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A note on the reducibility of binary affine polynomials

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Abstract Stickelberger–Swan Theorem is an important tool for determining parity of the number of irreducible factors of a given polynomial. Based on this theorem, we prove in this note that every affine polynomial $A(x)$ over $\mathbb{F}_2$ with degree >1, where $A(x) = L(x) + 1$ and $L(x) = \sum_{i=0}^{n} x^{2^i}$ is a linearized polynomial over $\mathbb{F}_2$, is reducible except $x^2 + x + 1$ and $x^4 + x + 1$. We also give some explicit factors of some special affine pentanomials over $\mathbb{F}_2$.

Keywords Finite field · Pentanomial · Discriminant · Resultant

Mathematics Subject Classification (2000) 11T06

1 Introduction

Let $\mathbb{F}_q$ denote the finite field with q elements, where q is a prime power. A polynomial of the form $L(x) = \sum_i a_i x^{q^i}$ with coefficients in an extension field $\mathbb{F}_{q^m}$ of $\mathbb{F}_q$ is called a linearized polynomial over $\mathbb{F}_{q^m}$, and $A(x) = L(x) - \alpha$, where $L(x)$ is a linearized polynomial over $\mathbb{F}_{q^m}$ and $\alpha \in \mathbb{F}_{q^m}$, is called an affine polynomial over $\mathbb{F}_{q^m}$. The weight of a polynomial is the number of its non-zero coefficients. It is well known that if the weight of $f(x) \in \mathbb{F}_q[x]$ is small, then the multiplication in $\mathbb{F}_{q^n}$ can be sped up considerably [10]. Fast arithmetic in finite fields is important for the efficient implementation of error-correcting codes and discrete logarithm cryptosystems. The arithmetic in binary field can be efficiently implemented in both

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hardware and software, and so it has been received special attention. Many researchers have studied the irreducibility of polynomials over $\mathbb{F}_2$. Swan [12] determined the parity of the number of factors of trinomials over $\mathbb{F}_2$, here a trinomial we mean a polynomial of the form $x^n + x^k + 1$. Blake et al. [3] listed all the irreducible trinomials of degree less than 2,000 for $k \leq \frac{n}{2}$. Hales et al. [7] presented a Swan-like theorem for binary tetranomials. Ahmadi et al. [2] studied the number of trace-one elements in a polynomial basis for $\mathbb{F}_{2^n}$, and listed all irreducible trinomials whose degree less than 1,000 and the corresponding basis has exactly one trace-one element. Seroussi [11] conjectured that there exists an irreducible pentanomial of degree n over $\mathbb{F}_2$ for each $n \geq 4$. By computer search, Ahmadi et al. [2] verified that for each $n \in [6, 4000]$ there exists irreducible pentanomial over $\mathbb{F}_2$. Using the Stickelberger–Swan Theorem, Kim et al. [8] established the relation between the discriminants of composed polynomial and the original ones, and then applied this to obtain some results concerning the parity of the number of irreducible factors for several special polynomials over finite fields.

In this note, we focus on studying affine polynomials $A(x) = L(x) + 1$, where $L(x) = \sum_{i=0}^{n} x^{2^i}$ is a linearized polynomial over $\mathbb{F}_2$. The remainder of this paper is organized as follows. In Section 2, we recall some propositions of discriminant and resultant of polynomials. For completeness, we also give some known results on the properties of low-weight affine polynomials over finite field. In Section 3, we prove that every pentanomial, the polynomial of weight five, is reducible over $\mathbb{F}_2$, and also give some explicit factors of some special pentanomials. In the final section, we also generalize our results to affine polynomials with arbitrary weights.

Throughout the paper, when we mention an affine polynomial over $\mathbb{F}_2$, we mean that it is the polynomial $A(x) = L(x) + 1$, where $L(x)$ is a linearized polynomial over $\mathbb{F}_2$.

It has to be pointed out that this paper owes its existence to the results of Swan [12] about the number of irreducible factors of polynomials over $\mathbb{F}_p$.

2 Preliminary results

In this section, we recall some results about the discriminant and the resultant of polynomials firstly, and then introduce Stickelberger–Swan Theorem. For the sake of completeness, we also list some known results about the properties of low-weight affine polynomials over finite field.

Let F be a field. Let $f(x)$ be a polynomial of degree $n > 1$ in $F[x]$ with leading coefficient $a \neq 0$. The discriminant of $f(x)$, denoted by $Disc(f)$, is defined by

$$Disc(f) = a^{2n-2} \prod_{i<j} (\alpha_i - \alpha_j)^2, \tag{1}$$

where $\alpha_0, \alpha_1, \ldots, \alpha_{n-1}$ are the roots of $f(x)$ (counted with multiplicity) in an extension of the ground field F. Since $Disc(f)$ is a symmetric function with respect to the roots of $f(x)$, $Disc(f) \in F$.

