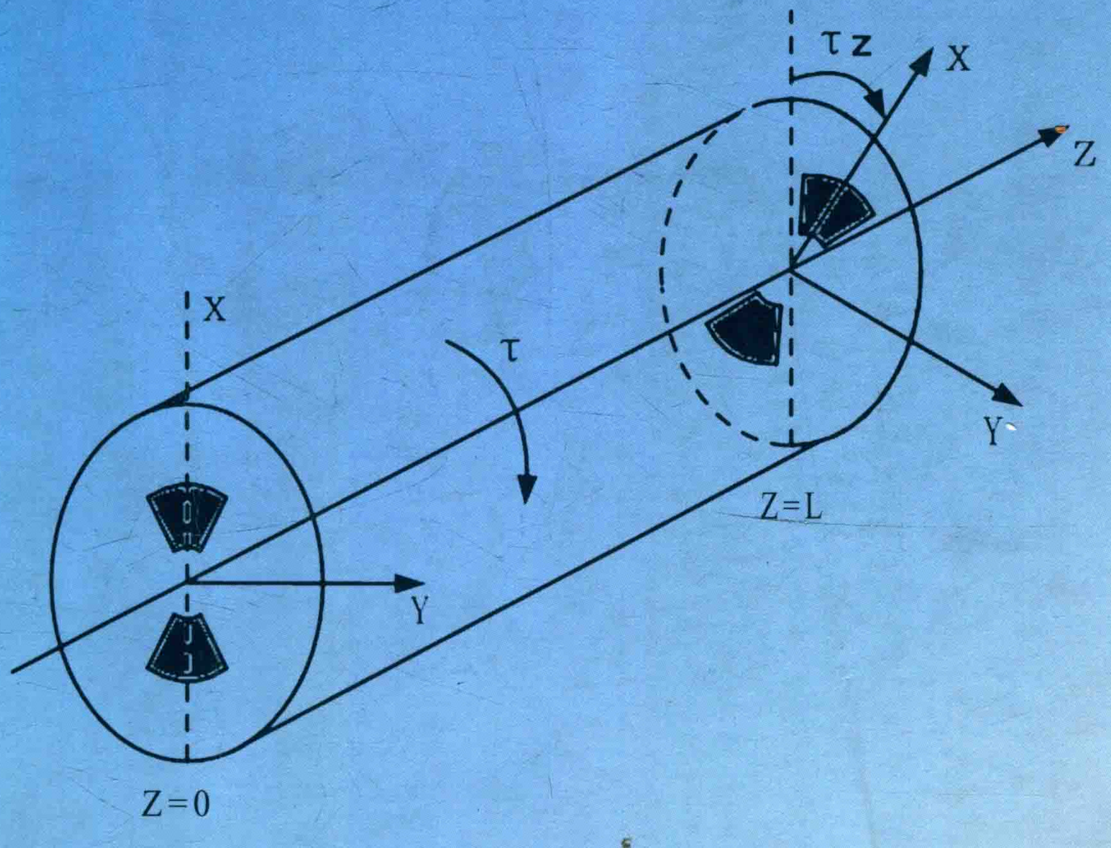
COLLECTED PAPERS OF COUPLED MODE THEORY IN OPTICAL FIBERS AND MICROWAVES



光纤和微波技术中耦合模理论论文集

钱景仁 著

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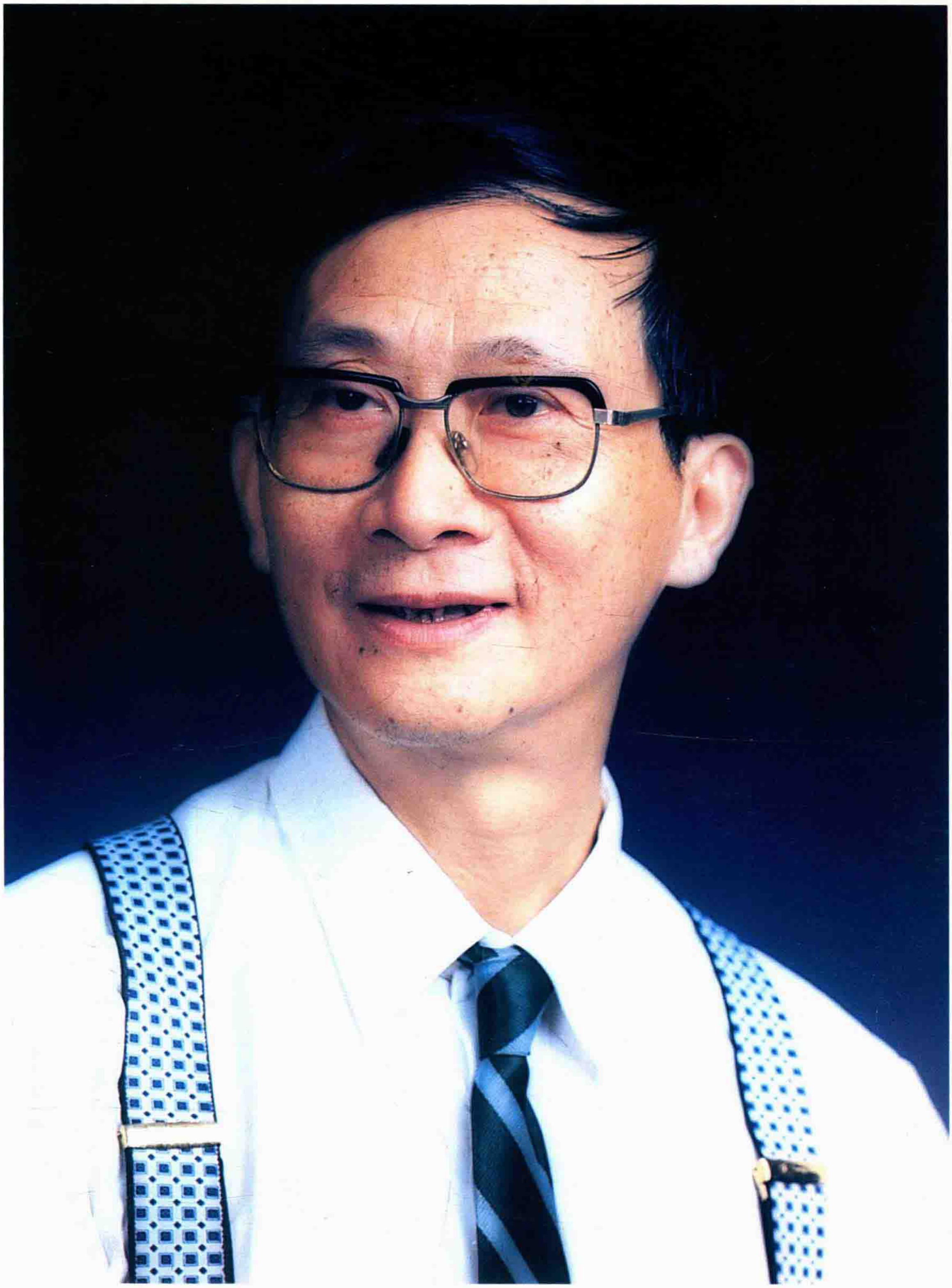

作者近影

自 序

本论文集收集了作者四十年来科研工作期间在公开刊物上所发表论文中的30篇代表作，其中不少是和科研工作的同事合作的。

由于作者早年从事微波理论和技术的科研工作，上世纪80年代以后又从事光纤技术方面的工作，因此收集的论文包括了这两个研究方向的内容。研究的对象虽然不同，但大部分的文章都贯穿着一条主线，即耦合模理论及其在光纤与微波技术中的应用。

耦合模理论是研究宇宙间两个或多个波动(或振动)间互相耦合、互相作用的客观规律。在上世纪40到50年代，用它成功地分析了微波管和金属波导中的各种模式耦合问题，特别是圆金属波导中TE_{01}波传输时的模式耦合问题。