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四元玉鉴

JADE MIRROR OF THE FOUR
UNKNOWNNS

II

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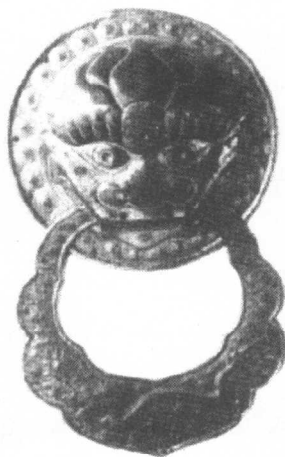
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Chinese-English

四元玉鉴

Jade Mirror of the Four Unknowns

II



[元] 朱世杰 著

郭书春 今译

陈在新 英译

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勾股测望 八问

1.

【原文】

今有直邑不知大小，各中开门。只云：南门外二百四十步有塔，人出西门行一百八十步见塔，复抹邑西南隅行一里二百四十步，恰至塔所。问：邑长、阔各几何？

答曰：长一里一百二十步，阔一里。

术曰：立天元一为邑半长，如积求之，得一十八亿六千六百二十四万为正实，一千五百五十五万二千为从方，二十七万为益上廉，四百八十为从下廉，一为正隅，三乘方开之，^[1]得二百四十步。倍之，即长。

又：立天元一为邑半阔，如积求之，得一十八亿六千六百二十四万为正实，二千七十三万六千为从方，二十七万为益上廉，三百六十为从下廉，一为正隅，三乘方开之，^[2]得一百八十步。倍之，即阔。合问。

【注释】

[1] 开方式的现代形式为： $x^4 + 480x^3 - 270000x^2 + 15552000x + 1866240000 = 0$ 。(陈)

[2] 开方式的现代形式为： $x^4 + 360x^3 - 270000x^2 + 20736000x + 1866240000 = 0$ 。(陈)

【今译】

今有长方形的城，不知大小，各在城墙的中间开门。只云：南门外240步有一座塔，人出西门西行180步，刚刚见到塔；此人擦过城的西南角走1里240步，恰好到塔的所在地。问：城的长、阔各为多少？

答：长1里20步，阔1里。

术：设天元一为城的长的 $\frac{1}{2}$ ，以如积方法求其解。得到1866240000作为常数项，15552000作为一次项系数，-270000作为二次项系数，



Gou Gu Ce Wang (Surveying with Right Triangles)

8 Problems

1. A city built in the form of a rectangle has a gate at mid-point of each side of the wall. At a distance of 240 *bu* outside of the south gate stands a pagoda. A person went out from the west gate and after walking 180 *bu* saw the pagoda. He turned and walking straight toward it found the distance to be 1 *li* and 240 *bu*. Find the length and the width of the city.

Ans. Length of the wall, 1 *li* 120 *bu*;

width of the wall, 1 *li*.

Process. Let the element *tian* be one-half of the length of the city wall. From the statement we have 1866240000 for the positive *shi*, 15552000 for the positive *fang*, 270000 for the negative upper *lian*, 480 for the positive lower *lian*, and 1 for the positive *yu*, an expression ^[1] of the fourth degree whose root, 240 *bu*, is the required number. Double this root is the length of the city wall. Again let the element *tian* be one-half of the width of the city wall. From the statement we have 1866240000 for the positive *shi*, 20736000 for the positive *fang*, 270000 for the negative upper *lian*, 360 for the positive lower *lian*, and 1 for the positive *yu*, a biquadratic expression^[2] whose root, 180 *bu*, is the required number. Double this root is the width of the city wall.

480 作为三次项系数, 1 作为最高次项系数, 开四次方, 得到 240 步。加倍, 就是长。又: 设天元一为城的阔的 $\frac{1}{2}$, 以如积方法求其解。得到 1866240000 作为常数项, 20736000 作为一次项系数, -270000 作为二次项系数, 360 作为三次项系数, 1 作为最高次项系数, 开四次方, 得到 180 步。加倍, 便得到阔。符合所问。

2.

【原文】

今有圆城不知大小, 各中开门。甲、乙俱从城心而出。甲出南门一十五步而立, 乙出东门四十步见甲。^[1] 问: 城周几何?

答曰: 一里。

术曰: 立天元一为城之半圆径, 如积求之, 得三十六万为正实, 六万六千为从方, 二千四百为从上廉, 一为益隅, 三乘方开之,^[2] 得半圆径六十步。倍而三之^[3], 即城周。合问。

【注释】

[1] 圆城心与甲、乙立处形成一个勾股形, 则 $a = r + 15$, $b = r + 40$ 。(郭)

[2] 开方式的现代形式为: $-x^4 + 2400x^2 + 66000x + 360000 = 0$ 。(陈)

[3] 取 $\pi = 3$, 故“三之”。(郭)

【今译】

今有圆城, 不知大小, 在城墙的东西南北中间开门。甲、乙二人同时从圆城的中心出发, 甲出南门 15 步站住。乙出东门 40 步, 恰好见到甲。问: 圆城的周长为多少?

