Stochastic Modelling and Applied Probability 64

Eckhard Platen
Nicola Bruti-Liberati

Numerical Solution of Stochastic Differential Equations with Jumps in Finance

金融数学中的带跳随机微分方程数值解

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Stochastic Models in Life Sciences

Stochastic Modelling and Applied Probability

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64

Eckhard Platen
Nicola Bruti-Liberati (1975–2007)
School of Finance and Economics
Department of Mathematical Sciences
University of Technology, Sydney
PO Box 123
Broadway NSW 2007
Australia
eckhard.platen@uts.edu.au

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Preface

This research monograph concerns the design and analysis of discrete-time approximations for stochastic differential equations (SDEs) driven by Wiener processes and Poisson processes or Poisson jump measures. In financial and actuarial modeling and other areas of application, such jump diffusions are often used to describe the dynamics of various state variables. In finance these may represent, for instance, asset prices, credit ratings, stock indices, interest rates, exchange rates or commodity prices. The jump component can capture event-driven uncertainties, such as corporate defaults, operational failures or insured events. The book focuses on efficient and numerically stable strong and weak discrete-time approximations of solutions of SDEs. Strong approximations provide efficient tools for simulation problems such as those arising in filtering, scenario analysis and hedge simulation. Weak approximations, on the other hand, are useful for handling problems via Monte Carlo simulation such as the evaluation of moments, derivative pricing, and the computation of risk measures and expected utilities. The discrete-time approximations considered are divided into regular and jump-adapted schemes. Regular schemes employ time discretizations that do not include the jump times of the Poisson jump measure. Jump-adapted time discretizations, on the other hand, include these jump times.

The first part of the book provides a theoretical basis for working with SDEs and stochastic processes with jumps motivated by applications in finance. This part also introduces stochastic expansions for jump diffusions. It further proves powerful results on moment estimates of multiple stochastic integrals. The second part presents strong discrete-time approximations of SDEs with given strong order of convergence, including derivative-free and predictor-corrector schemes. The strong convergence of higher order schemes for pure jump SDEs is established under conditions weaker than those required for jump diffusions. Estimation and filtering methods are discussed. The third part of the book introduces a range of weak approximations with jumps. These weak approximations include derivative-free, predictor-corrector, and simplified schemes. The final part of the research monograph raises questions on numerical stability and discusses powerful martingale representations and variance reduction techniques in the context of derivative pricing.

The book does not claim to be a complete account of the state of the art of the subject. Rather it attempts to provide a systematic framework for an understanding of the basic concepts and tools needed when implementing simulation methods for the numerical solution of SDEs. In doing so the book aims to follow up on the presentation of the topic in Kloeden & Platen (1999) where no jumps were considered and no particular field of application motivated the numerical methods. The book goes significantly beyond Kloeden & Platen (1999). It is covering many new results for the approximation of continuous solutions of SDEs. The discrete time approximation of SDEs with jumps represents the focus of the monograph. The reader learns about powerful numerical methods for the solution of SDEs with jumps. These need to be implemented with care. It is directed at readers from different fields and backgrounds.

The area of finance has been chosen to motivate the methods. It has been also a focus of research by the first author for many years that culminated in the development of the benchmark approach, see Platen & Heath (2006), which provides a general framework for modeling risk in finance, insurance and other areas and may be new to most readers. The book is written at a level that is appropriate for a reader with an engineer's or similar undergraduate training in mathematical methods. It is readily accessible to many who only require numerical recipes.

Together with Nicola Bruti-Liberati we had for several years planned a book to follow on the book with Peter Kloeden on the "Numerical Solution of Stochastic Differential Equations", which first appeared in 1992 at Springer Verlag and helped to develop the theory and practice of this field. Nicola's PhD thesis was written to provide proofs for parts of such a book. It is very sad that Nicola died tragically in a traffic accident on 28 August 2007. This was an enormous loss for his family and friends, his colleagues and the area of quantitative methods in finance.