本论文集中前面的几篇有这一实用背景，从分析方法上讲仍有普遍意义和参考价值。耦合模理论在光纤和光波导传输问题上得到了比金属波导中更为广泛的应用，并且在这个应用过程中，又发展和完善了耦合模理论本身。文集中有十几篇文章，分别研究了光纤中的扭转、拉锥、螺旋形变、光栅和外加磁场等作用下，单模光纤中的耦合模方程。这些文章有些虽已发表十多年，但仍有现实意义。特别要提到的是高线性双折射扭转光纤，它是作者在英国南安普敦大学光电子研究中心访问研究期间提出来的，1986年获英国专利。这种光纤采用耦合模理论来分析，成功地得到了优化设计参数，并为国际同行普遍认可。作者回国后，在南安普敦大学的英国同事应用这种光纤研制成光纤电流传感器。由于这一成果，以作者为首的四人获得英国IEE颁发的1991年测量奖。(该奖每年仅有一个，奖励在电气测量领域方面有重要贡献者，作者是获此奖的第一个从大陆去的访问学者)。在目前我国电力能源发展极为迅速的时期，高压电流互感器急需降低成本和简化装置，这一特种光纤最近又重新引起了人们的关注。

耦合模理论是分析光纤光栅最有效的方法之一，本文集包括了两篇作者的近期论文。

文集中还包括两篇研究电磁场小孔耦合的论文，这是作者当时的得意之作。特别是在1975年发表的有关小孔耦合能量守恒问题的文章，是在“文化革命”

