刘彦佩选集

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第十编 目录

Embeddability in Graphs

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Preface

The concept of embeddability is motivated by the planarity that is one of the most important topics in graph theory and was firstly studied by K. Kuratowski and soon after by H. Whitney and S. MacLane in the thirties. Each of them founded his own distinguished theory which is still active in the areas related to graphs today. This is well known for at least combinatoricists. In this book, we would rather like to present another kind of theories which seem to be novel in literature.

In the fifties, Wu Wen-tsun (W. T. Wu), a Chinese mathematician, discovered a criterion for testing the planarity of graphs in view of cohomology theory in algebraic topology. On the other hand, more than ten years later, W. T. Tutte found another one in view of the theory of chain groups over the real field which was introduced by himself in the fifties as well. However, in the eighties, the present author simplified the two criteria into the same, which is called the Wu - Tutte theorem in §5.2, in view of the theory of spaces over the finite field GF(2) of two elements.

Since L. Auslander and S. V. Parter published the first paper dealing with the idea of an algorithm for testing the planarity of a graph by using computers at the beginning of sixties, there have appeared hundreds of papers in literature to discuss the classical topic in this way upto the first linear time algorithm of J. Hopcroft and R. Tarjan. Wu also constructed a procedure by transforming the problem into solving a kind of modulo 2 equation systems in the seventies. Soon after that, the author improved his results to reach at the point of each equation only having at most two modulo 2 variables. Then, he found new criteria to show that the planarity problem, in view of the computing complexity, is equivalent to extracting a spanning tree in the planarity auxiliary graphs which are introduced in §5.3. The main theorems in §5.4 are the criteria.

Chapter 7 provides the further development of the basic theory described in Chapter 5. A complete set of forbidden configurations for the planarity and planar embeddings based on the orientation defined , or in short OD- trees and OD- immersions is revealed. This allows us to be able to find the planarity auxiliary graph with both its order and size as linear functions of the order and the size of the original one and leads also to reaching at the peak of the linearity of the computing complexity.

Chapters 2, 3, and 4 are organized as the basis of Chapters 5 and 7. In Chapter 2, we introduce the OD- trees and OD- cotrees with the properties which will be used in the context. Chapter 3 is about the spaces in a graph including cycle spaces , cocycle spaces and bicycle spaces and their constructions. Chapter 4 deals with the very important results on planar graphs such as those derived from the Euler formula, the uniqueness, the straight line and the convex representations. In particular, the discrete form of the Jordan curve theorem is also described. Of course, Chapter 1 serves all the necessary knowledges on sets, orders, graphs, groups and surfaces for the whole book.

In Chapter 6, we can see how the planarity is used to attach the Gaussian crossing problem on a closed curve in the plane which was firstly solved by M. Dehn in the thirties. But Dehn's solution is not as what was conjectured by Gauss. The Gauss Conjecture was really proved in different ways. However, a new one which looks simpler is presented here.

Chapters 8 and 9 contain a series of the results, most recently obtained by the author, on the rectilinear embiddability of mainly planar graphs with as few bends as possible even with no bend at all on each edge. Chapter 8 devotes the tri-, bi-, and uni- embeddabilities, or say those with at most 3, 2, and 1 bends on each edge allowable. In Chapter 9, the 0 - embeddability, or in other words, the net embeddability, the most difficult case is studied. The topics here are stimulated by trying to provide a theoretical basis for the layouts, or particularly for the routings of electronic circuits in the VLSI circuit design.

Chapter 10 is about the problem of determining if two polyhedra on a surface are isomorphic. The problem for planar graphs was polynomially solved by L. Weinberg first. Now, we have already known that it is linearly solvable according to J. Hopcroft and J. Wong. Here, we use the OD- tree technique to treat the problem for general polyhedra. Moreover, for non-planar graphs, the problem can be done in the similar manner.

Chapters 11 and 12 concentrate on discussing non-planar graphs. Chapter 11 is about decompositions which are divided into two categories. One is to decompose a graph into several parts such that each of them has higher connectivity. And the other is of each part planar, or rectilinearly planar with as many edges as possible. Chapter 12 is on the surface embeddability which consists of two important aspects. One is called the up- embeddability which is with the maximum genus. And the other, the down- embeddability which is related to the minimum genus. The former is constructively solved based on the OD- tree technique. Although the quotient embeddings have been used to show the down - embeddability of many kinds of graphs including the settlement of the Hilbert– Cohn-Vossen's thread problem which we shall see in this chapter, the latter is still far from being completely solved.

Several optimization problems are proposed and solved in Chapter 13. Among them, one might think that the minimum bend and minimum area problems are especially interesting to the people who are working on layouts in VLSI circuit design. Very recent development which has been made by the author is included.

As realized, matroids are the objects that play the role of the dual graph when the graph is non-planar. Chapter 14 devotes the characterizations of a matroid to be regular, graphic , or cographic. Those are considered as a kind of deep generalization of the planarity in graphs, that was firstly exposed by W. T. Tutte. Here, we also discuss them in our own way.

Finally, Chapter 15 concerns the knot problem from topology to combinatorics and shows two kinds of combinatorial invariants for planar graphs with binary weights on edges, from which the Jones polynomial and the bracket polynomial, the two kinds of topological invariants are also derived as special cases.