Let $g(x) \in F[x]$, and suppose $f(x) = a \prod_{i=0}^{n-1} (x - \alpha_i)$ and $g(x) = b \prod_{i=0}^{m-1} (x - \beta_i)$, where $\alpha_0, \alpha_1, \ldots, \alpha_{n-1}, \beta_0, \beta_1, \ldots, \beta_{m-1}$ are in an extension of F, and a, b are not zero. The resultant of two polynomials $f(x)$ and $g(x)$, denoted by $Res(f, g)$ [9], is defined by

$$Res(f, g) = a^m \prod_{i=0}^{n-1} g(\alpha_i) = (-1)^{nm} b^n \prod_{i=0}^{m-1} f(\beta_i). \tag{2}$$

By the definitions of discriminant and resultant, we have the following propositions.

Proposition 2.1 ([9]) *Let $f(x)$ be defined as above, and let $f'(x)$ be the derivative of it. Then*

$$Disc(f) = (-1)^{\frac{n(n-1)}{2}} a^{-1} Res(f, f'). \quad (3)$$

Proposition 2.2 ([12]) *Let $f(x)$, $g(x)$ be defined as above, and $g_1(x), g_2(x), q(x), r(x) \in F[x]$. Then we have*

(1) $Res(f, x) = (-1)^n f(0)$, $Res(f, -x) = f(0)$;
(2) *if c, d are constants not both 0, then $Res(c, d) = 1$;*
(3) $Res(f, g_1 g_2) = Res(f, g_1) Res(f, g_2)$;
(4) *if $f(x) = g(x)q(x)+r(x)$, then $Res(f, g) = b^{n-deg(r)} Res(r, g)$, where b is the leading coefficient of $g(x)$ and $deg(r)$ is the degree of $r(x)$.*

There is another definition of the resultant of two polynomials over field F, which is crucial for our prove of the main results. Let $f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0 \in F[x]$, and $g(x) = b_m x^m + b_{m-1} x^{m-1} + \cdots + b_1 x + b_0 \in F[x]$ be two polynomials with $a_n b_m \neq 0$. Then the resultant $Res(f, g)$ of the two polynomials is defined by the determinant

$$Res(f, g) = det \begin{pmatrix} a_n & a_{n-1} & \cdots & a_1 & a_0 & 0 & \cdots & 0 \\ 0 & a_n & a_{n-1} & \cdots & a_1 & a_0 & \cdots & 0 \\ & \cdots & & & & & \cdots & \\ 0 & \cdots & 0 & a_n & a_{n-1} & \cdots & a_1 & a_0 \\ b_m & b_{m-1} & \cdots & b_1 & b_0 & 0 & \cdots & 0 \\ 0 & b_m & b_{m-1} & \cdots & b_1 & b_0 & \cdots & 0 \\ & \cdots & & & & & \cdots & \\ 0 & \cdots & 0 & b_m & b_{m-1} & \cdots & b_1 & b_0 \end{pmatrix} \begin{matrix} \left.\vphantom{\begin{matrix}a\\a\\a\\a\end{matrix}}\right\} m \text{ rows} \\ \left.\vphantom{\begin{matrix}a\\a\\a\\a\end{matrix}}\right\} n \text{ rows} \end{matrix}$$

of order $m + n$ [9].

The next theorem, which is called Stickelberger–Swan Theorem, is our main tool for proving the reducibility of affine pentanomials over $\mathbb{F}_2$.

Theorem 2.3 (Swan[12]) *Let $f(x) \in \mathbb{F}_2[x]$, and suppose that $Disc(f) \neq 0$, that is, $f(x)$ has no multiple roots. Let t denote the number of irreducible factors of $f(x)$ over $\mathbb{F}_2$. Assume that $F(x) \in \mathbb{Z}[x]$ be any nomic lift to the integers. Then $t \equiv deg(f) (\mathrm{mod}\ 2)$ if and only if $Disc(F) \equiv 1 (\mathrm{mod}\ 8)$.*

Both in theory and in applications the linearized polynomials and affine polynomials over finite field are of importance. For completeness, some known results on the properties of low-weight affine polynomials over finite field will be listed. A trinomial is a polynomial with weight three. In what follows, we consider trinomials that are also affine polynomials.

Theorem 2.4 ([9]) *Let $a \in \mathbb{F}_q$ and let p be the characteristic of $\mathbb{F}_q$. Then the trinomial $x^p - x - a$ is irreducible in $\mathbb{F}_q[x]$ if and only if it has no root in $\mathbb{F}_q$.*

By the Hilbert's Theorem 90 and above theorem, we can get that the trinomial $x^p - x - a$ is irreducible in $\mathbb{F}_q[x]$ if and only if $T_{\mathbb{F}_q}(a) \neq 0$, where $T_{\mathbb{F}_q}$ is the trace from $\mathbb{F}_q$ to $\mathbb{F}_p$. If we consider more general trinomials of the above type for which the degree is a higher power of the characteristic, then these criteria need not be valid any more. In fact, the following decomposition formula can be established.