后期，空闲时间多，经过仔细推敲才写成。国外解决这个问题一直到1982年，R. E. Collin才发表类似的文章。

文集的最后几篇是近2～3年内和研究生合作的论文，也反映了作者近期的工作。

这一文集在一定程度上也反映了作者这四十多年的学术生涯是并非一帆风顺的。早年，在叶培大院士电磁场理论的启蒙教育下，又在黄宏嘉院士直接指导下，虽受到“极左”路线的影响，但总的来说，进步较快，因此1965年前的论文较多，收集在册的也有好几篇。在1966到1974年“文化革命”期间，研究工作严重受阻，没有写任何学术文章。改革开放后，迎来了科学的“春天”。中国科学技术大学又派遣作者到英国南安普敦大学访问研究，回国后又主持多个科研项目，本文集收集的论文以这一时期的较多。由于这一时期成绩显著，作者与徐善驾教授共获1993年中国科学院自然科学成果一等奖。如果没有后来的“从政”，只要再加一把劲，在学术上就可能再上一个层次。1993年，组织上安排兼任民主党派省里负责人和省政协副主席，各种会议和党派领导事务花费了作者主要的精力，这致使作者在1993年到1998年间有份量的论文显著减少。1998年“从政”下来后，在学术上逐渐恢复了元气，文章发表也多了，但要开辟一个新的局面，年龄和条件已不允许了，到底已经是接近古稀之年了，心有余而力不足。

作者要感谢合肥正阳光电公司董事长胡浩对本论文集出版的积极建议和全力支持，同时还要感谢作者所在课题组的同仁朱冰博士、杨利博士、罗家童博士、梁明博士和刘国祥博士等为该论文集的出版费心出力，在此一并表示感谢。最后，感谢亡妻王淳同志对作者前四十年工作的支持，她承揽了养育下一代和家务的主要工作，使作者得以专心于业务，在此表示深切的怀念。也要感谢现在的老伴王厚华同志，她的细心照料，使作者在这古稀之年“老有所为”，能多为国家做一点有益的工作。

钱景仁

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PHYSICS

THE CONCEPT OF IMPEDANCE-PERTURBATION AS APPLIED TO IMPERFECT WAVEGUIDES*

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ABSTRACT

In this paper, imperfect waveguides are analysed on the basis of the concept of impedance-perturbation introduced in a previous paper[1]. For a long-distance waveguide, the statistics of various waveguide imperfections are treated consistently by employing this unique concept. Maximum allowable mean-squared value of the impedance-perturbation is calculated for a specified added loss caused by random waveguide irregularities of an arbitrary form.

For the dielectric-coated waveguide and the helix, formulas are derived for computing the impedance-perturbations due to different forms of waveguide irregularities. With these, the calculated value of the maximum allowable impedance-perturbation sets the limits for various constructional defects appearing most frequently in waveguide practice.

As a preliminary study of the above subject, normal modes of a nonconventional waveguide with a pair of arbitrary wall impedances are investigated. An impedance parameter in terms of both of the anisotropic impedances is introduced, and the analysis is featured by its extensive use of the perturbation algebra in the derivation of approximate formulas, which are sufficiently accurate for many cases of practical interest.

I. INTRODUCTION

The problem of imperfect waveguides has long been a subject of extensive study. Among the various types of waveguides that have been devised, the dielectric-coated (or lined) waveguide and the helix stand out most promisingly in the development of long-distance transmission; so, naturally, in the majority of the published theoretical work, interest has been focused on these two waveguide types.

.The earliest work on dielectric-coated waveguide may be traced back to a paper by Buchholz[2]. In the last few years, the subject was taken up further in the papers by Малин[3] and Unger[4], yielding more practical results. In a paper by Каценеленбаум[5], the work was extended to the case of a dielectric-coated waveguide with small wall deformation. More recently, Janseen[6], Kreipe, and Unger[7] discussed the tolerance problem of the said waveguide type.

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Wave propagation along a helix structure has been a subject of theoretical and practical interest. The study of slow-wave helix structure dates back to the end of the last century. The work done in this field before 1954 was briefly reviewed in a paper by Senisper[8]. For the fast-wave helix structure——the real waveguide in the strict sense——it is only in recent years that the subject began to draw our attention. The work on fast-wave helix waveguide was first published by Morgan and Young[9], who used in their analysis the idealized boundary conditions originally introduced by Владимир and Pierce. In contrast to the method adopted by Morgan and Young, which is more mathematical, so to say, Каценеленбаум[5] adopted a more physical approach (the method of power flow), taking into account more constructional features of the helix, such as the wire form and spacing. Besides the above-quoted work, Karbowick[10], Piefke[11], Сивов[12], Исаенко, Малин and Малина[13], and some others have also made their contributions to the development of helix waveguide theories. In the literature, circular cylindrical coordinates have been universally employed for the study of helix waveguide, excepting one paper by Koo[14], who used the helical coordinates. Indeed his result is more exact mathematically, but nevertheless, the theory is not so practical for an immediate application to the actual complicated helix structure used in waveguide transmission. Recently, a more extensive investigation of the helix waveguide was carried out by Unger, who in his series of papers[4] used miscellaneous methods in treating waveguide irregularities of various kinds. In the meantime, a paper by Noda[15] on hybrid-mode transmission appeared, in which both the helix and the dielectric-coated waveguide were considered.

