刘彦佩

半闲数学集锦

Semi-Empty Collections in Mathemetics by Y.P.Liu

第十七编

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第十七编序

中国科学技术大学, 於 1958 年创建. 我是该校首届毕业生. 2008 年, 英文专著 [400] *Topological Theory on Graphs*(17.01—17.20) 被纳入中国科学技术大学校友文库. 在中国科学技术大学出版社的资助下, 出版(国家 "十一五" 重点图书), 以此纪念中国科学技术大学, 建校 50 周年.

本书将曲面(无边缘紧 2-流形)视为偶多边形, 使得每一对边都有到 {1,-1} 上的一个映射, 称为幂. 证明了曲面的拓扑分类就是由

$$\begin{cases}(Aaa^{-1}B) \Longleftrightarrow (AB);\\(AxByCx^{-1}Dy^{-1}E) \Longleftrightarrow (ADCBExyx^{-1}y^{-1});\\(AxBxC) \Longleftrightarrow (AB^{-1}Cxx)\end{cases}$$

或等价地,

$$\begin{cases}(ABxCx^{-1}) \Longleftrightarrow (BAxCx^{-1}),\\(AxByCx^{-1}Dy^{-1}) \Longleftrightarrow (ADxyBx^{-1}Cy^{-1});\\(AxBx) \Longleftrightarrow (AB^{-1}xx).\end{cases}$$

所确定的等价类. 提供了有效性实现定理. 因为在所接触到的拓扑学文献中, 从未见这种简单的形式, 也可视为对拓扑学研究的一个贡献.

将图视为多面形的集合. 在多面形上(而不是复形, 如拓扑学中惯用的), 引入 i-空间(i =0, 1 和 2, 前二者来自复形上的 0-链和 1-链), 通过他们之间的边缘与上边缘运算, 构建出同调空间与上同调空间. 由此建立图的平面性判准, 可以一并导出 Euler 必要条件的充分性, Whitney 的对偶性, McLane 的圈基, 以及 Lefschetz 的二重覆盖等判准. 在此基础上, 还导出了高斯猜想迄今最简单的一个证明, 和拟阵图性以上图性新判准.

有趣的是我的这个判准曾受到质疑. 例如 Abrums 和 Slilaty (An algebraic characterization of projective-planar graphs, Manuscript, 2002; 或 Jounal of Graph Theory, 42(2003), 320-331. MR2003k: 05041)曾给出一个"反例".

但他们忽略了定理的条件, 因为此例中有一个节点应为两个节点. 或用拓扑学的语言, 他们忽略了曲面的无边缘性. 或用组合学的语言, 他们忽略了双迁覆盖, 在每个节点处的可迁性.

另一方面, 在图上直接将 Wu(吴文俊)-Tutte 的 2-链群引伸到 2-空间. 然而, 当时只有上同调空间(与前述的不同)可资利用. 在此基础上, 使我能够将判别图的平面性问题, 转换为在一个派生图(后称平面性辅助图,或演生网)上, 求一个支撑树. 从而, 发现有效实现性(运行)定理.

经过在平面性辅助图上的一系列的变换, 最终发现一阶(即线性)运行定理.

应该指出, 我的第一个借助演生网的有效性实现定理, 是 1978 年问世的 参见 1.06[008]. 有趣的是, 20 年后, Archdeacon 和 Siran(Characterizing planarity using theta graphs, Journal of Graph Theory, 27(1998), 17–20) 用 θ-图的判准, 不能有效实现. 虽然 θ-图也是一类演生网, 由于过大而导致不能有效实现. 事实上, 我的平面性辅助图(演生网), 是他们的一个很小的子图.

在纵横(或正交)嵌入方面, 提出了三个可选用的曲面模型, 将我以前建立平面上的纵横嵌入理论, 拓广到一般曲面上.

在研究图论中的 Tutte 多项式的过程中, 发现一类更广的多项式. 这个多项式也是拓扑学中纽结的一个不变量, 并且由此直接导出 Jones 多项式.

基于这些, 使我不能不在专著 [141](3.11—3.29) 的理论基础上, 除了全面强化理论根基外，还提供了一些在亏格非 0 曲面上的新发展.

在文 17.21[387]—17.25[393] 中, 除文 17.22[388], 17.24[392] 和 17.25[393] 与专著 [400](17.01—17.20) 相关外, 其他的, 即文 17.21[387] 和 17.23[389] 都与地图计数有关.

刘彦佩
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第十七编目录

当代科学技术基础理论与前沿问题研究丛书

中国科学技术大学

校友文库

Topological Theory on Graphs

图的拓扑理论

刘彦佩 著

中国科学技术大学出版社

内容提要

本书不在于图的拓扑性质本身，而是着意以图为代表的一些组合构形为出发点，揭示与拓扑学中一些典型对象，如多面形、曲面、嵌入、纽结等的联系，特别是显示了定理有效化的途径对于以拓扑学为代表的基础数学的作用。同时，也提出了一些新的曲面模型，为超大规模集成电路的布线尝试构建多方面的理论基础。

本书可作为基础数学、应用数学、系统科学、计算机科学等专业高年级本科生和研究生的补充教材，也可供相关专业的教师和科研工作者参考。

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Preface

The subject of this book reflects new developments mainly by the author himself in company with cooperators most of them his former and present graduate students on the foundation established in Liu, Y.P.[33–34]. The central idea is to extract suitable parts of a topological object such as a graph not necessary to be with symmetry, as linear spaces which are all with symmetry for exploiting global properties in construction of the object. This is a way of combinatorizations and further algebraications of an object via relationship among their subspaces.