Theorem 2.5 ([9]) *For $x^q - x - a$ with a being an element of the subfield $K = \mathbb{F}_r$ of $F = \mathbb{F}_q$, we have the decomposition $x^q - x - a = \prod_{j=1}^{q/r} (x^r - x - \beta_j)$ in $\mathbb{F}_q[x]$, where the β_j are the distinct elements of $\mathbb{F}_q$ with $T_{F/K}(\beta_j) = a$.*

We note that the ordinary product of linearized polynomials need not be a linearized polynomial. However, the composition $L_1(L_2(x))$ of two linearized polynomials $L_1(x)$, $L_2(x)$ over $\mathbb{F}_{q^m}$ is again a linearized polynomial over $\mathbb{F}_{q^m}$. In fact, the set of linearized polynomials over $\mathbb{F}_q$ forms an integral domain under the operations of ordinary addition and composition. Using this fact, we can get many useful properties of linearized polynomial over $\mathbb{F}_q$. By the relation between the discriminants of composed polynomial and the original one, Kim et al. [8] determined the parity of the number of factors for $f(x^2+x+1)$, where $f(x) = x^n + a_1x^{n-1} + \cdots + a_{n-1}x + a_n \in \mathbb{F}_2[x]$. If $f(x)$ is a binary polynomial defined as above, we call $F(x, y) = x^n + a_1x^{n-1}y + \cdots + a_{n-1}xy^{n-1} + a_ny^n \in \mathbb{Z}[x, y]$ a homogeneous polynomial in two variables derived from $f(x)$.

Theorem 2.6 (Kim[8]) *Let $f(x) = x^n + a_1x^{n-1} + \cdots + a_{n-1}x + a_n \in \mathbb{F}_2[x]$ be a binary polynomial with no multiple roots. Then the composition $f(x^2+x+1) \in \mathbb{F}_2[x]$ also has no multiple roots. In this case assume that $f(x^2+x+1)$ has t irreducible factors over $\mathbb{F}_2$. Then t is even if and only if $(-1)^n F(3, 4) \equiv 1 (\mathrm{mod}\ 8)$, and $t \equiv n + a_1 (\mathrm{mod}\ 2)$ where F is a homogeneous polynomial corresponding to the monic lift of $f(x)$ to the integers.*

It is obvious that if $f(x) \in \mathbb{F}_2[x]$ is irreducible over $\mathbb{F}_2$, then the coefficient of x^{n-1} is equal to 1 if and only if the degree n of $f(x)$ is an even. Kim et al. applied above theorem to trinomials over $\mathbb{F}_2$ to obtain the following conclusion.

Theorem 2.7 (Kim[8]) *Let $f(x) = x^n + x^k + 1 \in \mathbb{F}_2[x]$. If $f(x)$ has no multiple roots, then the composition $f(x^2+x+1)$ has an even number of irreducible factors over $\mathbb{F}_2$ in the following two cases*

(1) *$n - k = 1$ and n is odd,*

(2) *$n - k \geq 2$ and n is even.*

3 The reducibility of affine pentanomials over $\mathbb{F}_2$

Let $A(x) = x^{2^{n_1}} + x^{2^{n_2}} + x^{2^{n_3}} + x^{2^{n_4}} + 1$ be an affine pentanomial over $\mathbb{F}_2$, where $n_1 > n_2 > n_3 > n_4 \geq 0$. If $n_4 \geq 1$, then $A(x)$ is a square of a pentanomial, so we may assume that $n_4 = 0$, that is, $A(x) = x^{2^{n_1}} + x^{2^{n_2}} + x^{2^{n_3}} + x + 1$. By the table of [11], there are no irreducible affine pentanomials over $\mathbb{F}_2[x]$ with $2^{n_1} \leq 10{,}000$, that is, $n_1 < 14$. In this section, we prove that every affine pentanomial over $\mathbb{F}_2$ is reducible. The next theorem is one of our main results.

Theorem 3.1 *Let $A(x) = x^{2^{n_1}} + x^{2^{n_2}} + x^{2^{n_3}} + x + 1$ be an affine pentanomial over $\mathbb{F}_2$, where $n_1 > n_2 > n_3 \geq 1$. Then $A(x)$ is always reducible over $\mathbb{F}_2$.*

Proof Since the coefficients of $A(x)$ are all equal to 0 or 1, let $\bar{A}(x) = A(x)$ be the lift of $A(x)$ to the integers. By Theorem 2.3 we know that if $Disc(\bar{A}) \equiv 1 (\mathrm{mod}\ 8)$, then $A(x)$ has an even number of irreducible factors over $\mathbb{F}_2$ and thus is reducible. Therefore in the next sequel, we concentrate on computing the discriminant of $\bar{A}(x)$. First, we assume that $n_3 \geq 3$. By Proposition 2.1, we have

$$Disc(\bar{A}) = Res(\bar{A}, \bar{A}') = Res(\bar{A}, 2^{n_1}x^{2^{n_1}-1} + 2^{n_2}x^{2^{n_2}-1} + 2^{n_3}x^{2^{n_3}-1} + 1). \tag{4}$$

Since $n_1 > n_2 > n_3 \geq 3$, we have $2^{n_1} \equiv 2^{n_2} \equiv 2^{n_3} \equiv 0 (\mathrm{mod} 8)$ and

$$Disc(\bar{A}) \equiv Res(\bar{A}, 1) \equiv 1 (\mathrm{mod}\ 8). \tag{5}$$

Using Theorem 2.3, we get that $A(x)$ is reducible over $\mathbb{F}_2$ when $n_3 \geq 3$.

