ECONOMIC DECISIONS UNDER UNCERTAINTY

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Preface

This book was first published in German as *Ökonomische Entscheidungen bei Ungewißheit*, Tübingen 1980. Thanks to the initiative of the editors of the present series, it is now available in English. Except for minor amendments, the English version closely follows the German original.

The translation would not have been accomplished without the help of Juli Irving-Leßmann, a charming professional economist, whom a benevolent fate sent from Australia to Mannheim. With great patience and care she read the manuscript in various stages of the translation process, corrected my mistakes and polished my style. I gratefully acknowledge her assistance.

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I have to record my intellectual debt to all those who gave their comments on the German version and on related papers. I should like to mention Elmar Helten, Peter Howitt, Heinz König, Wilhelm Krelle, Hans Schneeweiß, Ann Schwarz-Miller and, in particular, Hans Heinrich Nachtkamp, who has been a permanent source of encouragement and inspiration.

Finally, I owe much gratitude to my wife Gerlinde who not only bore more than her fair share of our family commitments while the book was being written, but also, as a fellow economist, gave unsparingly of her professional help. The English edition is dedicated to our children, who, albeit unwittingly, also bore some of the burden imposed.

Mannheim, November 1982 — Hans-Werner Sinn

Introduction

The Fundamental Issues Involved

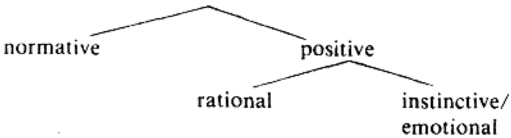
Why do we need a theory of uncertainty? It is a fact that almost all man's economic decisions are made under conditions of uncertainty, but this fact alone does not provide a strong enough argument for making the effort necessary to generalize ordinary preference theory designed for a world of perfect certainty. In accordance with *Occam's Razor*, the mathematician may well welcome a generalization of assumptions even if it does not promise more than a restatement of known results. The economist, however, will only be well disposed towards making the effort if he can expect to achieve new insights and interesting results, for he is interested in the techniques necessary for the generalization only as means to an end, not as ends in themselves. A stronger reason for developing a theory of uncertainty, therefore, seems to be the fact that there are kinds of economic activities to which the non-stochastic preference theory has no access or has access only through highly artificial constructions. Such activities include portfolio decisions of wealth holders, speculation, and insurance. These will be considered in detail in the last chapter of the book.

The main purpose of this book, however, is not to apply a theory of uncertainty to concrete economic problems, the purpose rather is to formulate such a theory. This may sound pretentious, especially in the light of the effort that has been involved in constructing the current theoretical edifice, starting from Cramer and Bernoulli and proceeding by way of von Neumann and Morgenstern. However, the aim of this book is comparatively modest. It is not to replace the current theoretical edifice by a new one but to examine it critically and to carry out some modifications, renovations, and extensions, which may improve its efficacy. Examples of such modifications are

- a rehabilitation of the Principle of Insufficient Reason (ch. I B 3),
- an improved argument for approximating nearly arbitrary von

Neumann-Morgenstern utility functions by indifference curves in a (μ, σ) diagram (ch. II D 2.2),
- the development of a preference hypothesis based on the psycho-physical law (ch. III A),
- the introduction of the 'blood out of a stone' rule (ch. III B),
- the use of a specific risk preference function in multiperiod planning problems (ch. IV B 2).

Although the theory of uncertainty presented here did not necessarily have to be limited to a specific range of economic activities, it was necessary to make a fundamental decision about its methodological direction. The following figure shows the alternatives that are available in principle.