答: 1 里。

术: 设天元一为城的半径, 以如积方法求其解。得到 360000 作为常数项, 66000 作为一次项系数, 2400 作为二次项系数, -1 作为最高次项系数, 开四次方, 得到圆半径 60 步。加倍, 乘以 3, 就是圆城周长。符合所问。

【 Notes 】

[1] The expression in modern form is the equation: $x^4 + 480x^3 - 270000x^2 + 15552000x + 1866240000 = 0$. (C)

[2] The expression in modern form is the equation: $x^4 + 360x^3 - 270000x^2 + 20736000x + 1866240000 = 0$. (C)

2. A city, built in the form of a circle, has four gates opening to the west, east, north, and south. Jia and Yi both started from the center of the city. *Jia* went out from the south gate and stopped at a distance of 15 *bu*. *Yi* went out from the east gate and after walking 40 *bu*, saw *Jia* at a distance. ^[1] Find the circumference of the city.

Ans. 1 *li*.

Process. Let the element *tian* be the radius of the city. From the statement we have 360000 for the positive *shi*, 66000 for the positive *fang*, 2400 for the positive upper *lian*, and 1 for the negative *yu*, an expression ^[2] of the fourth degree whose root, 60 *bu*, is the required radius. The radius multiplied by 3 gives the circumference of the city ^[3].

【 Notes 】

[1] A right triangle is formed by the center of the city, the places that Jia and Yi stopped. Then, $a=r + 15$, $b=r + 40$. (G)

[2] The expression in modern form is the equation: $-x^4 + 2400x^2 + 66000x + 360000=0$. (C)

[3] Use 3 for π , so the radius multiplied by 3. (G)



3.

【原文】

今有方城不知大小，各中开门。北门外九十步有邮亭一所，人于城中出西门外行一百六十步，却遥望参城隅见亭。^[1]问：城方几何？

答曰：二百四十步。

术曰：立天元一为城之半方面，如积求之，得一万四千四百为益实，一为正隅，平方开之，^[2]得一百二十步。倍之。合问。

【注释】

〔1〕此即邮亭、城之西北隅与人出西门遥望处三点共线。它与城之北门、西门形成两个勾股形，其勾分别为90、方城边长之半，其股分别为方城边长之半、160。（郭）

〔2〕开方式的现代形式为： $x^2 - 14400 = 0$ 。（陈）

【今译】

今有正方形的城，不知大小，在四面城墙的正中开门。北门外90步处有一所邮亭，有人从城中心出西门，走160步，恰好与城之西北角、邮亭三点共线。问：城的边长为多少？

答：240步。

术：设天元一为城的边长的 $\frac{1}{2}$ ，以如积方法求其解。得到-14400作为常数项，1作为最高次项系数，开平方，得到120步。加倍。符合所问。

4.

【原文】

今有圆城不知高、远。立两表各高一丈二尺，表间相去八十尺，令前表与后表参相直。于前表退行六十尺，人目薄地遥望乳头，与前表末参合。又从后表退行一百尺，人目薄地遥望乳头，与后表末参合。^[1]问：城高



3. A city built in the form of a square has a gate at the mid-point of each side of the wall. An arbor stands 90 *bu* from the north gate. A person started from the center of the city and after walking 160 *bu* beyond the west gate saw the arbor in the straight line with the corner of the city. ^[1] Find a side of the city.

Ans. 240 *bu*.

Process. Let the element *tian* be one-half of a side of the city. From the statement we have 14400 for the negative *shi*, and 1 for the positive *yu*, a quadratic expression ^[2] whose root, 120 *bu*, is the required number. Double this root is a side of the city.

【 Notes 】

[1] The arbor, the northwest corner, and the place where the person saw the arbor after he went out the west gate are in a straight line. Two right triangles are formed by the line, the north and west gates. Their *gou* are 90 and the half side of the city. Their *gu* are the half side of the city and 160. (G)

[2] The expression in modern form is the equation: $x^2 - 14400 = 0$. (C)

4. The distance of a round city and the height of its wall are unknown. Two measuring standards, 12 *chi* in length, are erected in a straight line with the city, the distance between them being 80 *chi*. At a point 60 *chi* from the first standard the surveyor lies on the ground and sees the tops of the wall and the first standard in a straight line. At a point 100 *chi* from the second standard he