For the sake of completeness, we need to study the remainder three cases, that is,

Case 1: $n_3 = 1, n_2 = 2$;
Case 2: $n_3 = 1, n_2 > 2$;
Case 3: $n_3 = 2$.

Case 1 Similar to $n_3 \geq 3$, we can get

$$Disc(\bar{A}) = Res(\bar{A}, \bar{A}') = Res(\bar{A}, 2^{n_1}x^{2^{n_1}-1} + 4x^3 + 2x + 1). \tag{6}$$

We note that $n_1 \geq 3$, (6) can be written as

$$Disc(\bar{A}) \equiv Res(\bar{A}, 4x^3 + 2x + 1)(\text{mod } 8). \tag{7}$$

In what follows, we focus our attention on computing a determinant. By the second definition of the resultant of two polynomials, we get a determinant

$$Res(\bar{A}, 4x^3 + 2x + 1) = det \begin{pmatrix} 1 & 0 & 0\,0\,0\,0 \cdots 0\,1\,0\,1\,1\,1 & 0 & 0 \\ 0 & 1 & 0\,0\,0\,0 \cdots 0\,0\,1\,0\,1\,1 & 1 & 0 \\ 0 & 0 & 1\,0\,0\,0 \cdots 0\,0\,0\,1\,0\,1 & 1 & 1 \\ 4 & 0 & 2\,1\,0\,0 \cdots 0\,0\,0\,0\,0\,0 & 0 & 0 \\ 0 & 4 & 0\,2\,1\,0 \cdots 0\,0\,0\,0\,0\,0 & 0 & 0 \\ 0 & 0 & 4\,0\,2\,1 \cdots 0\,0\,0\,0\,0\,0 & 0 & 0 \\ & \cdots & \cdots & \cdots & \\ 0 & 0 & 0\,0\,0\,0 \cdots 0\,0\,0\,0\,4\,0 & 2 & 1 \end{pmatrix} := \triangle$$

of order $2^{n_1} + 3$.

Multiplying the first row by 4 and adding it to the 4th row of $\triangle$, the element in the 4th row and the first column vanishes modulo 8, so we obtain

$$\triangle \equiv det \begin{pmatrix} 1 & 0 & 0\,0\,0\,0 \cdots 0\,1\,0\,1\,1\,1 & 0 & 0 \\ 0 & 1 & 0\,0\,0\,0 \cdots 0\,0\,1\,0\,1\,1 & 1 & 0 \\ 0 & 0 & 1\,0\,0\,0 \cdots 0\,0\,0\,1\,0\,1 & 1 & 1 \\ 0 & 0 & 2\,1\,0\,0 \cdots 0\,4\,0\,4\,4\,4 & 0 & 0 \\ 0 & 4 & 0\,2\,1\,0 \cdots 0\,0\,0\,0\,0\,0 & 0 & 0 \\ 0 & 0 & 4\,0\,2\,1 \cdots 0\,0\,0\,0\,0\,0 & 0 & 0 \\ & \cdots & \cdots & \cdots & \\ 0 & 0 & 0\,0\,0\,0 \cdots 0\,0\,0\,0\,4\,0 & 2 & 1 \end{pmatrix} (\text{mod } 8).$$

Multiplying the third row by 6 and adding it to the 4th row, the element in the 4th row and the third column vanishes modulo 8. Therefore, using this method step by step, we obtain an upper-triangular determinant with the elements in the main diagonal are all 1's modulo 8. Thus we have $Res(\bar{A}, 4x^3 + 2x + 1) \equiv 1(\text{mod}8)$.

Therefore, we have $Disc(\bar{A}) \equiv 1(\text{mod}8)$.

Case 2 Similarly, we have

$$Disc(\bar{A}) = Res(\bar{A}, \bar{A}') \equiv Res(\bar{A}, 2x + 1)(\text{mod } 8). \tag{8}$$

Similar to Case 1, we get a determinant

$$Res(\bar{A}, 2x + 1) = det \begin{pmatrix} 1 & 0 & 0\,0\,0 \cdots 0\,1\,0 \cdots 0\,0\,1 & 1 & 1 \\ 2 & 1 & 0\,0\,0 \cdots 0\,0\,0 \cdots 0\,0\,0 & 0 & 0 \\ 0 & 2 & 1\,0\,0 \cdots 0\,0\,0 \cdots 0\,0\,0 & 0 & 0 \\ 0 & 0 & 2\,1\,0 \cdots 0\,0\,0 \cdots 0\,0\,0 & 0 & 0 \\ & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0\,0\,0 \cdots 0\,0\,0 \cdots 0\,0\,0 & 2 & 1 \end{pmatrix}$$

which the order is $2^{n_1}+1$. Using above method step by step, we also obtain an upper-triangular determinant with the elements in the main diagonal are all 1's modulo 8. Thus we have $Res(\bar{A}, 2x+1) \equiv 1 (\text{mod} 8)$. Therefore, $Disc(\bar{A}) \equiv 1 (\text{mod} 8)$.