The first decision to be made is whether we want to explain how people behave (positive analysis), or to give them advice about 'right' behavior (normative analysis). If a decision is made in favor of positive analysis, the next question concerns the model of man to be used. Does man, after careful consideration, choose the best from a set of alternatives available to him (rational behavior), or is his behavior simply an automatic response to external stimuli (instinctive or emotional behavior)? The choice between these two models of man that MARCHAL (1949, p. 129) once appropriately characterized as *homme de Descartes* and *homme de Pavlov* remains a problem, even if attention is limited to the range of economic activities. The truth most probably lies between these extremes: man is, as ALBERT (1978, p. 65) says, a 'fehlbares theoretisches Tier' (fallible theoretical animal). By far the majority of the founders of economic theories solved the two-part methodological decision in a way that was designed to kill two birds with one stone. They assumed rationality, however defined, and saw their theories as tools which could be used both to identify wise decisions and to describe the actual behavior of man. In the present study the author has also adopted this solution.

However, doubts about this decision remain. They are concerned not so much with the usefulness of the theory presented here for normative analysis but with its usefulness for positive analysis.

Basically, the procedure of the economic theory of uncertainty is to derive concrete rules of behavior from a few plausible axioms by way of a rather complicated process of logical deduction. Suppose everyone accepts these axioms as devices for rational behavior. Does it necessarily follow that their implications will also be accepted? The fact that this question cannot be self-evidently answered in the affirmative is the reason for the above-mentioned doubts. Certainly KNIGHT (1921, p. 236) was right when he stated that the evolution of man is a development towards more rationality; rationality in fact, has turned out to be his evolutionary advantage compared to other species. But has man yet reached that high standard necessary for an unhesitatingly affirmative answer to be given? Probably not. The absurd implications of assuming that he has reached such a standard are graphically illustrated by SAVAGE (1954, p. 20): '... if anyone who believed the axioms of mathematics also believed all that they imply and nothing that they contradict, mathematical study would be superfluous for him; such a person would ... be able to state the ten-thousandth or any other term in the decimal expansion of π on demand.' Thus, from the point of view of positive theory, the usefulness of the axiomatic method in preference theory definitely has limits originating in the imperfect analytical ability, intelligence, or rationality of the economic actors considered.

This recognition, however, should not be interpreted as leaving no space whatsoever in an explanatory theory for the axiomatic method. The fact that economic decision makers *want* to behave rationally, gives rise to the hope that the axiomatic approach, although imperfect, will result in an approximately correct description of behavior, much like MARSCHAK's (1950, p. 111) saying that men 'cannot be "all fools all the time"'.

Contrary to the way it may seem at first glance, the decision for the *homme de Descartes* (alias *homo oeconomicus*) and against the *homme de Pavlov* is not a decision against psychology. This cannot be the case since, *ex definitione*, a preference theory is a psychological theory. But even where professional psychology is concerned, nothing would be further from the author's thoughts than an attempt to dissociate himself from its findings. The time when ROBBINS (1935, pp. 83-90) could banish psychology to the outskirts of economics as being 'the happy hunting-ground of minds averse to the effort of exact thought' and when he could state, with sympathetic indulgence, that even great economists like Gossen, Jevons, and Edgeworth were led astray by psychology are long since past. Instead, an attempt should be made to integrate the results of professional psychology into the model of the rationally calculating decision maker. Doing this should make it possible to substantially reduce the large degree of tolerance necessarily left over by using rationality axioms and thus make it possible to achieve more specific conclusions about human behavior, i.e., conclusions with a higher informational content. As mentioned above, this book makes an attempt in this direction by introducing the psychophysical law into the theory of economic decision making under uncertainty.

The structure of the book is as follows. The first chapter provides the basis for economic decision making under uncertainty. It is primarily concerned with the question of whether it makes sense to transform vague ideas about the possible outcomes of an economic decision into an objective probability distribution of such outcomes. Following that, the second chapter addresses the problem of a rational evaluation of objective probability distributions. Various decision criteria proposed in the literature are compared and assessed with respect to their usefulness. The framework for economic decisions produced in the first two chapters is rather general. Therefore, chapter three is devoted to the task of filling out this framework with two supplementary hypotheses about the structure of human preferences. The fourth chapter deals with special problems arising if, either simultaneously or sequentially, decisions have to be made about multiple risks. Finally, in chapter five, the general decision theory previously developed is applied to various economic problems.