The main purpose of this book is to provide a theoretical treatment of the problems related to the embeddability as mentioned above with constructing algorithms in mind. In order to save the space that has to be occupied , the last section of each chapter is arranged to be the notes in which the recent progress of related problems , background in theory and practice, and some historical remarks are briefly pointed. Meanwhile, a certain number of open problems with some suggestions that would be helpful for further research are also mentioned for the readers who want to hunt for novelty in the corresponding area. However, we are not allowed to discuss in every detail on algorithms even in natural extensions because it would double the number of pages the book is supposed to have. For the same reason, we have to omit all the theories on maps and their enumerations even their results flourished.

In conclusion, I should express my heartiest thanks to all the people who contribute themselves to this book in academic or technical work. Among them, I am only allowed to mention a few in the limited space. First of all, the beginning of the research in this topic was sprouted by the papers of Professor Wu Wenjun and was fertilized by those of Professor W.T.Tutte. Without their contributions, this book would never appear at the present time. Meanwhile, Professors Peter L. Hammer, K. C. Hsu, Fred S. Roberts, P. Rosenstiehl, Bruno Simeone, Z. Wan, Y. Wang, F. Wu, M. Y. Yue *et al.* are constantly concerned with what I have done and am doing in progress. Professors E. Aparo, Z. F. Ma, P. Marchioro, A. Morgana, R. Petreschi, W. X. Xu, J. Y. Yan *et al.* are often interested in discussing related problems. Audience including S. F. Cui, F. M. Dong, Y. Kang, X. G. Li, Yi. Liu, T. J. Lu (Ph.D), H. Ren, X. R. Sun (Ph.D), C. H. Velasquez

(Ph.D), Y. F. Xu (Ph.D), Z. Q. Yang, L. Yu, M. L. Zheng (Ph.D) *et al.* present the errata in parts or in the whole. Of course, the errors that remain are mine. Last but not least, I should especially appreciate the hospitalities from the Department of Combinatorics and Optimization, University of Waterloo, the RUTCOR, Rutgers University, the Department of Statistics, University "La Sapienza" of Rome and the financial supports from the N.S.F. and the A.F.O. in U.S.A., the C.N.R. in Italy and the N.N.S.F. in China.

Y. P. Liu

Beijing
March, 1995

Contents

Chapter 1

Preliminaries

For the sake of brevity we adopt, throughout this book, the usual logical conventions: disjunction, conjunction, negation, implication, equivalence, universal quantification and existential quantification denoted by the familiar symbols:$\vee, \wedge, \neg, \Rightarrow$, $\Leftrightarrow$, $\forall$ and $\exists$ respectively.

In the context, $(i.j.k)$ refers to item k of section j in chapter i.

A reference [k] refers to item k of the bibliographical references where k consists of the first few letters(initials) of last name(s) of the author(s) in alphabetical order followed by a number to distinguish publications of the same author(s).

1.1 Sets

A *set* is a collection of objects with some common property which might be numbers, points, symbols, letters or whatever even sets except itself to avoid paradoxes. The objects are said to be *elements* of the set. We always denote elements by italic lower letters and sets by capital ones. The statement " x is (is not) an element of M" is written as

$$x \in M (x \notin M).$$

A set is often characterized by a property. For example

$$M = \{x \mid x \le 4, \text{positive integer}\} = \{1, 2, 3, 4\}.$$

The *cardinality* of a set M (or the number of elements of M if finite) is denoted by $\mid M \mid$.

Let A, B be two sets. If $(\forall a)$ $(a \in A \Rightarrow a \in B)$, then A is said to be a *subset* of B which is denoted by $A \subseteq B$. Further, we may define the three main operations: union, intersection and subtraction respectively as

$$A \cup B = \{x \mid (x \in A) \vee (x \in B)\}; \tag{1.1.1}$$

$$A \cap B = \{x \mid (x \in A) \wedge (x \in B)\}; \tag{1.1.2}$$

$$A \setminus B = \{x \mid (x \in A) \wedge (x \notin B)\}. \tag{1.1.3}$$

If $B \subseteq A$, then $A \setminus B = A - B$ is denoted by $\overline{B}(A)$ which is said to be the *complement* of B in A. If all the sets are considered as subsets of Ω, then the complement of A in Ω is simply denoted by $\overline{A}$. The *empty* denoted by $\emptyset$ is the set without element. For those operations on subsets of Ω mentioned above, we have the following laws.

S1 Idempotent law. $\forall A \subseteq \Omega$,

$$A \cap A = A \cup A = A. \tag{1.1.4}$$

S2 Commutative law. $\forall A, B \subseteq \Omega$,

$$A \cup B = B \cup A; A \cap B = B \cap A. \tag{1.1.5}$$

S3 Associative law. $\forall A, B, C \subseteq \Omega$,

$$\begin{cases} A \cup (B \cup C) = (A \cup B) \cup C; \\ A \cap (B \cap C) = (A \cap B) \cap C. \end{cases} \tag{1.1.6}$$

S4 Absorption law. $\forall A, B \subseteq \Omega$,

$$A \cap (A \cup B) = A \cup (A \cap B) = A. \tag{1.1.7}$$

S5 Distributive law. $\forall A, B, C \subseteq \Omega$,

$$\begin{cases} A \cup (B \cap C) = (A \cup B) \cap (A \cup C); \\ A \cap (B \cup C) = (A \cap B) \cup (A \cap C). \end{cases} \tag{1.1.8}$$

S6 Universal bound law. $\forall A \subseteq \Omega$,

$$\emptyset \cap A = \emptyset,\ \emptyset \cup A = A; \Omega \cap A = A, \Omega \cup A = \Omega. \tag{1.1.9}$$