Similarly, we can get that $Disc(\bar{A}) \equiv 1 (\text{mod} 8)$ under condition $n_3 = 2$. In fact, we only need to compute the determinant

$$Res(\bar{A}, 4x^3+1) = det \begin{pmatrix} 1 & 0 & 0\,0\,0 \cdots 0\,1\,0\,0 \cdots 0\,1\,0\,0\,1\,1 & 0 & 0 \\ 0 & 1 & 0\,0\,0 \cdots 0\,0\,1\,0 \cdots 0\,0\,1\,0\,0\,1 & 1 & 0 \\ 0 & 0 & 1\,0\,0 \cdots 0\,0\,0\,1 \cdots 0\,0\,0\,1\,0\,0 & 1 & 1 \\ 4 & 0 & 0\,1\,0 \cdots 0\,0\,0\,0 \cdots 0\,0\,0\,0\,0\,0 & 0 & 0 \\ 0 & 4 & 0\,0\,1 \cdots 0\,0\,0\,0 \cdots 0\,0\,0\,0\,0\,0 & 0 & 0 \\ & \cdots & \cdots \qquad \cdots & \cdots & \\ 0 & 0 & 0\,0\,0 \cdots 0\,0\,0\,0 \cdots 0\,0\,0\,0\,4\,0 & 0 & 1 \end{pmatrix}$$

of order $2^{n_1}+3$, and the method of computation is the same as above.

By Stickelberger–Swan Theorem, the desired result follows and we complete our proof. □

Since every affine pentanomial over $\mathbb{F}_2$ is reducible, can we give some explicit factors of it over $\mathbb{F}_2$? In what follows, we answer this in the positive for some particular affine pentanomials over $\mathbb{F}_2$.

Obviously, $2^n \equiv 2 (\text{mod } 3)$ if n is odd, and $2^n \equiv 1 (\text{mod } 3)$ when n is even. Therefore, we can get that if n is an odd number, then $x^{2^n} \equiv x^2 (\text{mod } x^3+1)$ over $\mathbb{F}_2$, otherwise $x^{2^n} \equiv x (\text{mod } x^3+1)$. Hence we have following proposition.

Theorem 3.2 *Let $A(x) = x^{2^{n_1}} + x^{2^{n_2}} + x^{2^{n_3}} + x + 1$ be an affine pentanomial over $\mathbb{F}_2$, where $n_1 > n_2 > n_3 \geq 1$. If the number of odd numbers in n_1, n_2, n_3 is odd, then $A(x)$ has x^2+x+1 as an irreducible factor of $A(x)$ over $\mathbb{F}_2$.*

Proof If the number of odd numbers in n_1, n_2, n_3 is odd, then $A(x) \equiv x^2+x+1 (\text{mod } x^3+1)$ over $\mathbb{F}_2$. Since x^2+x+1 divides x^3+1, we can get our conclusion. □

Remark 3.1 When the number of odd numbers in n_1, n_2, n_3 is two or n_1, n_2, n_3 are all even numbers, the situation is complicated, sometimes $A(x)$ has x^4+x+1 as an irreducible factor, and we can find a necessary condition for $x^4+x+1 | A(x)$ but we cannot find an explicit factor of $A(x)$ in general. However, we have the following result.

Theorem 3.3 *Let $A(x) = x^{2^{n_1}} + x^{2^{n_2}} + x^{2^{n_3}} + x + 1$ be an affine pentanomial over $\mathbb{F}_2$, where $n_1 > n_2 > n_3 \geq 1$. If there exists integers t, t' with $0 \leq t < t'$ such that $n_1 \equiv n_2 \equiv n_3 \equiv 2^t (\text{mod } 2^{t'})$, then $x^{2^{2^t}} + x + 1$ is an factor of $A(x)$ over $\mathbb{F}_2$.*

Proof We can rewrite $A(x)$ as

$$A(x) = (x^{2^{n_1}} + x + 1) + (x^{2^{n_2}} + x + 1) + (x^{2^{n_3}} + x + 1), \tag{9}$$

Thus we only need to prove that $x^{2^{2^t}} + x + 1$ divides $x^{2^n} + x + 1$ over $\mathbb{F}_2$ if $n \equiv 2^t (\text{mod } 2^{t'})$.

Let $n = 2^{t'}m + 2^t$. Then the result is trivial when $m = 0$. Now

$$(x^{2^n} + x + 1) - (x^{2^{2^t}} + x + 1) = (x^{2^{2^{t'}m}} - x)^{2^{2^t}}. \tag{10}$$

Let $w(x)$ be an irreducible factor of $x^{2^{2^t}} + x + 1$ over $\mathbb{F}_2$ with degree s. By ([9], exercise 3.91), we know that s divides 2^{t+1}, and so s divides $2^{t'}m$. Therefore $w(x)$ divides $x^{2^{2^{t'}m}} - x$.