Contents

Chapter Five

Chapter One
The Object of Choice under Uncertainty

Section A
The Basic Decision-Theoretic Approach

1. *The Ordering of Alternatives*

It is the task of preference theory to indicate general criteria by which men choose, or should choose given their preferences, from a set of mutually exclusive action alternatives $(a_1, a_2, \ldots, a_m)$.

The economic approach[1] to a solution of this task is to search for an evaluation function $R(.)$, attaching to each of the action results $(e_1, e_2, \ldots, e_m)$ a real number with the property[2,3]

$$(1) \qquad R(e_i)\left\{\begin{smallmatrix} > \\ = \\ < \end{smallmatrix}\right\}R(e_j) \Leftrightarrow e_i\left\{\begin{smallmatrix} > \\ \sim \\ < \end{smallmatrix}\right\}e_j$$

such that the optimal alternative can in principle be found by selecting the highest number:

$$(2) \qquad \max_{a_i} R(e_i).$$

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[1] It was initially developed by PARETO (1906, p. 176). Cf. also the 'reconsideration' by HICKS and ALLEN (1934).

[2] In this book the symbols $>$, $<$, $\sim$, and $\Leftrightarrow$ mean respectively 'is better than', 'is worse than', 'is equally good as', and 'if, and only if'. The curved brackets indicate that those enclosed symbols which are at the same height belong together; to read them crosswise is not permitted.

[3] For all decision problems studied in this book, the function $R(.)$ is taken as given. Since, however, in the intertemporal part IV B, e_i is a time path of result variables, the possibility of changes in preferences *derived* for actual decisions is incorporated; indeed we shall find a very characteristic time dependence of derived preferences. Not all decision-theory approaches imply a given function $R(.)$. The Minimax-Regret Principle (cf. fn. 26 in section B) of NIEHANS (1948) and SAVAGE (1951), for example, has the property that the size of the opportunity set affects $R(.)$ an aspect that MILNOR (1954) was right in criticizing. (My present preference is apple $>$ pear $>$ sandwich. If my choice is between an apple and a pear, I take the apple. If it is between an apple, a pear, and a sandwich I still take the apple.)

The approach does not provide an answer to the question of which action to choose if there is no unique maximum, that is, if the highest value of $R(.)$ can be achieved in more than one way. In this case the decision can be made by plucking the petals of a daisy. The function $R(.)$ may be called a utility function, but it must be clearly recognized that utility in this context is only defined up to a strictly monotonic transformation of a subjective degree of satisfaction, which means that it is ordinal utility.

Of course, it is a necessary condition of this approach that the opportunity set contains only those alternatives which, by virtue of a preference ordering, can be given a unique value. We ensure this condition by the fundamental

Axiom of Ordering: *The decision maker has a complete weak ordering of all attainable action results.*

It primarily indicates[4]
- that, comparing two arbitrary achievable results, the decision maker is able to make the assessment 'not worse than' ($\gtrsim$) and
- that $e_i \gtrsim e_j$ and $e_j \gtrsim e_k$ imply $e_i \gtrsim e_k$ (transitivity).

It is easy to see that the Axiom of Ordering implies the existence of the preference function $R(e_i)$ although nothing more than weak ordering is required. Obviously

$$\text{(3)} \qquad \begin{aligned} R(e_i) > R(e_j) &\Leftrightarrow (e_i \gtrsim e_j \quad \text{and not} \quad e_j \gtrsim e_i), \\ R(e_i) = R(e_j) &\Leftrightarrow (e_i \gtrsim e_j \quad \text{and} \quad e_j \gtrsim e_i). \end{aligned}$$

Instead of a weak ordering of preferences being assumed, a strong ordering could have been postulated at the outset. Probably, however, the decision maker finds it easier to make an assessment of 'not worse

