中,一条路的目标函数定义为 k 个长度中的最大者。[2]证明了问题的 NP-C 完全性,给出了一个伪多项式算法和一个近似算法,并分析了近似算法的性能比;[3]中研究了最小最短路树问题,最短路树问题要求从开始点到达其它各点的唯一路线都是最短路,另一方面,最短路树又是一个树形图(arborescence),朱-刘算法是求解最小树形图的一个多项式算法。最小最短路树问题将最短路树问题和最小树形图问题进行了结合,即求出一个最短路树,其长度又是最小的。[3]给出了最小最短路树问题的一个多项式时间的好算法。双目标最短路问题也是一个重要的变形问题。[4]中证明了双目标最短路问题是 NP-C,并对控制一个目标函数,然后优化另一个目标函数的问题给出了一个对偶算法。[5,6]也分别研究了双目标最短路问题,并分别给出了算法。[7]研究了边长为向量形式的最短路问题。

上述双目标最短路问题的目标函数都是∑(求和)形式。在许多情况下,两个目标函数中,一个是路的长度,即∑形式,一个是路的瓶颈,即 max 形式。设 $G=\langle V,E,L,C\rangle$ 是个网络,V 是顶点集合,$|V|=n$, $v_1\in V$, $v_n\in V$; E 是边集合,$|E|=m$。用 $\tilde{P}$ 表示 G 中 $v_1\rightarrow v_n$ 路的集合。每条边 $\langle v_i,v_j\rangle$ 有两个非负参数 $l_{i,j}$ 和参数 $c_{i,j}$, $l_{i,j}$ 是长度参数,并对应着目标函数 $L(P)=\sum_{\langle v_i,v_j\rangle\in P} l_{i,j}$; $c_{i,j}$ 是一个瓶颈参数,并对应着目标函数 $C(P)=\max_{\langle v_i,v_j\rangle\in P} c_{i,j}$,其中 $P\in\tilde{P}$。因此本文提出一个关于目标函数为∑和 max 的一个双目标优化问题(BSP)如下:

$$\begin{aligned}&\min L(P)\\&\min C(P)\\&\text{s.t}\quad P\in\tilde{P}\end{aligned}\tag{1}$$

(1)式在网络设计、卫星通讯、拟阵覆盖中有着广泛的应用,见[8,9]。

关于(BSP)问题,本文首先给出了一个时间复杂性为 $O(m^2\log n)$ 的算法产生(BSP)问题的代表有效解集合(部分有效解)。然后研究了(BSP)问题的组合目标函数优化问题 $\min_{P\in\tilde{P}}\{\lambda\sum_{\langle v_i,v_j\rangle\in P} l_{i,j}+(1-\lambda)\max_{\langle v_i,v_j\rangle\in P}\{c_{i,j}\}\}$。对静态问题和动态问题,分别给出了一个时间复杂性为 $O(m^2\log n)$ 的算法,该算法

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