Although much work has been done in the development of the theories of imperfect waveguides, the exploitation of this broad field is yet far from exhaustive. In the previous work, different types of imperfections have been studied separately. Thus, while many types of waveguide irregularities have been tackled with success, others remain unsolved; and, by the usual approach to the problem, each type of waveguide irregularity will require a new solution of the very complicated boundary-value problem.

The purpose of this paper is to present a unified and consistent approach to the theory of imperfect waveguides. More specifically, not only are the perfect waveguide models described by a pair of surface impedances, but the waveguide imperfections of an arbitrary form are treated by utilizing the unique concept of impedance-perturbation[1]. By this approach, a single figure, namely, the maximum allowable mean-squared value of the impedance-perturbation for a given added average loss of the propagating mode, will be sufficient for a general specification of the tolerances of all waveguide types. In this manner, as was pointed out previously in the quoted paper, one is relieved of the tedious task of performing a new expansion each time when treating a new type of waveguide imperfection of any variety, and what is left in solving any specific waveguide problem is reduced to deriving a pair of surface impedances describing the nominal wall structure, and to deriving the relevant impedance-perturbations describing the waveguide irregularities.

In the following sections, normal mode solutions for nonconventional waveguides are first studied, and the impedance-perturbation formulas are derived for the above-mentioned two specific waveguide types. The maximum allowable impedance-perturbation is then calculated with a particular choice of the surface impedances. On the basis of this calculation, the tolerance problem for typical forms of waveguide imperfections is considered.

II. Transmission Characteristics of Nonsymmetrical Normal Modes in a Perfect Nonconventional Waveguide with a Pair of Arbitrary Impedances

As a preliminary study of imperfect waveguides, let us consider the normal modes of a nonconventional waveguide whose transmission property is completely described by a pair of wall impedances, i.e., the circumferential impedance $Z_{\varphi k}$ and the axial impedance Z_{zk}, k denoting a particular mode. The characteristic equation involving both of these impedances has been independently derived by Unger[4] and by Huang[1]. For nonsymmetrical modes, this characteristic equation can be conveniently put into the following form[1)]:

$$\frac{\kappa^2}{\chi_k^2}\left[\frac{J'_m(\chi_k a)}{J_m(\chi_k a)}\right]^2 + j\frac{\kappa}{\chi_k}\frac{J'_m(\chi_k a)}{J_m(\chi_k a)}\mathcal{Z}^{-1} + \left(\frac{m\gamma_k}{\chi_k^2 a}\right)^2 = 0, \qquad (2.1)$$

where a is the radius of the waveguide; m, the circumferential index; $\chi_k^2 = \kappa^2 + \gamma_k^2$ (χ_k is the separation constant and κ, γ_k are the free space and mode propagation constants respectively); and $\mathcal{Z}$, an impedance parameter defined by

$$\mathcal{Z} = \left(\frac{Z_{\varphi k}}{Z_0} + \frac{Z_0}{Z_{zk}}\right)^{-1}, \qquad (2.2)$$

where $Z_0 = \sqrt{\mu_0/\epsilon_0}$.

Approximate solution of Eq. (2.1) can be derived by utilizing the perturbation algebra, in case $\mathcal{Z}$ is very small or very large. The numerical results thus obtained are in good agreement with those obtained with an electronic digital computer.

1. *The Small $\mathcal{Z}$ Case*

When both $Z_{\varphi k}/Z_0$ and Z_{zk}/Z_0 are small, the impedance parameter $\mathcal{Z}$ is small. Since in a nonconventional waveguide the E_{mn}-limit modes can be

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1) The term $Z_{\varphi k}/Z_{zk}$ in the original equation has been omitted, under the restriction of $Z_{\varphi k}/Z_{zk} \ll (m\kappa a/\chi_k^2 a^2)^2$, which is the usual case in practice.

considered as perturbed E_{mn}-modes of an all-metallic waveguide when Z_{zk}/Z_0 is small, and the H_{mn}-limit modes, as perturbed H_{mn}-modes when both $Z_{\varphi k}/Z_0$ and Z_{zk}/Z_0 are small, it is possible to put $\chi_k a = p_{mn} + \Delta\chi a$, where p_{mn} for E_{mn}- and H_{mn}-modes are respectively the n-th roots of $J_m(x) = 0$ and $J'_m(x) = 0$. Substitution of this expression for $\chi_k a$ in Eq. (2.1) yields

$$\Delta\chi a = \frac{jp^3_{mn}[1 \pm \sqrt{1 + (2m\gamma_k a\mathfrak{Z}/\chi^2_k a^2)^2}]}{2\kappa a(p^2_{mn} - m^2)\mathfrak{Z} - jp^2_{mn}[1 \pm \sqrt{1 + (2m\gamma_k a\mathfrak{Z}/\chi^2_k a^2)^2}]} \qquad (2.3)$$

for perturbed H_{mn}-modes, and

$$\Delta\chi a = \frac{2\kappa a p_{mn}}{2\kappa a - j\mathfrak{Z}^{-1}p^2_{mn}[1 \pm \sqrt{1 + (2m\gamma_k a\mathfrak{Z}/\chi^2_k a^2)^2}]} \qquad (2.4)$$

for perturbed E_{mn}-modes, with χ_k and γ_k both taken as the unperturbed values. In Eqs. (2.3) and (2.4), $+$ or $-$ of the double-sign should be chosen according as which makes $\Delta\chi$ the smaller.

