A Concise Bilingual Course of Advanced Mathematics (Part Ⅰ)

高等数学简明双语教程

（上册）

平艳茹 姚海楼 编著

北京工业大学出版社

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内 容 简 介

本书按照教育部对理工类和金融类本科生高等数学课程的要求编写，满足高等数学课程的双语教学要求。本书分上、下两册。上册的内容是一元函数微积分，包括函数与极限、导数与微分、中值定理与导数的应用、不定积分、定积分。

本书可作为高校工科本科生一年级高等数学双语课程的教材，也可作为数学专业的专业英语教材，还可供相关专业的教师和科研人员参考。

图书在版编目（CIP）数据

高等数学简明双语教程．上册 / 平艳茹，姚海楼编著．
—北京：北京工业大学出版社，2014.9
ISBN 978－7－5639－4021－9

Ⅰ．①高…　Ⅱ．①平…②姚…　Ⅲ．①高等学校－双语数学－高等学校－教材　Ⅳ．013

中国版本图书馆 CIP 数据核字（2014）第 176826 号

高等数学简明双语教程（上册）

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封面设计：何　强
出版发行：北京工业大学出版社
（北京市朝阳区平乐园 100 号　邮编：100124）
010－67391722（传真）bgdcbs@ sina. com
出 版 人：郝　勇
经销单位：全国各地新华书店
承印单位：北京溢漾印刷有限公司
开　　本：787 mm×1092 mm　1/16
印　　张：18.5
字　　数：353 千字
版　　次：2014 年 9 月第 1 版
印　　次：2014 年 9 月第 1 次印刷
标准书号：ISBN 978－7－5639－4021－9
定　　价：35.00 元

Preface

Advanced Mathematics is a basic course for the college students , which is also a discipline with highly logical, abstract and rigorous academic curriculum system. At the same time, the level of acquiring the curriculum knowledge directly affects the college students' subsequent academic study. With the higher education becoming increasingly international and trying to be in line with international standards, it is extremely necessary to carry out bilingual teaching in colleges and universities. Because of the strong logicality in the Advanced Mathematics, the bilingual teaching can enable students to change the inherent passive exam – oriented thinking mode. Cultivating talents is the starting point and also the ultimate goal of bilingual teaching. In the world of natural science, knowledge updates at an alarming rate. The most part of information and materials on science and technology is published in English globally. A good command of English in terms of the mathematical knowledge helps to keep up with the latest achievements in the natural science abroad. By means of bilingual teaching the students can enlarge their vocabulary, especially the mathematical terms in foreign language, and show keen interest in learning. In this way, the students can apply the knowledge of English to math learning. What's more important, they can realize that learning is very practical, helping lay the foundation referring to data in foreign language in the future. Bilingual teaching of Advanced Mathematics can not only improve students' communication skills, but also stimulate their learning potentials to acquire mathematical knowledge when they use English as a tool. Offering the bilingual teaching in Advanced Mathematics helps cultivate the students'abilities both in Mathematics and English. The book is completed under this background.

This book is exclusively designed for undergraduates major in engineering as bilingual Advanced Mathematics course. It can also be used as a reference book for teachers and students of the similar level and interests.

The outstanding advantage of this textbook lies in its brevity and clarity. We try to combine the advantages of foreign original teaching materials and the domestic textbooks, making it possible for the students to learn with ease and interest. Because of our efforts in carefully selecting the materials with the proper level, the students can enhance their reading ability, the ability to think and the ability to solve practical problems. The textbook is also characterized by its integrity, being systematic as well as intuitive and practical.

The textbook emphasizes the basic ideas in calculus, such as the concept of local linearization, the method of approximation, the method of optimization, the micro - element method and variable substitutions. In order to cultivate the students' consciousness, interest and ability to solve practical problems, examples and exercises are chosen to relate to practical problems.