[4] A more complete list of the implications of this axiom can be given as follows. Let X be the Cartesian product attained by multiplying the set of all possible results by itself. Let Y denote the set of all pairs of results the decision maker is able to order by virtue of the relation $\gtrsim$, and let Y' be the converse of Y. Then we have:

$$\begin{aligned} & X \subsetneq Y \cup Y' && \text{(completeness),} \\ \text{in general it is not true that } & \forall e_i, e_j : e_i \gtrsim e_j \Rightarrow e_j \gtrsim e_i && \text{(non-symmetry),} \\ & e_i \gtrsim e_j \text{ and } e_j \gtrsim e_k \Rightarrow e_i \gtrsim e_k && \text{(transitivity).} \end{aligned}$$

An implication of completeness is the reflexivity of the relation $\gtrsim$, that is, $e_i \gtrsim e_i$. The set of ordered pairs for which the (reflexive, symmetrical, transitive) equivalence relation $\sim$ holds is $Y \cap Y'$. If $\bar{Y}'$ denotes the complement of Y' in Y then the set of all ordered pairs of results for which the strong preference relation $>$ is valid is $Y \cap Y'$. Since the relation $>$ is irreflexive it is also non-symmetric. Cf., e.g., NACHTKAMP (1969, pp. 66-81) and FISHBURN (1970, pp. 9-15).

than' than 'equally good as' or 'better than'. Moreover, from a behavioristic point of view, the above formulation has the advantage of only utilizing conclusions that can be drawn from observing what people actually choose. By observing a decision it is possible to conclude that the chosen alternative is not worse than the alternatives not chosen, but it is impossible to find out whether the decision maker liked it as much as or more than the others. Some alternative also has to be chosen in the case of indifference[5].

The Axiom of Ordering might appear innocuous and self-evident. However, from both the positive and the normative points of view, it is an idealization. Certainly no one is able to construct a completely consistent ordering of *all* the alternatives available to him in real life[6]. And even if it were possible, people might prefer to do something else from time to time rather than continually investigate preferences and order alternatives. From this, it is evident that a shortcoming of the Axiom of Ordering is its neglect of the *effort of ordering*.

In practical life the effort of ordering implies that the preference function $R(.)$ has a stochastic element[7], so that intransitivities are observable when the action results deviate only a little from one another. This can be explained as follows. Assume the decision maker has some prior information on the function $R(.)$ without knowing its exact value for the various alternatives. Then it is certainly possible that, when comparing e_i with e_j and e_j with e_k, he decides in both cases that one alternative is not worse than the other simply because the advantage he expects to gain from finding the better alternative is not worth the effort of ordering. Although $e_i \sim e_j$ and $e_j \sim e_k$ in this case, we must not conclude that $e_i \sim e_k$ as we could under transitive preferences. The reason is obvious. If the decision maker faces the task of making a decision between e_i and e_k, he is concerned about the advantage to be gained from knowing the true preference ordering between *these* two alternatives. This advantage may exceed that to be gained from knowing the ordering between e_i and e_j as well as that to be gained from knowing the ordering between e_j and e_k. Hence it may induce the decision maker to

[5] Thus LITTLE (1950, pp. 14–52) postulated that preference theory deal with acts rather than results. But it surely should be possible to gain insight into the state of mind of a decision maker by asking him about it.

[6] AUMANN (1962) therefore has tried to formulate a preference theory without the requirement of completeness.

[7] The first of the economists to consider stochastic preferences was GEORGESCU-ROEGEN (1936). In psychology, however, stochastic sensation functions have been discussed since the famous article of THURSTONE (1927).

calculate properly the ordering between e_i and e_k. The result of this calculation is very likely to be $e_i \nsim e_k$[8].

It is certainly desirable to develop an economic preference theory in which the precision of ordering itself is subject to an optimization process. But unfortunately such a theory is not available and cannot be offered here either[9].

2. *Action Results under Uncertainty*

In a world of certainty, the rule $\max_{a_i} R(e_i)$ for finding an optimal action can easily be interpreted. Here e_i is a particular result known with certainty. Its evaluation by use of the function $R(.)$ should not create fundamental problems. In the theory of the household, the result may be a bundle of consumption goods. In the theory of the firm, e_i can often be identified with the level of profit and hence the rule reduces to the well-known aim of profit maximization.

