

Graduate Texts in Mathematics

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Rainer Nagel

One-Parameter Semigroups for Linear Evolution Equations

线性发展方程的单参数半群



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One-Parameter Semigroups for Linear Evolution Equations

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Preface

The theory of one-parameter semigroups of linear operators on Banach spaces started in the first half of this century, acquired its core in 1948 with the Hille–Yosida generation theorem, and attained its first apex with the 1957 edition of *Semigroups and Functional Analysis* by E. Hille and R.S. Phillips. In the 1970s and 80s, thanks to the efforts of many different schools, the theory reached a certain state of perfection, which is well represented in the monographs by E.B. Davies [Dav80], J.A. Goldstein [Gol85], A. Pazy [Paz83], and others.

Today, the situation is characterized by manifold applications of this theory not only to the traditional areas such as partial differential equations or stochastic processes. Semigroups have become important tools for integro-differential equations and functional differential equations, in quantum mechanics or in infinite-dimensional control theory. Semigroup methods are also applied with great success to concrete equations arising, e.g., in population dynamics or transport theory. It is quite natural, however, that semigroup theory is in competition with alternative approaches in all of these fields, and that as a whole, the relevant functional-analytic toolbox now presents a highly diversified picture.

At this point we decided to write a new book, reflecting this situation but based on our personal mathematical taste. Thus, it is a book on semigroups or, more precisely, on one-parameter semigroups of bounded linear operators. In our view, this reflects the basic philosophy, first and strongly emphasized by A. Hadamard (see p. 152), that an autonomous deterministic system is described by a one-parameter semigroup of transformations.

Among the many continuity properties of these semigroups that were

already studied by E. Hille and R.S. Phillips in [HP57], we deliberately concentrate on strong continuity and show that this is the key to a deep and beautiful theory. Referring to many concrete equations, one might object that semigroups, and especially strongly continuous semigroups, are of limited value, and that other concepts such as integrated semigroups, regularized semigroups, cosine families, or resolvent families are needed. While we do not question the good reasons leading to these concepts, we take a very resolute stand in this book insofar as we put strongly continuous semigroups of bounded linear operators into the undisputed center of our attention. Around this concept we develop techniques that allow us to obtain

- a semigroup on an appropriate Banach space even if at first glance the semigroup property does not hold, and
- strong continuity in an appropriate topology where originally only weaker regularity properties are at hand.

In Chapter VI we then show how these constructions allow the treatment of many different evolution equations that initially do not have the form of a homogeneous abstract Cauchy problem and/or are not “well-posed” in a strict sense.

Structure of the Book

This is not a research monograph but an introduction to the theory of semigroups. After developing the fundamental results of this theory we emphasize spectral theory, qualitative properties, and the broad range of applications. Moreover, our book is written in the spirit of functional analysis. This means that we prefer abstract constructions and general arguments in order to underline basic principles and to minimize computations. Some of the required tools from functional analysis, operator theory, and vector-valued integration are collected in the appendices.

In Chapter I, we intentionally take a slow start and lead the reader from the finite-dimensional and uniformly continuous case through multiplication and translation semigroups to the notion of a strongly continuous semigroup.

To these semigroups we associate a generator in Chapter II and characterize these generators in the Hille–Yosida generation theorem and its variants. Semigroups having stronger regularity properties such as analyticity, eventual norm continuity, or compactness are then characterized, whenever possible, in a similar way. A special feature of our approach is the use of a rich scale of interpolation and extrapolation spaces associated to a strongly continuous semigroup. A comprehensive treatment of these “Sobolev towers” is presented by Simon Brendle in Section II.5.

In Chapter III we start with the classical Bounded Perturbation Theorem III.1.3, but then present a new simultaneous treatment of unbounded Desch–Schappacher and Miyadera–Voigt perturbations in Section III.3. In the remaining Sections III.4 and 5 it was our goal to discuss a broad range

of applications of the Trotter–Kato Approximation Theorem III.4.8.