The books learn from the original books, showing some strengths of foreign materials, considering the actual situation of the students, embodying the editors' rich experience in several rounds of bilingual teaching. But owing to our limitation in some aspects, the books are not perfect in a way. So, comments are welcomed from readers. At the same time, the books are on the way to be improved constantly through the editor's practice in bilingual teaching.

The books are supported by textbook publishing fund in Beijing University of Technology. In the end, we would like to acknowledge those who offer help to the completion of the books.

前言

高等数学是一门公共基础课，也是一门逻辑性很强、高度抽象且具有严谨课程体系的学科。同时，该课程知识掌握的程度直接关系到大学生后续课程的学习。随着高校国际化办学的日益发展，以及高等教育与国际接轨，在高校里开展双语教学显得尤为重要。由于高等数学的逻辑性强，通过双语教学能促进学生改变固有的、被动的、应试式的思维模式。培养优秀人才是双语教学的出发点，也是双语教学的最终目标。在自然科学领域，知识更新速度飞快，国际上的科技资料绝大部分是用英语发表的，掌握英语中有关数学的相关知识，有助于吸收国外优秀自然科学成果。高等数学双语教学不但能使学生在英语交流能力上获得提高，还可以使其以英语为工具获得数学知识，更能够激发学生的学习潜能。开展高等数学双语教学有助于数学与英语的相互促进，本书正是在这种背景下完成的。

本书主要是为高校工科本科生开设的高等数学双语课程而编写的，可作为高等数学双语课程的教材或参考用书。

本书借鉴了国外原版教材浅显易懂的优势，解决了国外教材课外阅读量大和国内教材偏难的问题，简洁易懂，帮助学生轻松进入阅读原版数学书籍的状态。本书在内容上紧紧围绕工科数学的特点，既照顾到数学内容的完整性与系统性，又照顾到工科学生要求教材讲解内容的直观性与实用性的特点。本教材强调了微积分的基本思想与方法：局部线性化方法、逼近的思想、最优化方法、微元法及变量代换的思想和方法等。为了培养学生应用数学解决实际问题的意识、兴趣和能力，本书中还挑选了一些与实际问题相关的例题和习题。

本书吸收了国外教材的一些长处，结合了学生的实际情况，融入了编者长期双语教学的经验。限于编者的水平，书中难免有不妥之处，恳请读者批评指正。同时，本书在编者今后的双语教学实践中有待不断完善与改进。

本书的出版得到了北京工业大学教材出版基金的资助。最后，衷心感谢为本书的完成作出奉献的人们。

目　　录

Chapter 1 Functions and Limits

1.1 Sets and Elementary Functions

1.1.1 Sets

Set is an important concept. Here we just have a quick review. A set is defined as the collection of all objects with some specified property. Sets are denoted by capital letters A, B, $\cdots$. Each of the objects in the set is called an element of the set, usually denoted by letter $a, b, \cdots$. The relation $a \in A$ means that a is an element of the set A, whereas $a \notin A$ means that a is not an element of the set A. A set containing no element is called the empty set, denoted by $\varnothing$. A subset B of A is a set which consists of some elements in A, denoted by $B \subseteq A$. Some specific set are listed as follows:

$\mathbb{N} = \{0, 1, 2, 3, \cdots\}$, the set of all natural numbers.

$\mathbb{Z}$, the set of all integers.

$\mathbb{Q}$, the set of all rational numbers.

$\mathbb{R}$, the set of all real numbers.

Intervals are a class of important number sets. Let a and b are real numbers, $a < b$. The number set

$$\{x \mid a < x < b\}$$

is called an open interval, denoted by (a, b), i. e.

$$(a, b) = \{x \mid a < x < b\},$$

a and b are called endpoints of the open interval. Here $a \notin (a, b)$ and $b \notin (a, b)$.

The closed interval $[a, b]$ is defined as the set

$$\{x \mid a \leqslant x \leqslant b\},$$

here the endpoints $a \in [a,b]$ and $b \in [a,b]$.