Thus $x^{2^{2^l}}+x+1$ divides $x^{2^{2^{l'}m}}-x$ and thus divides $x^{2^n}+x+1$, and this completes our proof. □

For any affine polynomial $A(x)=\sum_{i=0}^{s}x^{2^{n_i}}+1\ (n_s>n_{s-1}>\cdots>n_1>n_0=0, s\geq 1)$ over $\mathbb{F}_2$, it is obvious that when s is an even number, $A(1)=0$ and thus $A(x)$ is reducible over $\mathbb{F}_2$. If $s=1$, by the result of Swan [12], $A(x)$ is always reducible except x^4+x+1 and x^2+x+1. If $s\geq 3$ is an odd number, we can also claim that $A(x)$ is reducible by our methods above.

Corollary 3.4 *Let* $A(x)=\sum_{i=0}^{s}x^{2^{n_i}}+1$ *be an affine polynomial over* $\mathbb{F}_2$*, where* $n_s>n_{s-1}>\cdots>n_1>n_0=0$*, and* $s\geq 3$ *is an odd number. Then* $A(x)$ *is always reducible over* $\mathbb{F}_2$*.*

Proof Let $\bar{A}(x)=A(x)$ be the lift of $A(x)$ to the integers. First, we assume that $n_1\geq 3$. It is obvious that

$$Disc(\bar{A})=Res(\bar{A},\bar{A}')=Res(\bar{A},\sum_{i=1}^{s}2^{n_i}x^{2^{n_i}-1}+1)\equiv Res(\bar{A},1)\equiv 1(\text{mod } 8).$$

With the Stickelberger–Swan Theorem, $A(x)$ has an even number of irreducible factors over $\mathbb{F}_2$, and thus is reducible over $\mathbb{F}_2$.

In what follows, we study the remainder three cases, that is,

Case 1: $n_1=1, n_2=2$;
Case 2: $n_1=1, n_2>2$;
Case 3: $n_1=2$.

Case 1 Similar to Theorem 3.1,

$$Disc(\bar{A})=Res(\bar{A},\bar{A}')\equiv Res(\bar{A},4x^3+2x+1)(\text{mod } 8).$$

Using the second definition of resultant of two polynomials, we only need to compute the determinant $Res(\bar{A},4x^3+2x+1)$ of order $2^{n_s}+3$, and the method of computation is the same as in Case 1 of Theorem 3.1. Thus we can obtain that $Disc(\bar{A})\equiv 1(\text{mod } 8)$.

By the method of Theorem 3.1, we can get

Case 2

$$Disc(\bar{A})=Res(\bar{A},\bar{A}')\equiv Res(\bar{A},2x+1)\equiv 1(\text{mod } 8);$$

Case 3

$$Disc(\bar{A})=Res(\bar{A},\bar{A}')\equiv Res(\bar{A},4x^3+1)\equiv 1(\text{mod } 8).$$

By the Stickelberger–Swan Theorem, $A(x)$ is always reducible under the condition. □

With Theorem 3.1 and Corollary 3.4, we obtain that every affine polynomial with degree >1 over $\mathbb{F}_2$ is reducible except x^2+x+1 and x^4+x+1.

Remark 3.2 With the method of Theorem 3.2 and Theorem 3.3, we can get some explicit irreducible factors of some special affine polynomials over $\mathbb{F}_2$. For example, if the number of odd numbers in $n_1,n_2,\ldots,n_s$ is odd($s\geq 1$ is odd), then x^2+x+1 is an irreducible factor of $\sum_{i=0}^{s}x^{2^{n_i}}+1$ over $\mathbb{F}_2$.

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ENERGY-TRANSPORT LIMIT OF THE HYDRODYNAMIC MODEL FOR SEMICONDUCTORS

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This paper is mainly devoted to study the energy-transport limit of a non-isentropic hydrodynamic model with momentum relaxation time τ and energy relaxation time σ. Inspired by the Maxwell iteration, we construct a new approximation under the assumption $\tau\sigma = 1$, and show that periodic initial-value problems of a certain scaled hydrodynamic model have unique smooth solutions in a finite time interval independent of τ. Furthermore, it is also obtained that as τ tends to zero, the smooth solutions converge to the smooth solutions of energy-transport models at the rate of τ^2. The proof of these results is based on a continuation principle.

Keywords: Hydrodynamic model; energy-transport limit; continuation principle.

AMS Subject Classification: 35B25, 35L45, 35M20

1. Introduction

Accurate modeling of heat transport plays an important role in semiconductor science with the fast development of miniaturization devices. Their behavior is heavily affected by high-field phenomena (such as velocity overshoot effect, ballistic effect, etc.), while the traditional drift-diffusion model does not provide an adequate description of these effects. Consequently, several kinds of new macroscopic models exhibiting them are introduced, such as hydrodynamic models, energy-transport models and MEP models. With appropriate closure conditions, these models can be derived from the semiclassical Boltzmann equation coupled with a Poisson equation for the electric potential by a moment method or by a Hilbert expansion, see Refs. 3 and 19 for more explanation.