2. *The Large $\mathfrak{Z}$ Case*

By the large $\mathfrak{Z}$ case, it is meant that $Z_{\varphi k}/Z_0$ is again small, but Z_{zk}/Z_0 is large. In this case, solutions of Eq. (2.1) can be considered as perturbed modes of an "ideal helix" with $Z_{\varphi k} = 0$ and $Z_{zk} = \infty$ (i.e., $\mathfrak{Z} = \infty$). Then, it becomes legitimate to put $\chi_k a = \theta_{mn} + \Delta\chi a$ (θ_{mn} is the product of the guide radius and the cut-off wave number for either H_{mn}- or E_{mn}-modes, when the axial impedance, starting from zero, becomes infinity along a capacitive branch in the mapping of γ as a function of Z_z). When this expression is substituted for $\chi_k a$ in Eq. (2.1), we obtain

$$\Delta\chi a = \frac{1}{\dfrac{2}{\theta_{mn}} \pm j\dfrac{\theta_{mn}}{m}\dfrac{\kappa a}{\gamma_k a} - j\dfrac{2\kappa a}{\theta_{mn}}\mathfrak{Z}}, \qquad (2.5)$$

in which the $+$ and $-$ signs are for the perturbed E_{mn}- and H_{mn}-modes respectively.

To facilitate numerical computation of Eq. (2.5), the authors present a short table of the θ_{mn}-values (Table 1).

Now it is worthy of note that to say a given value of impedance is small or large is meaningful only in a relative sense. In fact, it depends on the value of κa and on the type of mode considered. On certain occasions, classification of the small $\mathfrak{Z}$ and the large $\mathfrak{Z}$ cases can not easily be ascertained before the mapping of the propagation constant as a function of the impedance parameter is somewhat exploded. In Section V.2, the above point will be made clearer by considering a practical example in the use of the above-derived perturbation formulas.

Table 1

A Short Table of Roots of Characteristic Equation (1.1) for the Large $\mathfrak{Z}$ Case. ($\kappa a = 23$)

(a) θ_{mn}–Values for the Electric Modes

$m \backslash n$	1	2	3	4
0	3.83171	7.01559	10.17347	13.32369
1	5.14048	8.42521	11.63964	
2	6.39222	9.77946	13.03980	
3	7.60982	11.09606		
4	8.80463	12.38520		
5	9.98306			
6	11.14919			
7	12.30601			

(b) θ_{mn}–Values for the Magnetic Modes

$m \backslash n$	1	2	3	4
1	2.40208	5.51486	8.64555	11.78038
2	3.82447	7.00233	10.15425	13.29849
3	5.12104	8.39332	11.58680	
4	6.35604	9.73414	12.96600	
5	7.55254	11.01246		
6	8.72180	12.26864		
7	9.87038			
8	11.00261			

III. Impedance-Perturbations Describing Certain Forms of Waveguide Irregularities for H_{0n}-Modes

For a long-distance waveguide, impedance-perturbation formulas for the H_{01}-mode will be needed the most. Certain formulas for the other modes are useful in case coupling coefficients involving the relevant modes caused by some forms of irregularities are required.

III.1. Nonuniform Irregularities of a Dielectric-Coated Waveguide

For the dielectric-coated waveguide, nonuniform irregularities are essentially of two types: the nonuniform wall deformation and the thickness irregularity.

1. *Wall Deformation*

Let $\rho = a + \delta$ be the radius of the deformed waveguide, a being the nominal radius, and δ, the small wall deformation. For the circular electric waves, the boundary condition at the nominal surface is

$$E_\varphi = j\omega\mu\delta H_z + Z_\varphi H_z, \tag{3.1}$$

Z_φ being the circumferential wall impedance, which may be taken to be zero for the all-metallic waveguide with a dielectric lining. Equation (3.1) may be written in the form of $E_\varphi/H_z = Z_\varphi + \Delta Z_{\varphi k}$, yielding the following simple impedance-perturbation formula for wall deformation:

$$\frac{\Delta Z_{\varphi k}}{Z_0} = j\kappa\delta. \tag{3.2}$$

2. *Irregular Thickness of the Dielectric Coat*

Let $\delta = \delta_0 + \Delta\delta$ be the thickness of the irregular dielectric layer, in which δ_0 and $\Delta\delta$ are respectively the nominal thickness and the deviation of the thickness from the nominal value. From the transmission-line equations, the impedance-perturbation with respect to the nominal surface is readily found to be

$$\Delta Z_{\varphi k} = j\frac{\omega\mu}{\chi_k}\frac{\left[\tan\chi_k^-\delta - \dfrac{\chi_k^-}{\chi_k}\tan\chi_k\Delta\delta\right]}{\dfrac{\chi_k^-}{\chi_k} + \tan\chi_k^-\delta\tan\chi_k\Delta\delta} - j\frac{\omega\mu}{\chi_k^-}\tan\chi_k^-\delta_0,$$

where the second term at the right side is the surface impedance for a uniform dielectric layer with thickness δ_0, and the first term, the impedance at the inner surface of the dielectric layer. $(\chi_k^-)^2 = \kappa^2\varepsilon_r + \gamma_k^2$.