The writing of such a book was not yet started at the time of Nicola's tragic death. I wish to express my deep gratitude to Katrin Platen, who then agreed to typeset an even more comprehensive book than was originally envisaged. She carefully and patiently wrote and revised several versions of the manuscript under difficult circumstances. The book now contains not only results that we obtained with Nicola on the numerical solution of SDEs with jumps, but also presents methods for exact simulation, parameter estimation, filtering and efficient variance reduction, as well as the simulation of hedge ratios and the construction of martingale representations.

I would like to thank several colleagues for their collaboration in related research and valuable suggestions on the manuscript, including Kevin Burrage, Leunglung Chan, Kristoffer Glover, David Heath, Des Higham, Hardy Hulley, Constantinos Kardaras, Peter Kloeden, Uwe Küchler, Herman Lukito, Remigius Mikulevicius, Renata Rendek, Wolfgang Runggaldier, Lei Shi and Anthony Tooman. Particular thanks go to Rob Lynch, the former Dean of the Faculty of Business at the University of Technology Sydney, who made the writing of the book possible through his direct support. Finally, I like to thank the Editor, Catriona Byrne, at Springer for her excellent work and her encouragement to write this book as a sequel of the previous book with Peter Kloeden.

It is greatly appreciated if readers could forward any errors, misprints or suggested improvements to: eckhard.platen@uts.edu.au. The interested reader is likely to find updated information about the numerical solution of stochastic differential equations on the webpage of the first author under "Numerical_Methods":

`http://www.business.uts.edu.au/`
`finance/staff/Eckhard/Numerical_Methods.html`

Sydney, January 2010 *Eckhard Platen*

Suggestions for the Reader

It has been mentioned in the Preface that the material of this book has been arranged in a way that should make it accessible to as wide a readership as possible. Prospective readers will have different backgrounds and objectives. The following four groups are suggestions to help use the book more efficiently.

(i) Let us begin with those readers who aim for a sufficient understanding, to be able to apply stochastic differential equations with jumps and appropriate simulation methods in their field of application, which may not be finance. Deeper mathematical issues are avoided in the following suggested sequence of reading, which provides a guide to the book for those without a strong mathematical background:

§1.1 → §1.2 → §1.3 → §1.4 → §1.5 → §1.6 → §1.7 → §1.8

↓

§5.1 → §5.2 → §5.3 → §5.4

↓

§6.1 → §6.2 → §6.3

↓

§7.1 → §7.2 → §7.3 → §7.4

↓

§8.1 → §8.2 → §8.3 → §8.4 → §8.5 → §8.6 → §8.8

↓

§11.1 → §11.2 → §11.3 → §11.4 → §11.5

↓

§12.1 → §12.2

↓

§13.1 → §13.2 → §13.3 → §13.4 → §13.7

↓

Chapter 14

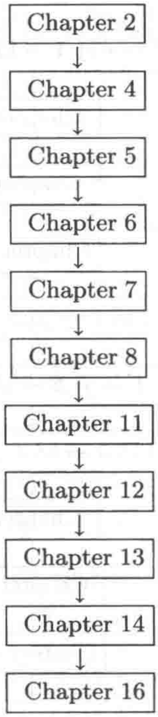
↓

§16.1

(ii) Engineers, quantitative analysts and others with a more technical background in mathematical and quantitative methods who are interested in applying stochastic differential equations with jumps, and in implementing efficient simulation methods or developing new schemes could use the book according to the following suggested flowchart. Without too much emphasis on proofs the selected material provides the underlying mathematics.

Chapter 1
↓
Chapter 2
↓
Chapter 4
↓
Chapter 5
↓
§6.1 → §6.2 → §6.3
↓
§7.1 → §7.2 → §7.3 → §7.4
↓
§8.1 → §8.2 → §8.3 → §8.4 → §8.5 → §8.6 → §8.8 → §8.9
↓
Chapter 11
↓
§12.1 → §12.2
↓
§13.1 → §13.2 → §13.3 → §13.4 → §13.7
↓
Chapter 14
↓
Chapter 16

(iii) Readers with strong mathematical background and mathematicians may omit the introductory Chap. 1. The following flowchart focuses on the theoretical aspects of the numerical approximation of solutions of stochastic differential equations with jumps while avoiding well-known or applied topics.



(iv) Financial engineers, quantitative analysts, risk managers, fund managers, insurance professionals and others who have no strong mathematical background and are interested in finance, insurance and other areas of risk management will find the following flowchart helpful. It suggests the reading for an introduction into quantitative methods in finance and related areas.

