高等学校土木工程专业卓越工程师教育培养计划系列规划教材

Fundamentals of Elastic and Plastic Mechanics (Edition for Bilingual Teaching)

弹塑性力学基础（双语教学版）

·平台课课程群·

丁建国　编著　　范　进　主审

WUHAN UNIVERSITY PRESS
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教学实践表明，有效地利用数字化教学资源，对于学生学习能力以及问题意识的培养乃至怀疑精神的塑造具有重要意义。

通过对数字化教学资源的选取与利用，学生的学习从以教师主讲的单向指导的模式而成为一次建设性、发现性的学习，从被动学习而成为主动学习，由教师传播知识而到学生自己重新创造知识。这无疑是锻炼和提高学生的信息素养的大好机会，也是检验其学习能力、学习收获的最佳方式和途径之一。

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本书基本数字教学资源及读者信息反馈表请登录www.stmpress.cn下载，欢迎您对本书提出宝贵意见。

丛书序

土木工程涉及国家的基础设施建设，投入大，带动的行业多。改革开放后，我国国民经济持续稳定增长，其中土建行业的贡献率达到1/3。随着城市化的发展，这一趋势还将继续呈现增长势头。土木工程行业的发展，极大地推动了土木工程专业教育的发展。目前，我国有500余所大学开设土木工程专业，在校生达40余万人。

2010年6月，中国工程院和教育部牵头，联合有关部门和行业协(学)会，启动实施"卓越工程师教育培养计划"，以促进我国高等工程教育的改革。其中，"高等学校土木工程专业卓越工程师教育培养计划"由住房和城乡建设部与教育部组织实施。

2011年9月，住房和城乡建设部人事司和高等学校土建学科教学指导委员会颁布《高等学校土木工程本科指导性专业规范》，对土木工程专业的学科基础、培养目标、培养规格、教学内容、课程体系及教学基本条件等提出了指导性要求。

在上述背景下，为满足国家建设对土木工程卓越人才的迫切需求，有效推动各高校土木工程专业卓越工程师教育培养计划的实施，促进高等学校土木工程专业教育改革，2013年住房和城乡建设部高等学校土木工程学科专业指导委员会启动了"高等教育教学改革土木工程专业卓越计划专项"，支持并资助有关高校结合当前土木工程专业高等教育的实际，围绕卓越人才培养目标及模式、实践教学环节、校企合作、课程建设、教学资源建设、师资培养等专业建设中的重点、亟待解决的问题开展研究，以对土木工程专业教育起到引导和示范作用。

为配合土木工程专业实施卓越工程师教育培养计划的教学改革及教学资源建设，由武汉大学发起，联合国内部分土木工程教育专家和企业工程专家，启动了"高等学校土木工程专业卓越工程师教育培养计划系列规划教材"建设项目。该系列教材贯彻落实《高等学校土木工程本科指导性专业规范》《卓越工程师教育培养计划通用标准》和《土木工程卓越工程师教育培养计划专业标准》，力图以工程实际为背景，以工程技术为主线，着力提升学生的工程素养，培养学生的工程实践能力和工程创新能力。该系列教材的编写人员，大多主持或参加了住房和城乡建设部高等学校土木工程学科专业指导委员会的"土木工程专业卓越计划专项"教改项目，因此该系列教材也是"土木工程专业卓越计划专项"的教改成果。

土木工程专业卓越工程师教育培养计划的实施，需要校企合作，期望土木工程专业教育专家与工程专家一道，共同为土木工程专业卓越工程师的培养作出贡献！

是以为序。

2014年3月于同济大学四平路校区

前 言

工程技术人员在工程实践中经常遇到弹塑性力学问题，当设计土木工程结构时需要对结构进行地震作用下的抗震性能分析和评估，当机械构件受各种外载作用时要判断其工作状态，并作出既安全又经济的设计，这些都需要弹塑性力学的基本理论和分析方法。由于塑性状态下的结构并没有完全丧失承载能力，当进行工程设计时，有时还需让一部分结构先进入塑性状态消耗地震能量，以确保整体工程结构有足够的抗震能力。因此学习并掌握弹塑性力学基础具有重要的现实意义。

为了响应教育部有关在高等学校中推广双语教学课程的号召，作者有幸承担了"弹塑性力学基础"的双语教学工作，在经过了多年对该课程的双语教学实践并在经验总结的基础上，按照精练、简洁原则编写了可适用于我国土木工程、水利工程以及机械工程等本科专业学生使用的《弹塑性力学基础》英文教材。