Under the assumption $\chi_k\Delta\delta \ll 1$, the above equation can be simplified to the following form:

$$\frac{\Delta Z_{\varphi k}}{Z_0} \simeq j\left(\frac{\tan\chi_k^-\delta_0}{\chi_k^-}\right)^2\kappa^3\Delta\delta(\varepsilon_r - 1), \tag{3.3}$$

which may be further simplified to

$$\frac{\Delta Z_{\varphi k}}{Z_0} \simeq j\kappa^3\delta_0^2\Delta\delta(\varepsilon_r - 1), \tag{3.4}$$

when $\chi_k^-\delta_0 \ll 1$, and to

$$\frac{\Delta Z_{\varphi k}}{Z_0} \approx j\tan^2[\kappa(\varepsilon_r - 1)^{1/2}\delta_0]\kappa\Delta\delta, \tag{3.5}$$

when $\kappa a \gg 1$, so that $(\chi_k^-)^2 \approx \kappa^2(\varepsilon_r - 1)$. In all the above formulas, ε_r is the relative dielectric constant of the coating.

III.2. The Imperfect Helix Waveguide[1)]

1. *Deformation of the Helix Structure*

The first type of irregularity to be treated is the wire deformation of the helices. Usually, such irregularity form makes the largest contribution to the total impedance-perturbation. To derive the impedance-perturbation formulas for the H_{01}-modes, we again employ the approximate boundary condition (3.1), in which Z_φ stands for the nominal surface impedance of a perfect helix model. Since the term involving an arbitrary surface impedance is cancelled in the final result, the impedance-perturbation formula for a helix waveguide turns out to be of the same form as Eq. (3.2). It is obvious that these formulas are in fact applicable to wall deformation of other waveguide structures as well, irrespective of the values of the nominal wall impedance of the particular waveguide structure.

2. *Irregular Pitch Angle*

The rest of the irregularity forms to be studied are all peculiar to the helix structure. In order to estimate the contribution of irregular pitch angle ψ to the total impedance-perturbation, we assume that the pitch angle is very small (this, of course, is usually the case in practice), and the nominal circumferential impedance can be neglected. Then, with the boundary condition $E_\varphi^+ = - E_z^- \tan\psi$, we immediately have $\Delta Z_{\varphi k} = -\tan\psi \dfrac{E_{zk}^+}{H_{zk}^+}$, and thence[2)]

$$\frac{\Delta Z_{\varphi k}}{Z_0} = \frac{Z_{zk}^+}{Z_0}\psi^2, \quad (m=0) \tag{3.6}$$

in which $\psi = \psi_0 + \sum_m \theta_m \begin{smallmatrix}\sin\\ \cos\end{smallmatrix} m\varphi$, ψ_0 being the nominal pitch angle, and θ_m, the coefficients of a Fourier expansion.

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1) The meaning of some of the symbols used in describing a helix is explained in the Appendix at the end of this paper.

2) The axial and the circumferential impedance-perturbations for the nonsymmetrical modes can be derived in a similar manner as:

$$\frac{\Delta Z_{zk}}{Z_0} = \pm \frac{\gamma_k s_k \left(\frac{\chi_k}{\kappa}\right)^2 \Pi_k^*}{\frac{\partial \Pi_k}{\partial \rho} - \frac{s_k \gamma_k^2}{\kappa^2 a}\frac{\partial \Pi_k^*}{\partial \rho}} \cdot \frac{Z_{zk}}{Z_0}\psi, \tag{3.7}$$

$$\frac{\Delta Z_{\varphi k}}{Z_0} = \mp \frac{j\kappa\psi\Pi_k}{s_k\gamma_k\Pi_k^*} \quad m \neq 0, \tag{3.8}$$

where the upper sign on the right-hand side of both equations is to be used when k refers to a forward mode, and the lower sign is to be used when k refers to a backward mode.