Spectral theory is the core of our approach, and in Chapter IV we discuss in great detail under what conditions the so-called spectral mapping theorem is valid. A first payoff is the complete description of the structure of periodic groups in Theorem IV.2.27.

On the basis of this spectral theory we then discuss in Chapter V qualitative properties of the semigroup such as stability, hyperbolicity, and mean ergodicity. Inspired by the classical Liapunov stability theorem we try to describe these properties by the spectrum of the generator. It is rewarding to see how a combination of spectral theory with geometric properties of the underlying Banach space can help to overcome many of the typical difficulties encountered in the infinite-dimensional situation.

Only at the end of Chapter II do differential equations and initial value problems appear explicitly in our text. This does not mean that we neglect this aspect. On the contrary, the many applications of semigroup theory to all kinds of evolution equations elaborated in Chapter VI are the ultimate goal of our efforts. However, we postpone this discussion until a powerful and systematic theory is at hand.

In the final chapter, Chapter VII, Tanja Hahn and Carla Perazzoli try to embed today's theory into a historical perspective in order to give the reader a feeling for the roots and the *raison d'être* of semigroup theory.

Furthermore, we add to our exposition of the mathematical theory an epilogue by Gregor Nickel, in which he discusses the philosophical question concerning the relationship between semigroups and evolution equations and the philosophical concept of “determinism.” This is certainly a matter worth considering, but regrettably not much discussed in the mathematical community. For this reason, we encourage the reader to grapple and come to terms with this genuine philosophical question. It is enlightening to see how such questions were formulated and resolved in different epochs of the history of thought. Perhaps a deeper understanding will emerge of how one's own contemporary mathematical concepts and theories are woven into the broad tapestry of metaphysics.

Guide to the Reader

The text is not meant to be read in a linear manner. Thus, the reader already familiar with, or not interested in, the finite-dimensional situation and the detailed discussion of examples may start immediately with Section I.5 and then proceed quickly to the Hille–Yosida Generation Theorems II.3.5 and II.3.8 via Section II.1. To indicate other shortcuts, several sections, subsections, and paragraphs are given in small print.

Such an individual reading style is particularly appropriate with regard to Chapter VI, since our applications of semigroup theory to the various evolution equations are more or less independent of each other. The reader should select a section according to his/her interest and then continue with the more specialized literature indicated in the notes. Or, he/she may

even start with a suitable section of Chapter VI and then follow the back references in the text in order to understand our arguments.

The exercises at the end of each section should lead to a better understanding of the theory. Occasionally, we state interesting recent results as an exercise marked by *.

The notes are intended to identify our sources, to integrate the text into a broader picture, and to suggest further reading. Inevitably, they also reflect our personal perspective, and we apologize for omissions and inaccuracies. Nevertheless, we hope that the interested reader will be put on the track to uncover additional information.

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Klaus-Jochen Engel
Rainer Nagel
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Prelude

An Excerpt from *Der Mann ohne Eigenschaften* (*The Man Without Qualities*) by Robert Musil*

in German, followed by the English Translation

Es läßt sich verstehen, daß ein Ingenieur in seiner Besonderheit aufgeht, statt in die Freiheit und Weite der Gedankenwelt zu münden, obgleich seine Maschinen bis an die Enden der Erde geliefert werden; denn er braucht ebensowenig fähig zu sein, das Kühne und Neue der Seele seiner Technik auf seine Privatseele zu übertragen, wie eine Maschine imstande ist, die ihr zugrunde liegenden Infinitesimalgleichungen auf sich selbst anzuwenden. Von der Mathematik aber läßt sich das nicht sagen; da ist die neue Denklehre selbst, der Geist selbst, liegen die Quellen der Zeit und der Ursprung einer ungeheuerlichen Umgestaltung.