§1.1 → §1.2 → §1.3 → §1.4 → §1.5 → §1.6 → §1.7 → §1.8

↓

Chapter 2

↓

Chapter 3

↓

Chapter 5

↓

§6.1 → §6.2 → §6.3

↓

§7.1 → §7.2 → §7.3 → §7.4

↓

§8.1 → §8.2 → §8.3 → §8.4 → §8.5 → §8.6 → §8.8

↓

Chapter 9

↓

Chapter 10

↓

Chapter 11

↓

§12.1 → §12.2

↓

§13.1 → §13.2 → §13.3 → §13.4 → §13.7

↓

Chapter 14

↓

Chapter 15

↓

Chapter 16

↓

Chapter 17

Basic Notation

μ_X	mean of X
σ_X^2, $\mathrm{Var}(X)$	variance of X
$\mathrm{Cov}(X, Y)$	covariance of X and Y
$\inf\{\cdot\}$	greatest lower bound
$\sup\{\cdot\}$	smallest upper bound
$\max(a, b) = a \vee b$	maximum of a and b
$\min(a, b) = a \wedge b$	minimum of a and b
$(a)^+ = \max(a, 0)$	maximum of a and 0
$\boldsymbol{x}^\top$	transpose of a vector or matrix $\boldsymbol{x}$
$\boldsymbol{x} = (x^1, x^2, \ldots, x^d)^\top$	column vector $\boldsymbol{x} \in \Re^d$ with ith component x^i
$\lvert\boldsymbol{x}\rvert$	absolute value of $\boldsymbol{x}$ or Euclidean norm
$\boldsymbol{A} = [a^{i,j}]_{i,j=1}^{k,d}$	$(k \times d)$-matrix $\boldsymbol{A}$ with ijth component $a^{i,j}$
$\det(\boldsymbol{A})$	determinant of a matrix $\boldsymbol{A}$
$\boldsymbol{A}^{-1}$	inverse of a matrix $\boldsymbol{A}$
$(\boldsymbol{x}, \boldsymbol{y})$	inner product of vectors $\boldsymbol{x}$ and $\boldsymbol{y}$
$\mathcal{N} = \{1, 2, \ldots\}$	set of natural numbers

∞	infinity
(a, b)	open interval $a < x < b$ in $\Re$
$[a, b]$	closed interval $a \leq x \leq b$ in $\Re$
$\Re = (-\infty, \infty)$	set of real numbers
$\Re^+ = [0, \infty)$	set of nonnegative real numbers
$\Re^d$	d-dimensional Euclidean space
Ω	sample space
$\emptyset$	empty set
$A \cup B$	the union of sets A and B
$A \cap B$	the intersection of sets A and B
$A \backslash B$	the set A without the elements of B
$\mathcal{E} = \Re \backslash \{0\}$	$\Re$ without origin
$[X, Y]_t$	covariation of processes X and Y at time t
$[X]_t$	quadratic variation of process X at time t
$n! = 1 \cdot 2 \cdot \ldots \cdot n$	factorial of n
$[a]$	largest integer not exceeding $a \in \Re$
i.i.d.	independent identically distributed
a.s.	almost surely
f'	first derivative of $f : \Re \to \Re$
f''	second derivative of $f : \Re \to \Re$
$f : Q_1 \to Q_2$	function f from Q_1 into Q_2
$\frac{\partial u}{\partial x^i}$	ith partial derivative of $u : \Re^d \to \Re$
$\left(\frac{\partial}{\partial x^i}\right)^k u$	kth order partial derivative of u with respect to x^i
$\exists$	there exists
$F_X(\cdot)$	distribution function of X
$f_X(\cdot)$	probability density function of X
$\phi_X(\cdot)$	characteristic function of X
$\mathbf{1}_A$	indicator function for event A to be true