及前表去城各几何？

答曰：城高三丈六尺，表去城一百二十尺。

术曰：立天元一为城高，如积求之，得一千四百四十为正实，四十为益方。上实，下法而一，^[2]得城高。求表去城者，以前表退行乘表间为实，两表退行差为法，实如法而一。^[3]合问。

【注释】

[1] 此为用重差术的重表法求圆城之高、远。记测望物的高、远、两表间距、表高、前表退行（或日影）、后表退行（或日影）分别为 h, l, d, a, b_1, b_2 ，刘徽在《九章算术注》序中提出公式 $h = \frac{da}{b_2 - b_1} + a$ 。（郭）

[2] 开方式的现代形式为： $-40x + 1440 = 0$ 。（陈）

[3] 刘徽提出求远公式： $l = \frac{b_1 d}{b_2 - b_1}$ 。此即应用求远公式。（郭）

【今译】

今有圆城，不知其高、远。树立两支表，高各为1丈2尺，表间相距80尺，令前表、后表与城三者共线。从前表退行60尺，人目贴地遥望城头，恰与前表端三者共线。又从后表退行100尺，人目贴地遥望城头，恰与后表端三者共线。问：城高及前表距城各为多少？

答：城高3丈6尺，前表距城120尺。

术：设天元一为城高，以如积方法求其解。得到1440作为常数项，-40作为一次项系数。常数项作为被除数，一次项系数作为除数，除之，得到城高。如果求前表距城，以前表退行乘表间相距作为被除数，两表退行之差作为除数，除之，就是前表距城。符合所问。



sees the tops of the wall and the second standard in a straight line.^[1] What is the height of the wall and the distance of the first standard from the city?

Ans. Height of the wall, 36 *chi*;

distance from the first standard, 120 *chi*.

Process. Let the element *tian* be the height of the city wall. From the statement we have 1440 for the positive *shi* and 40 for the negative *fang*, a linear expression^[2] whose root is the required height. To find the distance of the first standard from the city, use the product of the distance between the standards and the distance of the point of the observer from the first standard as the dividend, and the difference between the distance of the second point of the observer from the second standard, and the first point of the observer from the first standard as the divisor.^[3] The quotient is the required number.

【 Notes 】

[1] It means that the height of the wall and the distance from the first standard were solved by the *chong biao fa* (the method of two standards or gnomons) of the *chong cha* method (the method of double differences). Let the height of the wall be h , the distance from the first standard l , the distance between the two standards d , the height of the standard a , the first standard backward (or sun shadow) b_1 , and the second standard backward (or sun shadow) b_2 . The formula given by Liu Hui in the introduction of *The Nine Chapters of Mathematical Procedures* is as follows: $h = \frac{da}{b_2 - b_1} + a$. (G)

[2] The expression in modern form is the equation: $-40x + 1440 = 0$. (C)

[3] Liu Hui gave the formula for solving the distance: $l = \frac{b_1 d}{b_2 - b_1}$. The formula is used here. (G)

5.

【原文】

今有方城，上有戍楼，不知高、远。立两表，齐高一丈五尺，表间相去八十步，令前表与后表参相直。人目高四尺，于前表退行三十步，遥望楼岑，与前表末参合。复望楼足，入表五尺六寸。又从后表退行五十步，遥望楼岑，与后表末参合。^[1]问：城、楼各高几何？

答曰：楼高二丈八尺，城高三丈一尺。

术曰：立天元一为楼高，如积求之，得二千八百为益实，一百为从方。开无隅平方而一，^[2]得楼高。求城高者，置表高，减人目及入表，余，乘表间为实。以两表退行相多为法。实如法而一。合问。^[3]

【注释】

[1] 此问是刘徽《海岛算经》之望松类问题，楼高即松高。记松高、山高、表间、表高、前表退行、后表退行、人目高、入表分别为 $h, H, d, a, b_1, b_2, c_1, c_2$,

刘徽提出松高公式： $h = \frac{dc_2}{b_2 - b_1} + c_2$ 。(郭)

[2] 开方式的现代形式为： $100x - 2800 = 0$ 。(陈)

[3] 此即： $H = \frac{d(a - c_1 - c_2)}{b_2 - b_1}$ 。(郭)

【今译】

今有正方形的城，上面有戍楼，不知其高、远。树立两支表，高同为1丈5尺，表间相距80步，令前表、后表与城三者共线。人目高4尺。从前表退行30步，遥望楼顶，恰与前表端三者共线。再望楼底，入表5尺6寸。又从后表退行50步，遥望楼顶，恰与后表端三者共线。问：城高、楼高各为多少？

答：楼高2丈8尺，城高3丈1尺。

术：设天元一为楼高，以如积方法求其解。得到-2800作为常数项，