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3. *General Irregular Winding*

For circular electric waves, impedance-perturbation due to nonuniformity of the helix winding, e.g., the nonuniformity of pitch and wire spacing, the nonuniformity of the cross-sectional form and size of the helix wire, etc., can be derived in a straightforward manner from Eq. (A.10b) in the Appendix. In general, the "filling coefficient" of the wiring is quite large (say, $q > 0.5$), and we have the following simple form for the circumferential impedance-perturbation:

$$\frac{\Delta Z_{\varphi k}}{Z_0} = \frac{j}{2}\kappa(\Delta l_2 + \Delta l_3), \quad m = 0, \tag{3.9}$$

where Δl_2 and Δl_3 are respectively deviations of the parameters l_2 and l_3 caused by the nonuniformity in periodicity and wire diameter, as well as by the small variation in wire form, of the helical structure. Figure 1 shows explicitly, for typical values of the nominal period ($p = 0.2$ mm) and nominal wire diameter ($\phi = 0.15$ mm), the value of ΔZ_φ *versus* the deviations in p and ϕ.

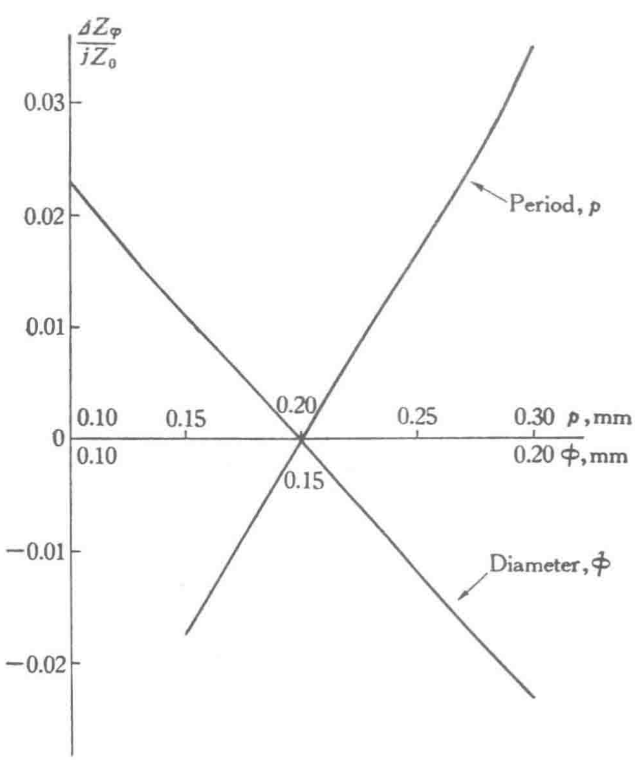

Fig. 1. ΔZ_φ *versus* diviations in p and ϕ (nominal values of p and ϕ are 0.2 mm and 0.15 mm respectively).

4. *Irregular Shielding*

Physical reasoning reveals that radiation through the wire spacing and the oblique helix wire are the two main sources responsible for the field outside the helix. Consequently, each of these two sources contributes its share to the circumferential impedance-perturbation resulting from irregularity of the shielding.

The former part of the impedance-perturbation can be derived from Eq. (A.10b):

$$\frac{\Delta Z_{\varphi k}}{Z_0}=j\frac{\kappa^3(l_3-l_2)^2}{4}\cdot\frac{(\sec\chi_k^- d_0)^2}{\left[\frac{\kappa}{\chi_k^-}\tan\chi_k^- d_0+\frac{\kappa(l_2+l_3)}{2}\right]^2}\sum_m d_m\,{}^{\sin}_{\cos}\,m\varphi, \quad (3.10)$$

where d_0 is the average distance from the shield to the helix wire, and d_m is the m-th component in the Fourier expansion of the variation in the distance from the average value.

For the latter part (due to oblique winding), substitution of Eq. (A.11) into Eq. (3.6), with d replaced by its perturbed value, yields

$$\frac{\Delta Z_{\varphi k}}{Z_0}=j\frac{(\chi_k^-\psi)^2}{\kappa\varepsilon_r}\cdot\frac{(\csc\chi_k^- d_0)^2}{\left[\operatorname{ctg}\chi_k^- d_0-\frac{1+\varepsilon_r}{\varepsilon_r}\chi_k^- l_1\right]^2}\sum_m d_m\,{}^{\sin}_{\cos}\,m\varphi, \quad (3.11)$$

the meaning of d_m being the same as in Eq. (3.10).

IV. Mode Couplings Due to Waveguide Imperfections

In this section we shall show how the coupling coefficients due to various forms of waveguide imperfections can be derived very simply from the set of impedance-perturbation formulas already obtained. From this, one will easily appreciate that the impedance-perturbation approach is featured not only by its consistency in the method of treatment, but also by its simplicity in the mathematical manipulation involved.

