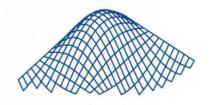
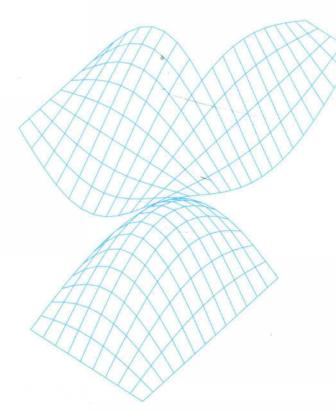


# Structural Dynamics

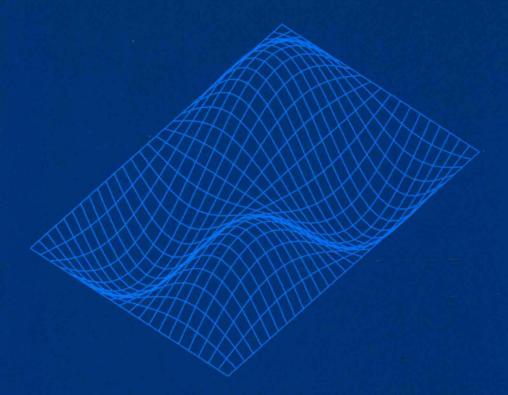
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# **Structural Dynamics**

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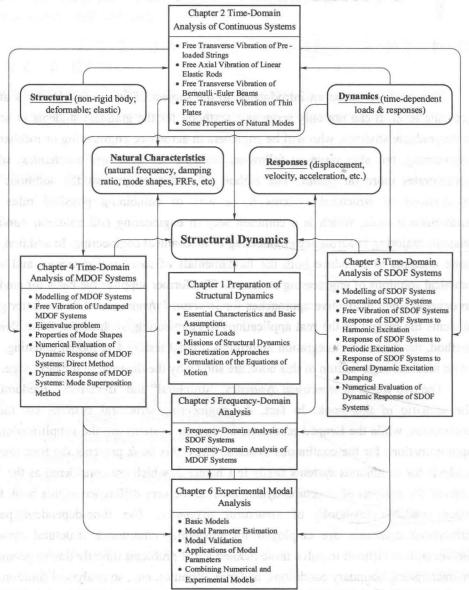
# Preface

This book serves as an introduction to the subject of *structural dynamics* at the graduate level. It can not only serve as a textbook for the graduate students or senior undergraduate students, who will be engineers in aerospace engineering or mechanical engineering, but also serve as follow-up contents for *structural mechanics*, which concentrates more on *statics*. The authors of this book present the solutions and applications of structural systems by a way of combining physical rules and mathematical tools, which is a common way in engineering and common mind for students majoring in aerospace engineering or mechanical engineering. In addition, this book attempts to introduce both the fundamentals of *structural dynamics* and some practical solutions of engineering structures. The former supplies the basis for students to develop some innovative approaches for *structural dynamics*, while the latter gives students the insight of the real applications in engineering, such as the finite element method, the numerical integration algorithms, modal testing and modal updating. The basic idea and organization of this book are shown by the diagram in the next page.

The two words in *structural dynamics*, "structural" and "dynamics", predominate the scenario of this book. In fact, the engineering structural systems are mostly continuous, while the lumped-parameter or discrete systems are the simplifications or approximations for the continuous systems. Thus, this book presents the time-domain analysis for continuous systems firstly in Chapter 2, which are considered as the 'raw objects' for analysis of structural dynamics. It is the very difference of this book from other available textbooks of *structural dynamics*. The time-dependent partial differential equations are employed for modelling continuous structural systems. However, it is difficult to solve these mathematical problems directly due to geometric characteristics, boundary conditions, material attributes, etc., so analytical solutions for continuous structural systems are very limited to some special cases, such as strings, rods, uniform beams and plates with regular boundary conditions.