This paper gives the asymptotic relation between non-isentropic hydrodynamic models and energy-transport models. Denote by $n = n(t, x)$, $\mathbf{u}(t, x)$ and $T(t, x)$ the electron density, electron velocity and electron temperature, respectively. $\Phi = \Phi(t, x)$

represents the electrostatic potential generated by the Coulomb force from the electrons and background ions. After a re-scaling of time, these variables satisfy the following non-isentropic hydrodynamic model for semiconductors

$$\begin{cases} \partial_t n + \frac{1}{\tau}\operatorname{div}(n\mathbf{u}) = 0, \\ \partial_t(n\mathbf{u}) + \frac{1}{\tau}\operatorname{div}(n\mathbf{u}\otimes\mathbf{u}) + \frac{1}{\tau}\nabla(nT) = \frac{1}{\tau}n\nabla\Phi - \frac{1}{\tau^2}n\mathbf{u}, \\ \partial_t\left(\frac{n|\mathbf{u}|^2}{2} + \frac{nT}{\gamma-1}\right) + \frac{1}{\tau}\operatorname{div}\left[\left(\frac{n|\mathbf{u}|^2}{2} + \frac{\gamma nT}{\gamma-1}\right)\mathbf{u}\right] \\ \quad = \frac{1}{\tau}n\mathbf{u}\nabla\Phi - \frac{1}{\tau\sigma}\left[\frac{n|\mathbf{u}|^2}{2} + \frac{n(T-T_L(x))}{\gamma-1}\right], \\ \Delta\Phi = n - b(x) \quad \text{for } (t,x)\in[0,+\infty)\times\mathbf{T}^d, \end{cases} \tag{1.1}$$

where $\mathbf{T}^d$ $(d \geq 2)$ is the d-dimensional torus. The dimensionless parameters τ, σ are the respective momentum relaxation-time and energy relaxation-time, here, we assume $0 < \tau \leq 1, \sigma > 0$ for simplicity. $T_L(x) > 0$ is a given lattice temperature of semiconductor device, and $b(x) > 0$ stands for the density of fixed, positively charged background ions (doping profile). Note that the scaling

$$t = \tau\tilde{t}$$

converts (1.1) back into the original non-isentropic model[19] with $\tilde{t}$ as its time variable.

To show our approach, we need to rewrite the momentum equation in (1.1) as

$$\begin{aligned} n\mathbf{u} &= \tau n\nabla\Phi - \tau\nabla(nT) - \tau\operatorname{div}(n\mathbf{u}\otimes\mathbf{u}) - \tau^2\partial_t(n\mathbf{u}), \\ &= \tau n\nabla\Phi - \tau\nabla(nT) + O(\tau^2). \end{aligned} \tag{1.2}$$

Set $\tau\sigma = 1$, and let us substitute the truncation $n\mathbf{u} = \tau n\nabla\Phi - \tau\nabla(nT)$ into the mass equation and energy equation in (1.1) respectively, yields

$$\begin{cases} \partial_t n = \Delta(nT) - \operatorname{div}(n\nabla\Phi), \\ \partial_t\left(\frac{nT}{\gamma-1}\right) + \operatorname{div}\left[\frac{\gamma T}{\gamma-1}(n\nabla\Phi - \nabla(nT))\right] \\ \quad = [n\nabla\Phi - \nabla(nT)]\nabla\Phi - \frac{n(T-T_L)}{\gamma-1} + O(\tau^2). \end{cases} \tag{1.3}$$

Then, we immediately obtain the energy-transport model for semiconductors

$$\begin{cases} \partial_t n = \Delta(nT) - \operatorname{div}(n\nabla\Phi), \\ \partial_t\left(\frac{nT}{\gamma-1}\right) + \operatorname{div}\left[\frac{\gamma T}{\gamma-1}(n\nabla\Phi - \nabla(nT))\right] \\ \quad = [n\nabla\Phi - \nabla(nT)]\nabla\Phi - \frac{n(T-T_L)}{\gamma-1}, \\ \Delta\Phi = n - b, \end{cases} \tag{1.4}$$

which is a system of diffusion equations for the electron density and energy, and maintains the parabolic–elliptic character.

Marcati and Natalini[18] first established the relation between the isentropic hydrodynamic models which the energy equation in (1.1) is replaced with a smooth function of $n(t,x)$ and drift-diffusion models rigorously, via the zero-relaxation-time limit. Subsequently, this kind of limit problem has been investigated by various authors for entropy weak solutions,[7,10–15] and for smooth solutions.[1,2,5,16,21,23] To the best of our knowledge, energy-transport limit of hydrodynamic models is hardly studied. Up to now, only partial results are available. Under the assumptions that the global weak entropy solutions exist, Gasser and Natalini[7] studied the convergence from the system (1.1) to (1.4) in the compensated compactness framework. Independently, Y. Li[16] investigated the same convergence for *small* smooth solutions by virtue of the Aubin–Lions compactness lemma,[20] however, their results did not indicate the definite rates of convergence.

The main aim of our paper is to justify the above Maxwell iteration procedure (1.2)–(1.4) rigorously. This idea is from Yong in Ref. 23, where he proved the convergence from isentropic hydrodynamic models to drift-diffusion models at the rate of τ^2.