Graphs are dealt with three vector spaces over GF(2), the finite field of order 2, generated by 0(dimensional)-cells, 1(dimensional)-cells and 2(dimensional)-cells. The first two spaces were known from, *e.g.*, Lefschetz, S.[2] by taking 0-cells and 1-cells as, respectively, vertices and edges. Of course, a graph is only a 1-complex without 2-cells.

Since the fifties of last century, in Wu, W.J.[1] and Tutte, W.T.[4, 16], the chain groups generated by 0-cells and 1-cells over, respectively, GF(2) and the real field were independently used for describing a graph. And they both then after ten years adopted nonadjacent pair of edges as a 2-cell for which the cohomology on a graph was allowed to be established.

Their results especially in Wu, W.J.[1–6] enabled the present author to create a number of types of planarity auxiliary graphs induced from the graph considered for the study of the efficiency of theorems in Liu, Y.P.[1,2,19,22,42] as one approach. Another approach can be seen in Liu, Y.P.[23–25,43].

More interestingly, two decades later than Liu, Y.P.[1], in Archdeacon, D. and J. Siran[1] a theta graph(network) was used for characterizing the planarity of a given graph. The theta graph can be seen to be a type of planarity auxiliary graph(network) because our planarity auxiliary graphs are subgraphs of the theta graph. However, in virtue of the order of theta network upper bounded by an exponential function of the size of given graph and that of planarity auxiliary network by a quadratic polynomial of the size of given graph, theorems deduced from a theta network are all without efficiency while those from a planarity auxiliary network are all with efficiency.

The effects of planarity auxiliary graphs are reflected in Chapters 8, 10, 11, 12 and 13 with a number of extensions.

On the other hand, in Liu, Y.P.[31] a graph was dealt with a set of polyhedra via double covering the edge set by travels under certain condition so that travels were treated as 2-cells. These enable us to discover homology and another type of cohomology for showing the sufficiency of Eulerian necessary condition in this circumstance. Further, all the results for the planarity of a graph in Whitney, H.[7] on the duality, MacLane, S.[1–2] on a circuit basis and Lefschetz, S.[1] on a circuit double covering have a universal view in this way. In fact, our polyhedra are all on such surfaces, *i.e.*, 2-dimensional compact manifolds without boundary. If a boundary is allowed on a surface, the Eulerian necessary condition is not always sufficient in general. Some person used to have missing the boundary condition in Abrams, L. and C.D. Slilaty[1].

The effects of this theoretical thinking are reflected in Chapters 4,5,7 and 14.

Because of the clarification of the joint tree model of a polyhedron in Liu, Y.P.[35–36] by the present author recently on the basis of Liu, Y.P.[8–9], we are allowed to write a chapter for brief description of the theories of surfaces and polyhedra each in Chapters 2 and 3 with related topics in Chapters 6, 9 and 15.

Although quotient embeddings(current graph and its dual, voltage graph) were quite active in constructing an embedding of a graph on a surface with its genus minimum in a period of decades, this book has no space for them. One reason is that some books have mentioned them such as in White, A.T.[1], Ringel, G.[3] and Liu, Y.P.[33–34], *etc.* Another reason is that only graphs with higher symmetry are suitable for quotient embeddings, or for employing the covering space method whence this book is for general graphs without such a limitation of symmetry.

In spite of refinements and simplifications for known results, this book still contains a number of new results such as in §5.2, the sufficiency in the proof of Theorem 5.2.1, §9.4, §11.3–4, §13.1–2, §13.4–5 *etc.*, only name a few. Researches were partially supported by NNSF in China under Grants No.60373030 and No.10571013.

Y. P. Liu
Beijing
December 2007

Contents

Chapter 1
Preliminaries

Throughout for the sake of brevity, the usual logical conventions are adopted: disjunction, conjunction, negation, implication, equivalence, universal quantification and existential quantification denoted, respectively, by the familiar symbols: $\vee$, $\wedge$, $\neg$, $\Rightarrow$, $\Leftrightarrow$, $\forall$ and $\exists$. And, $\S x.y$ is for the section y in Chapter x.

In the context, $(i.j.k)$ refers to item k of section j in chapter i.

A reference $[k]$ refers to item k of the corresponding author(s) in the bibliography where k is a positive integer to distinguish publications of the same author(s).

1.1 Sets and relations

A *set* is a collection of objects with some common property which might be numbers, points, symbols, letters or whatever even sets except itself to avoid paradoxes. The objects are said to be *elements* of the set. We always denote elements by italic lower letters and sets by capital ones. The statement " x is (is not) an element of M" is written as $x \in M (x \notin M)$. A set is often characterized by a property. For example

$$M = \{x \mid x \leq 4, \text{ positive integer}\} = \{1, 2, 3, 4\}.$$

The *cardinality* of a set M (or the number of elements of M if finite) is denoted by $\mid M \mid$.

Let A, B be two sets. If $(\forall a)$ $(a \in A \Rightarrow a \in B)$, then A is said to be a *subset* of B which is denoted by $A \subseteq B$. Further, we may define the three main operations: union, intersection and subtraction