Can we only solve these special cases of the "raw objects" from the continuous

systems? Of course not. The scientists and engineers of structural analysis never give up finding out various simplified or approximate solutions for the engineering structures, although more or less errors or biases are possible. This book introduces the time-domain analysis for single-degree-of-freedom (SDOF) systems in Chapter 3 and the time-domain analysis for multiple-degree-of-freedom (MDOF) systems in Chapter 4.



Chapter 3 introduces the theories about SDOF systems, which are the simplest systems in *structural dynamics*. At the beginning of this chapter, the continuous

structural systems are transformed into the generalized SDOF systems equivalently by the Rayleigh method. As a result, for an SDOF system, the non-rigid property of structures is presented by springs and dampers, while the inertia and time-dependent loads and responses represent the properties of dynamics, solutions for various cases of SDOF systems, such as the free vibration and the force vibration under the different dynamic loads. Meanwhile, the natural characteristics are discussed, such as natural frequency and damping. SDOF systems are very simple, even impossible to be used in a real engineering application, but they supply an initial and fundamental insight of structural dynamics.

In Chapter 4, continuous structural systems are transformed into the corresponding MDOF systems by the Rayleigh-Ritz's method, which is considered as a great improvement from the Rayleigh method. However, the Ritz functions, or named assumed shape functions, are very difficult to prescribed globally. Thus, the finite element method is introduced, which assumes the shape functions in a single element locally and overcomes the difficulty of the original Rayleigh-Ritz's method. The finite element method supplies a good and flexible balance between the computational cost and the accuracy. Based on the discretized systems, i.e. the MDOF systems, the natural characteristics, such as natural frequencies, mode shapes, damping ratios and some important properties of these characteristics, are discussed. For the solutions of responses, the direct numerical integration and the mode superposition are introduced, respectively. Combining the discretization by the finite element method and the numerical integration is a common way, even the final solution in engineering nowadays. The mode superposition supplies a good way of model reduction through the modal decoupling and the modal truncation.

Frequency-domain analysis and time-domain analysis are two arms of analysis for linear systems. In Chapter 5, we introduce the theories of SDOF systems and MDOF systems in the frequency domain. Frequency analysis is not only a very useful tool for the design of *structural dynamics*, but also supplies the concise models for modal analysis analytically and experimentally.

Modal analysis is a very important part in *structural dynamics*, which is a powerful tool for structural analysis and testing and supplies a kind of fundamental for linking tests and numerical models. In Chapter 6, we introduce the experimental modal analysis and its applications.

Chapter 5 and Chapter 6 highly refer to the co-author, Prof. Heylen's modal book, *Modal Analysis Theory and Testing*, published by KU Leuven.

In closing, the authors wish to express their sincere thanks and appreciation to the

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# **Introduction to Structural Dynamics**

This is an introductory chapter for *structural dynamics* in this book. This chapter mainly discusses the questions listed below. The two keywords, "structural" and "dynamics," are not only highlighted in this chapter but also in this whole book, since these two words mostly present the domain and the essence of *structural dynamics*. They also identify the structural dynamics from relational disciplines, such as vibration theory and mechanical vibration. The basic assumptions are also introduced in this chapter, as well as the three subjects in *structural dynamics*, the structural system, the excitation (dynamic loads on the objective structural system) and the response. Based on these three subjects, four main kinds of missions of structural dynamics are described, including the response analysis, system identification, load identification and structural dynamic control. *Structural dynamics* is a historical, extensive and profound discipline, so it is hardly possible to cover all knowledge in a single book. To supply the fundamental for sequential chapters, the final part of this chapter introduces the kinds of classic theories for building mathematical models of dynamical systems.

#### **Ouestions to Answer**

- What is structural dynamics?
- What are the basic assumptions that structural dynamics bases on?
- Why can a structure have motion?
- How can we use the knowledge from structural dynamics?
- How to apply structural dynamics theory on practical problems?
- How to build a mathematical model for a physical structural system?

#### 1.1 Essential Characteristics and Basic Assumptions

This section discusses "What is *structural dynamics*?" and "What are the basic assumptions that *structural dynamics* is based upon?" in a definitive way. Of course, considering the various theoretical researches and applications in the general realm of

structural dynamics, this book limits its discussion to a common understanding about the contents of structural dynamics.