美国等西方发达国家在《弹塑性力学基础》的内容安排方面与我国有较大的差异，并且国外原版教材比较昂贵，如要求我国学生直接选用国外原版教材并不合适，因此作者在参阅了多本英文版经典教材后，从中选用合适的部分并按我国的内容体系重新安排，这样既保持了双语教材的原汁原味，又使其教材内容体系符合我国国情。

本书内容共分八章，其内容分别为力系统(Force Systems)、应力(Stress)、应变(Strain)、应力-应变关系(Stress-strain Relations)、弹性理论基本方程(Basic Equations of Elasticity)、弹性理论的平面应变和平面应力问题(Problems in Plane Strain and Plane Stress from the Theory of Elasticity)、塑性理论(Plasticity)、弹塑性分析实例(The Examples of Elastoplastic Analysis)。本书还配有大量的英文习题。

本书在编写过程中得到了南京理工大学范进教授的建议与帮助，南京理工大学土木工程系一些研究生也给了本人许多帮助，特向他们致以衷心的感谢。

由于作者水平有限，书中难免有不足之处，欢迎读者批评指正。

编 者

2013 年 12 月

Foreword

Engineers often encounter elastic-plastic mechanics problems. The seismic performance of structures during earthquakes must be analyzed and estimated when some building structures are designed. Engineers have to analyze the working state of mechanical components under various external loads to make safe and economical designs. Therefore, the basic theories and analytic methods for elastic-plastic mechanics are essential. Because plastic state does not necessarily refer to a loss of load bearing capacity, engineers allow plastic deformation in some components to dissipate earthquake energy and to increase the overall structural seismic capability. The fundamentals of elastic and plastic mechanics are significant in these designs.

To respond to the Ministry of Education of PRC's promotion of Bilingual Teaching in colleges and universities, the author had the opportunity to bilingually teach a "Fundamentals of Elastic and Plastic Mechanics" course. After years of teaching practice and experience, I have compiled "Fundamentals of Elastic and Plastic Mechanics" in a concise way. This book is suitable for students majoring in civil engineering, hydraulic engineering and mechanical engineering in China.

There are many differences between the contents of a foundational course in elastic and plastic mechanics in our country and in other developed Western countries, such as the United States. The original foreign books are expensive and not suitable for Chinese students. After referring to some classic English books, I chose the suitable parts and rearranged them according to the content system used in China. Therefore, this book maintains the quintessence of the original foreign books while making the content suitable for Chinese students.

This book contains eight chapters: Force Systems, Stress, Strain, Stress-strain Relations, Basic Equations of Elasticity, Problems in Plane Strain and Plane Stress from the Theory of Elasticity, Plasticity, The Examples of Elastoplastic Analysis. Meanwhile, a large number of English language exercises are provided.

Throughout the process of writing this book, I have obtained the advice and help of Professor Jin Fan and several graduate students from the Civil Engineering Department of Nanjing University of Science and Technology. I extend my heartfelt thanks to them.

Deficiencies are inevitable because of the limited level of me, and I will appreciate your comments.

Jianguo Ding

December 2013

Contents

1 Force Systems

1.1 Introduction

In our studies of rigid-body mechanics, the deformation of bodies was of no significance in the problems we were able to solve. You will recall that, in such problems, Newton's law was all we needed in order to compute certain unknown forces acting on bodies in equilibrium. The study of statics did lead, however, to problems where the use of Newton's law alone was insufficient for the handling of the problem even though the bodies involved seemed quite "rigid" from a physical point of view. For those problems, called statically indeterminate problems, the deformation, however small, was significant for the determination of the desired forces. As an illustration, consider the simply supported beam shown in Figure 1-1(a) with the freebody diagram of the beam shown in Figure 1-1(b). You may readily solve for the supporting forces A, B_x, and B_y by the method of rigid-body mechanics, provided that there is little change of position of the external loads as they are applied to the beam. We can accomplish this because we know that the resultant force system on the free body is of zero value. By setting this resultant equal to zero while using the undeformed geometry of the problem we can easily solve for the three unknowns. Suppose next that there are three supports for the beam instead of two as shown in Figure 1-2. Clearly, the deformation of the beam can be expected to be even smaller in extent than for the two-support system, so there should be no difficulty in this problem arising from a changing geometry. The total supporting force system from the two supports is in accordance with the dictates of rigid-body mechanics. However, we cannot, in this latter problem, determine the value of supporting forces A, C, B_x and B_y uniquely since there are an infinite number of combinations of values that will give the precise total resultant force required by rigid-body mechanics. To choose the proper combination of values requires the consideration of the deformation of the beam, small as this deformation might be. We see that the rigid-body considerations afford us a necessary requirement for the resultant of the supporting force system whereas the deformation analysis supplies the additional information sufficient to determine the values of each supporting force. Statically indeterminate problems akin to the one discussed will be one of the classes of problems that we shall undertake to solve.