A purely physical reasoning would reveal that the couplings of modes must be proportional to the amount of impedance-perturbations concerned. In the previous paper[1], the coupling coefficients originating from an arbitrary impedance-perturbation have been derived. With reference to Section IV of the quoted paper (attention being called particularly to Eqs. 4.15, 4.17, and a foot-note on page 772), the coupled-wave equations together with the coupling coefficients can be put into the following form:

$$\frac{dA_k^\pm}{dz}=\mp\gamma_k A_k^\pm\mp\sum_k(\kappa_{ki}^+A_i^\pm+\kappa_{ki}^-A_i^\mp),$$

$$\kappa_{ki}^\pm=\frac{a}{2j}\frac{\sqrt{\gamma_i\gamma_k}}{\kappa^3}s_is_k\chi_i^2\chi_k^2\int_0^{2\pi}\frac{\Delta Z_{\varphi i}}{Z_0}\Pi_k^*\Pi_i^*d\varphi\,\pm$$

$$\pm\frac{ja}{2}\frac{\kappa}{\sqrt{\gamma_i\gamma_k}}\int_0^{2\pi}\frac{\Delta Z_{zi}}{Z_0}\left[\frac{\partial\Pi_k}{\partial\rho}-s_k\left(\frac{\gamma_k}{\kappa}\right)^2\frac{\partial\Pi_k^*}{a\partial\varphi}\right]\left[\frac{\partial\Pi_i}{\partial\rho}-s_i\left(\frac{\gamma_i}{\kappa}\right)^2\frac{\partial\Pi_i^*}{a\partial\rho}\right]d\varphi. \quad (4.1)$$

If the index i denotes a H_{0n}-mode, then Eq. (4.1) is reduced to

$$\kappa_{ki}^{+} = \kappa_{ki}^{-} = \frac{a}{2j}\frac{\sqrt{\gamma_i\gamma_k}}{\kappa^3} s_i s_k \chi_i^2\chi_k^2 \Lambda_i\Lambda_k J_0(\chi_i a) J_m(\chi_k a)\int_0^{2\pi}\frac{\Delta Z_{\varphi i}}{Z_0}\cos m\varphi\, d\varphi, \tag{4.2}$$

where + and − refer to forward and backward waves, $\Delta Z_{\varphi k}/Z_0$ and $\Delta Z_{zk}/Z_0$ are normalized circumferential and axial impedance-perturbations respectively.

The above form of the coupling coefficients is applicable to the general case when neither Z_φ nor Z_z selected for the reference waveguide is vanishing, and equally to the special case when $Z_\varphi = 0$, $Z_z \neq 0$. For the latter case, ΔZ_φ in the first term on the right-hand sides of Eqs. (4.1) and (4.2) should be replaced, of course, by the total circumferential impedance Z_φ of the actual waveguide.

Let us first consider the couplings due to wall deformation of a dielectric-coated all-metallic waveguide, or of a helix winding. For couplings involving the H_{0n}-modes, substituting Eq. (3.2) for $\Delta Z_{\varphi i}$ in Eq. (4.2), we obtain

$$\begin{aligned}\kappa_{ki}^{\pm} &= \frac{a}{2}\frac{\sqrt{\gamma_i\gamma_k}}{\kappa^2} s_i s_k \chi_i^2\chi_k^2 \Lambda_i\Lambda_k J_0(\chi_i a) J_m(\chi_k a)\int_0^{2\pi}\delta\cos m\varphi\, d\varphi,\\ &= j\frac{D_{ki}}{\pi}\kappa\int_0^{2\pi}\delta\cos m\varphi\, d\varphi,\end{aligned} \tag{4.3}$$

the latter form being more convenient for numerical calculation. For a description of the symbols involved in Eq. (4.3), the Appendix at the end of the paper may be consulted; D_{ki} is expressed by Eq. (5.5).

Now consider the nonuniform lining of a dielectric-coated waveguide. From Eqs. (3.3) and (4.2), the following formula is directly obtained:

$$\begin{aligned}\kappa_{ki}^{\pm} &= \frac{a}{2}\sqrt{\gamma_i\gamma_k}\chi_i^2\chi_k^2\Lambda_i\Lambda_k s_i s_k\left[\frac{\tan\chi_k^-\delta_0}{\chi_k^-}\right]^2(\varepsilon_r-1)J_0(\chi_i a)J_m(\chi_k a)\cdot\\ &\quad\cdot\int_0^{2\pi}\Delta\delta\cos m\varphi\, d\varphi,\\ &= j\left[\frac{\tan\chi_k^-\delta_0}{\chi_k^-}\right]^2\frac{\kappa^3}{\pi}(\varepsilon_r-1)D_{ki}\int_0^{2\pi}\Delta\delta\cos m\varphi\, d\varphi.\end{aligned} \tag{4.4}$$

For a helix waveguide, coupling coefficients originating from different sources of waveguide imperfections can be derived by putting Eq. (3.7) through Eq. (3.11) into Eqs. (4.1), (4.2) and performing the integration. The following is a summary of the results:

For nonuniform pitch angle, we have

$$\kappa_{ki}^{\pm} = \pm\frac{a}{2}\sqrt{\frac{\gamma_i}{\gamma_k}}\frac{\chi_i^2\chi_k^2}{\kappa^2}s_i\Lambda_i\Lambda_k J_0(\chi_i a)J_m(\chi_k a)\int_0^{2\pi}\psi\sin\varphi\, d\varphi, \tag{4.5}$$