Wenn es die Verwirklichung von Urträumen ist, fliegen zu können und mit den Fischen zu reisen, sich unter den Leibern von Bergriesen durchzubohren, mit göttlichen Geschwindigkeiten Botschaften zu senden, das Unsichtbare und Ferne zu sehen und sprechen zu hören, Tote sprechen zu hören, sich in wundertätigen Genesungsschlaf versenken zu lassen, mit lebenden Augen erblicken zu können, wie man zwanzig Jahre nach seinem Tode aussehen wird, in flimmernden Nächten tausend Dinge über und unter dieser Welt zu wissen, die früher niemand gewußt hat, wenn Licht, Wärme, Kraft, Genuß, Bequemlichkeit Urträume der Menschheit sind,—dann ist die

* Rowohlt Verlag, Hamburg 1978, by permission.

heutige Forschung nicht nur Wissenschaft, sondern ein Zauber, eine Zereemonie von höchster Herzens- und Hirnkraft, vor der Gott eine Falte seines Mantels nach der anderen öffnet, eine Religion, deren Dogmatik von der harten, mutigen, beweglichen, messerkühlen und -scharfen Denklehre der Mathematik durchdrungen und getragen wird.

Allerdings, es ist nicht zu leugnen, daß alle diese Urträume nach Meinung der Nichtmathematiker mit einemmal in einer ganz anderen Weise verwirklicht waren, als man sich das ursprünglich vorgestellt hatte. Münchhausens Posthorn war schöner als die fabriksmäßige Stimmkonserve, der Siebenmeilenstiefel schöner als ein Kraftwagen, Laurins Reich schöner als ein Eisenbahntunnel, die Zauberwurzel schöner als ein Bildtelegramm, vom Herz seiner Mutter zu essen und die Vögel zu verstehen schöner als eine tierpsychologische Studie über die Ausdrucksbewegung der Vogelstimme. Man hat Wirklichkeit gewonnen und Traum verloren. Man liegt nicht mehr unter einem Baum und guckt zwischen der großen und der zweiten Zehe hindurch in den Himmel, sondern man schafft; man darf auch nicht hungrig und verträumt sein, wenn man tüchtig sein will, sondern muß Beefsteak essen und sich rühren. (...). Man braucht wirklich nicht viel darüber zu reden, es ist den meisten Menschen heute ohnehin klar, daß die Mathematik wie ein Dämon in alle Anwendungen unseres Lebens gefahren ist. Vielleicht glauben nicht alle diese Menschen an die Geschichte vom Teufel, dem man seine Seele verkaufen kann; aber alle Leute, die von der Seele etwas verstehen müssen, weil sie als Geistliche, Historiker, Künstler gute Einkünfte daraus beziehen, bezeugen es, daß sie von der Mathematik ruiniert worden sei und daß die Mathematik die Quelle eines bösen Verstandes bilde, der den Menschen zwar zum Herrn der Erde, aber zum Sklaven der Maschine macht. Die innere Dürre, die ungeheuerliche Mischung von Schärfe im Einzelnen und Gleichgültigkeit im Ganzen, das ungeheure Verlassensein des Menschen in einer Wüste von Einzelheiten, seine Unruhe, Bosheit, Herzensgleichgültigkeit ohnegleichen, Geldsucht, Kälte und Gewalttätigkeit, wie sie unsre Zeit kennzeichnen, sollen nach diesen Berichten einzig und allein die Folge der Verluste sein, die ein logisch scharfes Denken der Seele zufügt! Und so hat es auch schon damals, als Ulrich Mathematiker wurde, Leute gegeben, die den Zusammenbruch der europäischen Kultur voraussagten, weil kein Glaube, keine Liebe, keine Einfalt, keine Güte mehr im Menschen wohne, und bezeichnenderweise sind sie alle in ihrer Jugend- und Schulzeit schlechte Mathematiker gewesen. Damit war später für sie bewiesen, daß die Mathematik, Mutter der exakten Naturwissenschaft, Großmutter der Technik, auch Erzmutter jenes Geistes ist, aus dem schließlich auch Giftgase und Kampfflieger aufgestiegen sind.