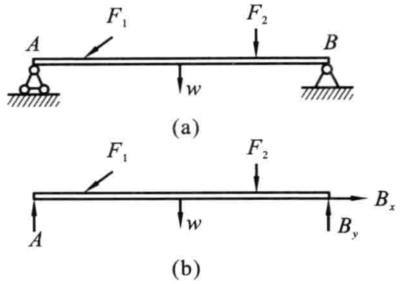

Figure 1-1 The simply supported beam

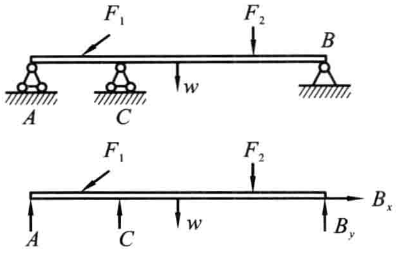

Figure 1-2 The beam of three supports

Next, consider some arbitrary solid in equilibrium as shown in Figure 1-3. Suppose we pass a hypothetical plane M through the body as shown in the diagram. We wish to determine the force distribution that is transmitted from one portion of the body A to the other portion B through this interface. Considering part B as a free body (Figure 1-4), we can find by the methods of rigid-body mechanics a force and couple at some position in the section that is the correct resultant force system for the desired distribution, provided, of course, that the applied forces have not appreciably changed their initial known orientation as a result of deformation. But just like the supports of the indeterminate beam problem, there are an infinite number of distributions that can yield this resultant force system. We must investigate the deformation of the entire body in order to obtain sufficient additional information for establishing a unique force distribution.

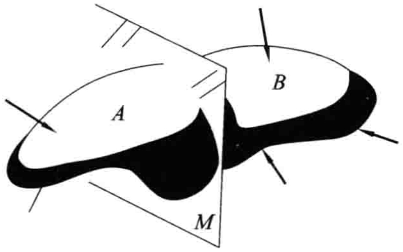

Figure 1-3 Some arbitrary solid in equilibrium

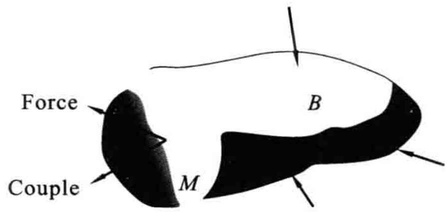

Figure 1-4 The free body diagram of some arbitrary solid

The knowledge of force distribution in solids is of vital importance to the design of most systems. It might be pointed out that the spectacular increase in the range of single-stage chemical rockets in recent years from the 200-mile range of the German V-2 to the 1000-mile-plus range of the IRBM's has been the result largely of structural improvements rather than of propulsion gains.

We shall therefore be interested in internal distribution of forces in solids as well as the computation of certain discrete forces in statically indeterminate structural problems like the beam problem.

Until now, we have been centering our attention on the computation of certain discrete forces and force distributions for problems requiring the consideration of the deformation of the body. It should be clear that sometimes the information of the deflection itself may be of paramount interest and not that of the force distributions. Because we shall limit ourselves to problems involving small deformation, we shall be able to determine the deflection of the statically determinate two-support beam by first computing the supporting forces using rigid-body mechanics, and then proceeding with considerations of deformation. And in the statically indeterminate three-support beam, we shall compute the supporting forces as well as the deflection at the same time since the deformation is intrinsically connected with both computations.