Let (n,T,Φ) solves the energy-transport model (1.4), inspired by the Maxwell iteration, we construct

$$\begin{cases} n_\tau = n(t,x), \\ \mathbf{u}_\tau = \tau\nabla\Phi - \tau\dfrac{\nabla(nT)}{n}, \\ T_\tau = T(t,x), \\ \Phi_\tau = \Phi = \Delta^{-1}(n-b) \end{cases} \tag{1.5}$$

as an approximation for the solution $(n^\tau, \mathbf{u}^\tau, T^\tau, \Phi^\tau)$ to the system (1.1) with initial data

$$(n^\tau, \mathbf{u}^\tau, T^\tau)(0,x) = (n_\tau, \mathbf{u}_\tau, T_\tau)(0,x). \tag{1.6}$$

Note that the initial data are well-prepared. Then, with the aid of the classical hyperbolic energy method, we can prove the validity of the approximation (1.5) and establish the following result.

Theorem 1.1. *Let $s > 1 + d/2$ be an integer. Assume that $T_L = T_L(x)$, $b = b(x)$ satisfy conditions*

$$b(x),\ T_L(x) \in H^{s+1}(\mathbf{T}^d) \tag{1.7}$$

and the energy-transport equations (1.4) *has a solution*

$$(n,T) \in \mathcal{C}([0,\mathbb{T}_*], H^{s+3}(\mathbf{T}^d)) \cap \mathcal{C}^1([0,\mathbb{T}_*], H^{s+1}(\mathbf{T}^d))$$

with positive lower bounds. Then, for sufficiently small τ, the system (1.1) *with periodic initial data* (1.6) *has a unique solution $(n^\tau, \mathbf{u}^\tau, T^\tau)$ satisfying $(n^\tau, \mathbf{u}^\tau, T^\tau) \in \mathcal{C}([0,\mathbb{T}_*], H^s(\mathbf{T}^d))$. Furthermore, there exists a constant $K > 0$, independent of τ but dependent* *on* $\mathbb{T}_* \in (0,\infty)$, *such that*

$$\sup_{t\in[0,\mathbb{T}_*]} \|(n_\tau - n^\tau, \mathbf{u}_\tau - \mathbf{u}^\tau, T_\tau - T^\tau)\|_{H^s(\mathbf{T}^d)} \le K\tau^2. \tag{1.8}$$

Remark 1.1. From (1.8) and Lemma 2.1 in the next section it simply follows that

$$\sup_{t\in[0,\mathbb{T}_*]} \|\nabla\Phi^\tau - \nabla\Phi_\tau\|_{H^s(\mathbf{T}^d)} \le K'\tau^2, \tag{1.9}$$

where $K' > 0$ is a constant independent of τ.

Remark 1.2. From (1.5) and (1.8)–(1.9) we can see that, for sufficiently small τ, the exact smooth solution $(n^\tau, \mathbf{u}^\tau, T^\tau, \nabla\Phi^\tau)$ to the system (1.1) exhibits the following asymptotic expression:

$$\begin{cases} n^\tau(t,x) = n(t,x) + O(\tau^2), \\ \mathbf{u}^\tau(t,x) = \tau\nabla\Phi(t,x) - \tau\dfrac{\nabla(nT)}{n} + O(\tau^2), \\ T^\tau(t,x) = T(t,x) + O(\tau^2), \\ \nabla\Phi^\tau(t,x) = \nabla\Phi(t,x) + O(\tau^2), \end{cases}$$

for $(t,x) \in [0,\mathbb{T}_*] \times \mathbf{T}^d$. Therefore, Theorem 1.1 characterizes the limiting behaviors more precisely than previous results[7,16] where the convergence was proved but no convergence rates were given. Moreover, *no smallness condition* on the initial data is required by Theorem 1.1.

Remark 1.3. Theorem 1.1 only deals with the case where the initial data are well-prepared. For more general periodic initial data, the initial layers will occur and similar results of form (1.8)–(1.9) may still be verified by using the matched expansion methods, e.g. see Ref. 24. This will be shown in a forthcoming paper. On the other hand, it is not difficult to see that our arguments and results hold true for the *bipolar* non-isentropic hydrodynamic models.

The rest of the paper is arranged as follows. In Sec. 2, we rewrite the hydrodynamic models as a symmetrizable hydrodynamic system and review some Moser-type calculus inequalities in Sobolev space and a continuation principle.[4,23] The approximation solution (1.5) is discussed in Sec. 3 and Sec. 4 is devoted to the proof of Theorem 1.1. In the Appendix, we present another drift-diffusion limit of the system (1.1) derived from the Maxwell iteration, which is similar to the proof of Theorem 1.1.

Notations. Throughout this paper, C is a generic positive constant independent of τ. $H^s(\mathbf{T}^d)$ $(s \ge 0)$ is the usual Sobolev space on the d-dimensional unit torus $\mathbf{T}^d = (0,1]^d$ whose distribution derivatives of order $\le s$ are all in $L^2(\mathbf{T}^d)$. We use the notation $\|U\|_s$ $(\|U\|_0 := \|U\|)$ as the space norm. Finally, we denote by $\mathcal{C}([0,\mathbb{T}], X)$ (resp., $\mathcal{C}^1([0,\mathbb{T}], X)$) the space of continuous (resp., continuously differentiable) functions on $[0,\mathbb{T}]$ with values in a Banach space X.