In Unkenntnis dieser Gefahren lebten eigentlich nur die Mathematiker selbst und ihre Schüler, die Naturforscher, die von alledem so wenig in ihrer Seele verspüren wie Rennfahrer, die fleißig darauf los treten und nichts in der Welt bemerken als das Hinterrad ihres Vordermanns. Von Ulrich dagegen konnte man mit Sicherheit sagen, daß er die Mathematik liebte,

wegen der Menschen, die sie nicht ausstehen mochten. Er war weniger wissenschaftlich als menschlich verliebt in die Wissenschaft. Er sah, daß sie in allen Fragen, wo sie sich für zuständig hält, anders denkt als gewöhnliche Menschen. Wenn man statt wissenschaftlicher Anschauungen Lebensanschauung setzen würde, statt Hypothese Versuch und statt Wahrheit Tat, so gäbe es kein Lebenswerk eines ansehnlichen Naturforschers oder Mathematikers, das an Mut und Umsturzskraft nicht die größten Taten der Geschichte weit übertreffen würde. Der Mann war noch nicht auf der Welt, der zu seinen Gläubigen hätte sagen können: Stehlt, mordet, treibt Unzucht— unserer Lehre ist so stark, daß sie aus der Jauche eurer Sünden schäumend helle Bergwässer macht; aber in der Wissenschaft kommt es alle paar Jahre vor, daß etwas, das bis dahin als Fehler galt, plötzlich alle Anschauungen umkehrt oder daß ein unscheinbarer und verachteter Gedanke zum Herrscher über ein neues Gedankenreich wird, und solche Vorkommnisse sind dort nicht bloß Umstürze, sondern führen wie eine Himmelsleiter in die Höhe. Es geht in der Wissenschaft so stark und unbekümmert und herrlich zu wie in einem Märchen. Und Ulrich fühlte: die Menschen wissen das bloß nicht; sie haben keine Ahnung, wenn man sie neu denken lehren könnte, würden sie auch anders leben.

Nun wird man sich freilich fragen, ob es denn auf der Welt so verkehrt zugehe, daß sie immerdar umgedreht werden müsse? Aber darauf hat die Welt längst selbst zwei Antworten gegeben. Denn seit sie besteht, sind die meisten Menschen in ihrer Jugend für das Umdrehen gewesen. Sie haben es lächerlich empfunden, daß die Älteren am Bestehenden hingen und mit ihrem Herzen dachten, einem Stück Fleisch, statt mit dem Gehirn. (...). Dennoch haben sie, sobald sie in die Jahre der Verwirklichung gekommen sind, nichts mehr davon gewußt und noch weniger wissen wollen. Darum werden auch viele, denen Mathematik oder Naturwissenschaft einen Beruf bedeuten, es als einen Mißbrauch empfinden, sich aus solchen Gründen wie Ulrich für eine Wissenschaft zu entscheiden.

*The Man Without Qualities**

It is understandable that an engineer should be completely absorbed in his speciality, instead of pouring himself out into the freedom and vastness of the world of thought, even though his machines are being sent off to the ends of the earth; for he no more needs to be capable of applying to his own personal soul what is daring and new in the soul of his subject than a machine is in fact capable of applying to itself the differential calculus on which it is based. The same thing cannot, however, be said about mathematics; for here we have the new method of thought, pure intellect, the

* From *The Man Without Qualities* by Robert Musil, trans. Sophie Wilkins.
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very wellspring of the times, the *fons et origo* of an unfathomable transformation.

If the realization of primordial dreams is flying, traveling with the fishes, boring one's way under the bodies of mountain-giants, sending messages with godlike swiftness, seeing what is invisible and what is in the distance and hearing its voice, hearing the dead speak, having oneself put into a wonder-working healing sleep, being able to behold with living eyes what one will look like twenty years after one's death, in glimmering nights to know a thousand things that are above and below this world, things that no one ever knew before, if light, warmth, power, enjoyment, and comfort are mankind's primordial dreams, then modern research is not only science but magic, a ritual involving the highest powers of heart and brain, before which God opens one fold of His mantle after another, a religion whose dogma is permeated and sustained by the hard, courageous, mobile, knife-cold, knife-sharp mode of thought that is mathematics.