Unlike your studies of rigid-body continua in which you consider statics and dynamics of rigid bodies, we shall, in our studies of deformable solids, restrict ourselves essentially to statical problems. However, there is an important class of non-statical problems for which the formulations of this text can be applied. Suppose that, in response to a given force system, a body moves while undergoing little change in shape so that by rigid-body mechanics we can compute the motion of the body at any instant using the undeformed shape of the body. We then may employ the methods of this text to compute the deformation of the body resulting from the combined action of the given forces and computed inertial forces. However, there is an important proviso that the aforementioned forces vary slowly with time. This requirement is imposed because we shall compute the deformation of the body at time t as if the applied force system and the inertial force distribution computed from the rigid-body motion of the body at time t were static loads on the body. Clearly such an approach for a rapid time-varying force distribution would have little meaning. The case of the rotating disk is an important example of the kind of problem that may be suitable for the approach described here.

The dynamics of deformable solids is reserved for more advanced course, where you may consider the vibrations of beams, plates, and shells; or wave propagation in solids; or, possibly, shock loading of structures. Others may study the stresses in machines such as jet engines, or the vibrations induced in rockets by the propulsion system. These are fascinating problems beyond the scope of this text. Nevertheless, they require the understanding of the fundamentals that we shall stress in our studies here.

Since we shall be dealing with forces to a great extent, it is now useful to make certain classification.

1.2 Types of Force Distributions

We shall at this time set forth classifications of force distributions. Force fields which act throughout a body, i. e. , force fields which exert influence on the mass distributed throughout the body, are called body-force distributions. The force of gravity and inertial forces described in the previous section are the chief examples of such distributions. Body-force distributions are expressed per unit mass or per unit volume of the body elements they directly influence. Thus if $\boldsymbol{B}(x,y,z,t)$ is a body-force distribution per unit mass, the force on element of mass $\mathrm{d}m$ would be

$$\mathrm{d}\boldsymbol{F}=\boldsymbol{B}(x,y,z,t)\,\mathrm{d}m \tag{1-1}$$

In addition to body-force distributions, we also have surface-force distributions. These force distributions act on the boundary of a body that we may want to consider. The surface-force distribution is given per unit of area of the boundary acted on. A surface-force distribution might consist of a force distribution acting on the outside surface of the body shown in Figure 1-3 or might equally well include also the distribution on a surface exposed by a hypothetical plane, such as M in the diagram. The force and couple shown in Figure 1-4 are the resultant force system of such a distribution. We sometimes wish to distinguish between surface-force distribution at actual boundaries and surface-force distribution at hypothetically exposed boundaries, such as plane M. When this is the case, we call the surface-force distribution at the actual boundary the surface traction. We shall have occasion later in the text to wish to use this distinction.

1.3 Closure

In this chapter, we have attempted to show some connections between earlier rigid-body mechanics and studies to be undertaken in this text. Also we presented certain definitions of force distribution that will help us in our study of deformable solids. We shall next consider carefully, force distributions on internal surfaces which are hypothetically exposed by the use of free-body diagrams.

2 Stress

2.1 Introduction

In the previous chapter, we discussed two types of force distributions, namely, body-force distributions and surface-force distributions. We shall now consider the latter in greater detail. You will recall that surface-force distributions may be found on an actual boundary, in which case they are called surface tractions, or they may be considered at internal sections of a body exposed by the technique of free bodies. Clearly any internal interface of a body may in this way be exposed so as to have a surface-force distribution associated with it. By this reasoning we can think of surface-force distributions pervading the entire body. Indeed, it is by such a viewpoint that we are able to describe quantitatively how external loads are transmitted through a body.

Consider a small area δA of a hypothetical interface of a body in equilibrium, as shown in Figure 2-1. Notice that the rigid-body resultant force F_R and couple M_R have been shown for the whole interface. For the area element there will also be a resultant force δF and a resultant couple δM as indicated in Figure 2-2.

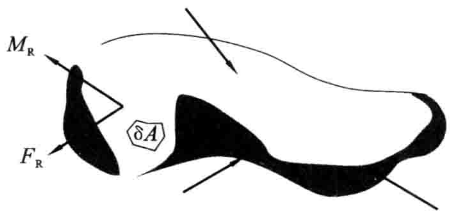

Figure 2-1 A body in equilibrium

If this area element is decreased to infinitesimal size, the couple can be neglected because the force distribution across the area then approaches that of a uniform and parallel distribution which can be replaced by a single force, as we learned in rigid-body mechanics. We will then delete the couple δM from the ensuing remarks since we shall soon go to the infinitesimal limit. It is convenient to decompose δF into a set of orthogonal components, as shown in Figure 2-3 in which one of the components δF_n is normal to the area element whereas the other components δF_{s_1} and δF_{s_2} are tangent to the area element.