Any open interval with center a and radius δ, where $\delta > 0$ is called a δ-neighborhood of a point a, denoted by $U(a,\delta)$, i. e.

$$U(a,\delta) = \{x \mid a-\delta < x < a+\delta\}.$$

Sometimes, we need to delete the center of a neighborhood, which is said a deleted neighborhood, denoted by

$$\mathring{U}(a,\delta) = \{x \mid 0 < |x-a| < \delta\},$$

where $0 < |x-a|$ represent $x \neq a$. For convenience, the open interval $(a-\delta, a)$ is called the left δ-neighborhood of the point a, whereas the open interval $(a, a+\delta)$ is called the right δ-neighborhood of the point a.

1.1.2 Functions

Definition 1.1.1 (Function) Suppose that both x and y are variables, if D is a number set, and for every $x \in D$, there is only one $y \in \mathbb{R}$ corresponding to x according to some determined rule f, then f is called a function defined on set D, denoted by

$$y = f(x), x \in D.$$

where x is called an independent variable, and y is called a dependent variable, the set D is called the domain of function $f(x)$, denoted by D_f, whereas the set $\{y \mid y = f(x), x \in D\}$ is called the range of function $f(x)$, denoted by R_f.

For example, $y = x^2$, $y = \sin x$, $y = \ln x$, $y = e^x$ and so on, which are all familiar functions we have all known.

Definition 1.1.2 (One to One Function, Inverse Function) Suppose that $y = f(x)$ is a function with domain D and range R. If for each $y \in R$, there is only one $x \in D$ such that $f(x) = y$. Then we can define a function from R to D, such that $x = g(y)$ if and only if $y = f(x)$. The function $x = g(y)$ is called the Inverse Function of $y = f(x)$, denoted by $x = f^{-1}(y)$, and the domain of function $x = g(y)$, denoted by D_g, is just the range of the function $y = f(x)$ and vice versa, that is, the range of $x = g(y)$ is just the domain of function $y = f(x)$.

For example, $y = \sin x, x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ has an inverse function $y = \arcsin x, x \in [-1,1]$.

Definition 1.1.3 (Composite) The composition of two functions $y = f(u)$ and $u = g(x)$ is defined as

$$(f\circ g)(x)=f(g(x)),$$

denoted by $f\circ g$.

For example, if $f(x)=x^2$ and $g(x)=\sin x$, then $(f\circ g)(x)=f(g(x))=f(\sin x)=\sin^2 x$, and $(f\circ g)(x)=g(f(x))=\sin x^2$. It is clear that composition is not commutative.

There are four possible ways to represent a function:

(1) verbally, by a description in words;

(2) numerically, by a table of values;

(3) visually, by a graph;

(4) algebraically, by an explicit formula.

The analytic representation and graphs are widely used.

Example 1.1.1 The function $y=\operatorname{sgn} x=\begin{cases}-1, x<0;\\ 0, x=0;\\ 1, x>0.\end{cases}$ is called Sign Function, which is a piecewise defined function, a function consists of several components on different subsets of the domain of the function.

Example 1.1.2 (the Greatest Integer Function) The function whose value at any real number x is the largest integer less than or equal to x is called the greatest integer function, denoted by $[x]$, $x\in(-\infty,+\infty)$. For instance, $[-1.2]=-2$, $[1.4]=1$.

1.1.3 Properties of Functions

Now that functions play a very important role in mathematics, it is necessary to understand their properties. Now, we introduce some primary properties of functions.

Definition 1.1.4 (Bounded) Suppose that $y=f(x)$ is a function with domain D. If for any $x\in D$, there exists a number $M>0$, such that $|f(x)|\leqslant M$, then we say that $f(x)$ is bounded on D.

Definition 1.1.5 (Monotonicity) Suppose that $y=f(x)$ with domain D, and $A\subseteq D$. If $\forall$ two points $x_1, x_2\in A$ and $x_1<x_2$, there holds

$$f(x_1)\leqslant f(x_2)\ (f(x_1)\geqslant f(x_2))$$

then $f(x)$ is said to be monotonic increasing (monotonic decreasing) on A. If

$$f(x_1)<f(x_2)\ (f(x_1)>f(x_2))$$

$f(x)$ is called to be strictly monotonic increasing (strictly monotonic decreasing) on A.