Admittedly, it cannot be denied that in the nonmathematician's opinion all these primordial dreams were suddenly realized in quite a different way from what people had once imagined. Baron Münchhausen's post-horn was more beautiful than mass-produced canned music, the Seven-League Boots were more beautiful than a motor-car, Dwarf-King Laurin's realm more beautiful than a railway-tunnel, the magic mandrake-root more beautiful than a telegraphed picture, to have eaten of one's mother's heart and so to understand the language of birds more beautiful than an animal psychologist's study of the expressive values in bird-song. We have gained in terms of reality and lost in terms of the dream. We no longer lie under a tree, gazing up at the sky between our big toe and second toe; we are too busy getting on with our jobs. And it is no good being lost in dreams and going hungry, if one wants to be efficient; one must eat steak and get a move on. (...). There is really no need to say much about it. It is in any case quite obvious to most people nowadays that mathematics has entered like a daemon into all aspects of our life. Perhaps not all of these people believe in that stuff about the Devil to whom one can sell one's soul; but all those who have to know something about the soul, because they draw a good income out of it as clergy, historians, or artists, bear witness to the fact that it has been ruined by mathematics and that in mathematics is the source of a wicked intellect that, while making man the lord of the earth, also makes him the slave of the machine. The inner drought, the monstrous mixture of acuity in matters of detail and indifference as regards the whole, man's immense loneliness in a desert of detail, his restlessness, malice, incomparable callousness, his greed for money, his coldness and violence, which are characteristic of our time, are, according to such surveys, simply and solely the result of the losses that logical and accurate thinking has inflicted on the soul! And so it was that even at that time, when Ulrich became a mathematician, there were people who were prophesying the collapse of European civilization on the grounds that there was no longer

any faith, any love, any simplicity or any goodness left in mankind; and it is significant that these people were all bad at mathematics at school. This only went to convince them, later on, that mathematics, the mother of the exact natural sciences, the grandmother of engineering, was also the arch-mother of that spirit from which, in the end, poison-gases and fighter aircraft have been born.

Actually, the only people living in ignorance of these dangers were the mathematicians themselves and their disciples, the natural scientists, who felt no more of all this in their souls than racing-cyclists who are pedaling away hard with no eyes for anything in the world but the back wheel of the man in front. As far as Ulrich was concerned, however, it could at least definitely be said that he loved mathematics because of the people who could not endure it. He was not so much scientifically as humanly in love with science. He could see that in all the problems that came into its orbit science thought differently from the way ordinary people thought. If for "scientific attitude" one were to read "attitude to life," for "hypothesis" "attempt" and for "truth" "action," then there would be no considerable natural scientist or mathematician whose life's work did not in courage and revolutionary power far outmatch the greatest deeds in history. The man was not yet born who could have said to his disciples: "Rob, murder, fornicate—our teaching is so strong that it will transform the cesspool of your sins into clear, sparkling mountain-rills." But in science it happens every few years that something that up to then was held to be error suddenly revolutionizes all views or that an unobtrusive, despised idea becomes the ruler over a new realm of ideas; and such occurrences are not mere upheavals but lead up into the heights like Jacob's ladder. In science the way things happen is as vigorous and matter-of-fact and glorious as in a fairy-tale. "People simply don't know this," Ulrich felt. "They have no glimmer of what can be done with thinking. If one could teach them to think in a new way, they would also live differently."

Now someone is sure to ask, of course, whether the world is so topsy-turvy that it is always having to be turned up the other way again. But the world itself long ago gave two answers to this question. For ever since it has existed most people have in their youth been in favor of turning things upside-down. They have always felt that their elders were ridiculous in being so attached to the established order of things and in thinking with their heart—a mere lump of flesh—instead of with their brains. (...). Nevertheless, by the time they reach years of fulfillment they have forgotten all about it and are far from wishing to be reminded of it. That is why many people for whom mathematics or natural science is a job feel it is almost an outrage if someone goes in for science for reasons like Ulrich's.

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