What, however, is the result of an action under uncertainty? Think of an entrepreneur who, despite uncertainty about the future revenue, has to choose one from a set of mutually exclusive investment projects. Could the results we are speaking of be the profit observable *ex post*? This would not make much sense for the decision about the investment project has to be made before knowing how profitable it will be. The basis for a decision, therefore, can only be a result visualized *ex ante*. Such a result has an element of vagueness in it; it can only be represented as a 'random vector' of possible '*ex post* results' or 'subresults':

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[8] Cf. SCHNEEWEISS (1967a, pp. 35 f. and 81–84) and KRELLE (1957, p. 637; 1961, pp. 112–116; 1968, pp. 21–24). These authors discuss the problem of calculation costs and the possibility of intransitivities being caused by sensation thresholds. The above reasoning unites both aspects since it explains sensation thresholds through calculation costs. For a theoretical explanation of specious intransitivities in terms of automatons see RÖDDING and NACHTKAMP (1978, 1980).

[9] The postulate should not be confused with the aim of the aspiration-level theory, as formulated by SIMON (1957, pp. 241–260), SIEGEL (1957), SAUERMANN and SELTEN (1962), STARBUCK (1963a and b), and others, which includes in the optimization problem the process of information gathering undertaken in order to find the opportunity set. Contrary to first impressions, this theory does not contradict the Axiom of Ordering. This becomes clear if the various possibilities for information gathering are considered as additional actions within the opportunity set. The inclusion of information gathering creates a sequential decision problem, but at each point in time there is a given opportunity set of alternatives, one of which has to be chosen. This is completely in accordance with the Axiom of Ordering. To interpret this choice as if the decision maker were merely trying to achieve an aspiration level below the 'true' optimum is a little bit misleading.

(4) $$e_i = (e_{i1}, e_{i2}, \ldots, e_{in}).$$

For this reason TINTNER (1941, p. 301) has called the evaluation function $R(.)$ 'preference functional'. A concrete example of such a random vector is a lottery ticket.

In order to find out what the result vector e_i may be, the decision maker has to take into account the fact that the single *ex post* result depends not only on his own actions, but also on various environmental influences that he can neither manipulate nor perfectly foresee[10]. For the purpose of elucidation, the decision problem may therefore be represented in the form of a case study that can easily be carried out with the aid of the following 'decision' or 'result matrix' originating from VON NEUMANN and MORGENSTERN (1947).

Table 1

action \ class of states of the world	Z_1	…	Z_j	…	Z_n
a_1	e_{11}	…	e_{1j}	…	e_{1n}
⋮	⋮		⋮		⋮
a_i	e_{i1}	…	e_{ij}	…	e_{in}
⋮	⋮		⋮		⋮
a_m	e_{m1}	…	e_{mj}	…	e_{mn}

Here the symbols $(Z_1, \ldots, Z_n)$ denote mutually exclusive classes of states of the world that the decision maker wants to distinguish[11]. The decision maker knows that if he chooses action a_i and the environment dictates class Z_j the subresult e_{ij} will obtain. However, he does not know into which class the true state of the world will fall; this is the particular aspect of the decision problem that emerges under uncertainty.

The matter becomes more complicated if the problem of time is taken into account. In a non-random world, time does not change the nature of the decision problem very much. Action a_i describes a time path of the decision maker's activity that is uniquely associated with a time path of results. Once the optimal activity path is chosen in advance, the individual will stick to it without making new decisions. Things are

[10] Cf. VON NEUMANN and MORGENSTERN (1947, pp. 10 f.).

[11] Note that we have to consider classes of states of the world rather than completely described states themselves. The decision maker will classify the states of the world according to those criteria he is interested in, but not, of course, according to *all* criteria. This distinction is of some importance for the discussion of Bayes's Theorem which occurs below.