Theorem 1.1.1 A strictly monotonic increasing monotonic (decreasing) function $y=$

$f(x)$ with domain D_f range R_f has a strictly monotonic increasing (decreasing) inverse function $x = f^{-1}(y)$ with domain R_f and range D_f.

Definition 1.1.6 (Even and Odd Functions) Let function $y = f(x)$ with domain D, and if $\forall x \in D$, there holds

$$f(-x) = f(x),$$

then f is called an even function, whereas if $\forall x \in D$, there holds

$$f(-x) = -f(x),$$

then $f(x)$ is called an odd function.

Graphically, an even function is symmetric along y-axis, whereas an odd function is symmetric about the origin.

Definition 1.1.7 (Periodicity) If a function $y = f(x)$ with the real number set $\mathbb{R}$ as its domain and if there is a constant $T \neq 0$ $(T > 0)$, such that $f(x+T) = f(x)$, then $y = f(x)$ is called a periodic function with period T.

Example 1.1.3 There is a function $f(x) = \begin{cases} 0, x \text{ is an irrational number;} \\ 1, x \text{ is a rational number.} \end{cases}$ which is periodic but without the smallest positive period.

Solution It is easy to check that any positive rational number is a period of the function but there is no smallest positive rational number.

Definition 1.1.8 (Basic Elementary Functions) The following functions listed below: Power Function ($y = x^{\alpha}$), Exponential Function ($y = a^x$, $a > 0$ and $a \neq 1$), Logarithm Function ($y = \log_a x$), Trigonometric Functions ($y = \sin x$, $y = \cos x$, $y = \tan x$, $y = \cot x$, $y = \sec x$, $y = \csc x$) and Inverse Trigonometric Functions ($y = \arcsin x$, $y = \arccos x$, $y = \arctan x$, $y = \operatorname{arccot} x$), each a is a constant, the five types of functions are called by a joint name Basic Elementary Functions.

Definition 1.1.9 (Elementary Functions) A function that is made up of Basic Elementary Functions and constants by a finite number of arithmetic operation steps and compositions of functions which can be expressed by a single expression is called an Elementary Function.

For example, $y = \sqrt{1-x^2}$, $y = \ln(x + \sqrt{1-x^2})$, $y = \sin^3 x$ are all Elementary Functions. However, the sign function is not an Elementary Function, because it can not be expressed by a single analytic expression.

Exercise 1.1

1. Determine whether the function is even, odd, or neither.

(1) $f(x)=\ln(x+\sqrt{1+x^2})$; (2) $f(x)=\dfrac{2^x+2^{-x}}{2}$;

(3) $f(x)=\dfrac{a^x-a^{-x}}{2}$; (4) $y=x(x-1)(x+1)$;

(5) $f(x)=\sin x-\cos x+2$.

2. Prove that any function $f(x)$ with the real number set $\mathbb{R}$ as its domain can be expressed as a sum of an even and an odd function.

3. Let $f(x)=\begin{cases}1, & |x|<1;\\ 0, & |x|=1;\\ -1, & |x|>1.\end{cases}$ $g(x)=e^x$. Find $f(g(x))$ and $g(f(x))$, and sketch the graphs of the two functions.

1.2 Limits of Functions

In this section we see how limits arise when finding the tangent to a curve or the velocity of an object.

1.2.1 The Tangent and Velocity Problems

For a circle we could simply follow Euclid and say that a tangent is a line that intersects the circle once and only once as in Figure 1.2.1(a). For more complicated curves this definition is inadequate. Figure 1.2.1(b) shows two line l and t passing through a point P on a curve C. Though the line l intersects C only once, it is obvious that l does not look like what we think of as a tangent. The line t, on the other hand, looks like a tangent but it intersects C twice.

Now we take a tangent as the limit of the secant lines, we will give detailed explanation in the following example.