$N(\cdot)$	Gaussian distribution function
$\Gamma(\cdot)$	gamma function
$\Gamma(\cdot;\cdot)$	incomplete gamma function
$(\bmod c)$	modulo c
$\mathcal{A}$	collection of events, sigma-algebra
$\underline{\mathcal{A}}$	filtration
$E(X)$	expectation of X
$E(X \mid \mathcal{A})$	conditional expectation of X under $\mathcal{A}$
$P(A)$	probability of A
$P(A \mid B)$	probability of A conditioned on B
$\in$	element of
$\notin$	not element of
$\neq$	not equal
$\approx$	approximately equal
$a \ll b$	a is significantly smaller than b
$\lim_{N\to\infty}$	limit as N tends to infinity
$\liminf_{N\to\infty}$	lower limit as N tends to infinity
$\limsup_{N\to\infty}$	upper limit as N tends to infinity
$\imath$	square root of -1, imaginary unit
$\delta(\cdot)$	Dirac delta function at zero
$\boldsymbol{I}$	unit matrix
$\operatorname{sgn}(x)$	sign of $x \in \Re$
$\mathcal{L}_T^2$	space of square integrable, progressively measurable functions on $[0,T] \times \Omega$
$\mathcal{B}(U)$	smallest sigma-algebra on U
$\ln(a)$	natural logarithm of a
MM	Merton model
MMM	minimal market model

GIG	generalized inverse Gaussian
GH	generalized hyperbolic
VG	variance gamma
GOP	growth optimal portfolio
EWI	equi-value weighted index
ODE	ordinary differential equation
SDE	stochastic differential equation
PDE	partial differential equation
PIDE	partial integro differential equation
$I_\nu(\cdot)$	modified Bessel function of the first kind with index ν
$K_\lambda(\cdot)$	modified Bessel function of the third kind with index λ
Δ	time step size of a time discretization
$\binom{i}{l} = \frac{i!}{l!(i-l)!}$	combinatorial coefficient
$\mathcal{C}^k(\mathcal{R}^d, \mathcal{R})$	set of k times continuously differentiable functions
$\mathcal{C}^k_P(\mathcal{R}^d, \mathcal{R})$	set of k times continuously differentiable functions which, together with their partial derivatives of order up to k, have at most polynomial growth

Letters such as K, K_1, ..., $\tilde{K}$, C, C_1, ..., $\tilde{C}$, ... represent finite positive real constants that can vary from line to line. All these constants are assumed to be independent of the time step size Δ.

Motivation and Brief Survey

Key features of advanced models in many areas of application with uncertainties are often event-driven. In finance and insurance one has to deal with events such as corporate defaults, operational failures or insured accidents. By analyzing time series of historical data, such as prices and other financial quantities, many authors have argued in the area of finance for the presence of jumps, see Jorion (1988) and Ait-Sahalia (2004) for foreign exchange and stock markets, and Johannes (2004) for short-term interest rates. Jumps are also used to generate the short-term smile effect observed in implied volatilities of option prices, see Cont & Tankov (2004). Furthermore, jumps are needed to properly model credit events like defaults and credit rating changes, see for instance Jarrow, Lando & Turnbull (1997). The short rate, typically set by a central bank, jumps up or down, usually by some quarters of a percent, see Babbs & Webber (1995). Models for the dynamics of financial quantities specified by stochastic differential equations (SDEs) with jumps have become increasingly popular. Models of this kind can be found, for instance, in Merton (1976), Björk, Kabanov & Runggaldier (1997), Duffie, Pan & Singleton (2000), Kou (2002), Schönbucher (2003), Glasserman & Kou (2003), Cont & Tankov (2004) and Geman & Roncoroni (2006). The areas of application of SDEs with jumps go far beyond finance. Other areas of application include economics, insurance, population dynamics, epidemiology, structural mechanics, physics, chemistry and biotechnology. In chemistry, for instance, the reactions of single molecules or coupled reactions yield stochastic models with jumps, see, for instance, Turner, Schnell & Burrage (2004), to indicate just one such application.

Since only a small class of jump diffusion SDEs admits explicit solutions, it is important to construct discrete-time approximations. The focus of this monograph is the numerical solution of SDEs with jumps via simulation. We consider pathwise scenario simulation, for which strong schemes are used, and Monte Carlo simulation, for which weak schemes are employed. Of course, there exist various alternative methods to Monte Carlo simulation that we only consider peripherally in this book when it is related to the idea of discrete-time