We may now define the normal stress σ and the shear stress τ_s by the following limiting processes

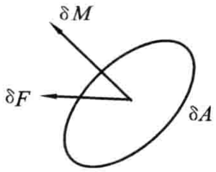

Figure 2-2 The area element of a body

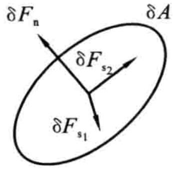

Figure 2-3 A set of orthogonal components in the area element of a body

$$\left.\begin{aligned}\sigma&=\lim_{\delta A\to 0}\frac{\delta F_{\mathrm{n}}}{\delta A}=\frac{\mathrm{d}F_{\mathrm{n}}}{\mathrm{d}A}\\ \tau_{\mathrm{s}_1}&=\lim_{\delta A\to 0}\frac{\delta F_{\mathrm{s}_1}}{\delta A}=\frac{\mathrm{d}F_{\mathrm{s}_1}}{\mathrm{d}A}\\ \tau_{\mathrm{s}_2}&=\lim_{\delta A\to 0}\frac{\delta F_{\mathrm{s}_2}}{\delta A}=\frac{\mathrm{d}F_{\mathrm{s}_2}}{\mathrm{d}A}\end{aligned}\right\}\tag{2-1}$$

We see that shear and normal stresses are intensities of force components given per unit area. Note that they are scalar quantities. Giving shear and normal stress distributions is our way of describing a force distribution over a plane interface and, as we shall soon see the distribution of force through a body.

2.2 Stress Notation

In the previous section, we showed how we could describe a force distribution over a plane interface inside a body. We can in this way set forth normal and shear stresses for any interface at a point, hence, we can describe the manner in which external forces are transmitted throughout a body. With three stress components set forth for each interface at a point, it becomes imperative to formulate an effective and meaningful notation to identify the stresses. For this purpose, consider in a deformed body an infinitesimal element with faces parallel to Cartesian reference as shown in Figure 2-4 where each of these faces forms and infinitesimal rectangular parallelepiped. Stresses on three faces have been shown. Notice that a double index scheme has been utilized. The first subscript indicates the direction of the normal to the plane on which the stress is considered; the second subscript denotes the direction of the stress itself. The normal stress must then have one indice, since the stress direction and the normal to the plane on which the stress acts are collinear. The shear stresses, on the other hand, have mixed indices. For example, τ_{yx} is the shear stress acting on the plane parallel to the xz plane whose normal proceeds in the y direction whereas the stress itself is oriented in the x direction.

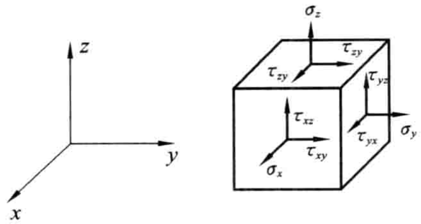

Figure 2-4 An infinitesimal element with faces parallel to Cartesian reference

As for the sign convention, we shall follow these rules: a stress acting on an area element whose outward normal points in the positive direction of any coordinate axis will be taken as positive if the stress itself also points in the positive direction of any coordinate axis(The axes for the area vector and stress need not be the same axes). A stress is positive also if both the area vector and the stress point in the negative direction of the same or different coordinate axes. Thus you will note on inspection that the stresses shown in Figure 2-4 are all positive stresses. If now the area vector and stress are not directed simultaneously in either positive or negative coordinate directions, the stress is negative.

We shall see, in the next section, that knowing the stresses on three orthogonal interfaces at a point, we can determine, by employing transformation formulas, stresses on any interface at that point. Therefore, specifying distributions corresponding to interfaces parallel to Cartesian set of axes is tantamount to specifying the stress distribution throughout the entire body. The notation that we have presented will then prove to be extremely helpful.

It should be clearly understood that stress is not restricted to solids. Our conclusions here and indeed throughout the entire chapter apply to any continuous medium exhibiting viscosity of rigidity.

We now proceed to develop the aforementioned transformation formulations for stress.

2.3 Transformation Formulations for Stress

Let us consider a small tetrahedron of a continuous medium as shown in Figure 2-5. The orthogonal edges of the tetrahedron are of length Δx, Δy and Δz respectively. Positive shear and normal stresses have been shown on the faces parallel to the reference planes. On the inclined surface whose normal has been indicated as $\boldsymbol{n}$, we have shown the normal stress σ_n and the total shear stress τ_{ns}. It is convenient to denote the direction co-