#### 1.1.1 Essential Characteristics

There are two key words in *structural dynamics*: "structural" and "dynamics" and *structural dynamics* concerns dynamics of structures, which is a branch in *structural mechanics* and involves varying engineering fields, including mechanical engineering, aerospace engineering, civil engineering, etc. The detailed explanation of these two words by comparing "statics" and "general systems" may clarify the boundaries of introducing the theories of *structural dynamics*.

#### 1.1.1.1 Statics vs. Dynamics

In *mechanics*, *statics* concerns the behaviors, phenomena, and mechanisms of forces on bodies at rest and the bodies could be rigid or deformable. The basic characteristics of *statics* are that all values of the objective body are invariant with time. In *dynamics*, inputs and outputs of systems are time-varying. In other words, loads and responses of dynamical systems are both functions with respect to time. The kinematic and kinetic behaviors, phenomena and mechanisms of bodies when dynamic loads act are concerned. In these two aspects of dynamics, *kinematics* and *kinetics*, the former focuses on the motion of bodies with some geometric constraints or relations between bodies, irrespective of the causes of the motion; the latter concerns motions of bodies and its causes, namely forces and torques.

#### 1.1.1.2 General Systems vs. Structural systems

In short, a structural system is a special realization of general systems. A system is defined by "an assembly or set of related elements" [1, 2]. "A structure refers to a system of connected parts used to support loads" [3]. In other words, the function of structures is to support payloads and to transfer loadings in context of engineering, such as the aerospace, mechanical and civil engineering in particular. To support the payloads and transfer loadings, structures have to deform to balance the internal force inside themselves, so structures are "non-rigid," deformable, or elastic and the evaluation of deformations and stresses under the action of applied loads is concerned [4].

#### 1.1.1.3 Structural Dynamics

After discussing relations and differences between *statics* and *dynamics* and between *general systems* and *structural systems* above, the question "What is structural dynamics?" can be addressed now. *Structural dynamics* concerns dynamic responses under time-varying loadings and dynamic characteristics of deformable objects, i.e. structures or named structural systems. In contrast, the rigid-body dynamics treats an

object as rigid body that undergoes motion without deformation under the dynamic loading, whose study has many applications, such as, the movement of machinery, the *flight dynamics* of an aircraft or a spacecraft and astrodynamics; *structural statics*, or *structural analysis in narrow sense*, focuses on the static responses and static characteristics of structural systems. Conclusively, *structural dynamics* keeps two main features: the time-variation of loads and responses, and the deformability of objects.

A simple example [5] as is shown in Figure 1.1 could release these two main features.

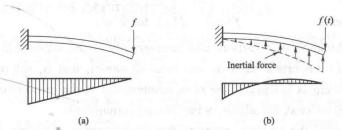


Figure 1.1 A cantilever and its corresponding bending moment diagram under (a) static and (b) dynamic loading, respectively

Firstly, since the applied loadings are time-varying as f(t) in Figure 1.1(b) and inertia exists in all physical objects, the inertial force with respect to the acceleration balances the applied loads and the internal force in statics case as shown in Figure 1.1(b). It is an important characteristic to identify *statics* and *dynamics*. Secondly, as shown in Figure 1.1, either for the static (a) or dynamic cases (b), the cantilever beam deflects; for the dynamic case, the inertial force is distributed to balance the distributed internal and external loads. In other words, the deformability of structures results in the distribution of physical mechanisms. It is an important characteristic to identify *rigid-body dynamics* and *structural dynamics*. In addition, other distributed dynamic mechanisms are also possible, such as damping.

#### 1.1.2 Basic Assumptions in Structural Dynamics of This Book

Section 1.1.1 generally answers the question "What is structural dynamics?" and introduces the essential characteristics of *structural dynamics*, which limits the discussion to a deterministic scope. However, *structural dynamics* is still a broad discipline even if these limitations are performed. Actually, this section will introduce a tip of structural dynamics' iceberg, which merely covers the fundamentals for *structural dynamics*. All introduction and discussion in this book obeys the three main assumptions as below.