普通高等院校航空航天双语教学用书

ROTARY WING STRUCTURAL DYNAMICS AND AEROELASTICITY

SECOND EDITION

旋翼飞行器结构动力学与气动弹性力学

（第2版）

●（双语教学精选译注版）

（美）理查德 ·L. 比拉瓦（Richard L. Bielawa） 著
刘 勇 邵 松 孙传伟 译注
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内 容 提 要

本书为普通高等院校航空航天双语教学精选译注版用书。《旋翼飞行器结构动力学与气动弹性力学》原版教材是 AIAA 教育系列丛书之一，曾作为“优秀旋翼技术中心”旋翼飞行器方向的研究生教材、宇航短训班教材，是旋翼飞行器结构动力学与气动弹性领域的一本重要的参考书。

本书围绕直升机动力学的基础理论、动力学设计、振动及控制等主题，选编了原著的部分章节对重要过程、重点结论以及相关重点词汇等进行了注释。本书适合我国高等院校航空航天等相关专业的学生和技术人员使用。

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出版说明

适合教学和专业发展的双语教材的编写、引进和出版，是我国高等院校双语教学示范课程建设的重要内容之一。针对目前我国高等院校推广的双语教学课程建设项目，中航出版传媒有限责任公司（航空工业出版社）作为国内航空航天领域领先的专业出版机构，与国内各航空航天院校积极探索，根据各院校的实际教学需求，对国外成熟的、优秀的航空航天教材进行了甄选，形成了独具特色的航空航天类双语版专业教材。其中部分优选出权威的、经典的教材已经在国内部分院校的教学实践中进行了使用，不但获得了教师和学生的肯定，而且取得了业内专家和学者的一致认同。

本套丛书所包含的双语教材均是由从事相关专业教学工作多年的一线教师根据教学实践内容编写、翻译或译注而成的。从出版形式上，既有中英文对照版，又有译注版。在中英文对照版中，教材作者又根据不同的教学安排对原版教材进行了取舍，集中精选了适合教学计划的内容编撰成双语教学精选版。译注版对原版教材中的要点进行了注释，这样可以使得学生在学习过程中更容易理清知识脉络，抓住重点，增加了注释的译注版教材基本保持了英文原版教材的结构和篇幅，不同的是，每章都增加了一部分提炼出来的知识要点。本套丛书所引进的原版教材，多是国外专业教材中的经典作品，被国外多所院校广泛采用，并经多次再版修订。此外，本套丛书基本保留了原版书的量和单位符号，公式中的矢量和标量等也大多沿用了原书的符号系统。

本套丛书的出版是我国航空航天专业教材出版领域的创新之举，得到了国内各航空相关院校的大力支持，由既熟悉原版教材，又具备丰富的双语教学经验和系统专业知识的任课教师担任丛书的作（译）者，他们在繁重的教学工作之余完成了各自书稿的编写和翻译工作，在此对他们的辛勤付出表示感谢！

由于出版工作繁杂，本套丛书难免会有疏漏、差错及不妥之处，敬请读者指正。

译者的话

理查德·L．比拉瓦先生的原著*Rotary Wing Structural Dynamics and Aeroelasticit, Second Edition*是AIAA教育系列丛书之一，曾作为“优秀旋翼技术中心”旋翼飞行器方向的研究生教材、宇航短训班教材，是旋翼飞行器结构动力学与气动弹性领域的一本重要的参考书目。从2010年开始，原著作为南京航空航天大学研究生院飞行器设计专业（直升机方向）以及直升机工程专业的研究生国际化教学的指定教材。

本教材围绕着直升机动力学的基础理论、动力学设计、振动及控制等主题，选编了原著的部分章节作为双语教学的主要内容。在选编过程中，重点对所讨论问题的数学或物理建模及求解方法等的背景与思路、典型问题的讨论过程以及一些重要的指导性、归纳性的结论等进行了注释。公式推演、分析计算等不作为注释的重点，只对其所涉及的关键词汇、专业词汇做了提示性说明。

由于编者水平有限，加之时间仓促，错漏之处在所难免，敬请读者谅解。

译者

2014年2月

目录①

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① 本书为双语教学精选译注版，按从原书中所选章节编排，保留了原版书中的章节序号。——译者注

Contents

1
Introduction 引言

At its basic roots, the subject matter of rotary wing structural dynamics and aeroelasticity is concerned with the following problem: How can structural integrity and passenger comfort be attained in the vibratory environment peculiar to rotary wing aircraft? From this broad problem definition has evolved the two principal areas of concern in rotary wing structural dynamics: *vibrations* and *aeroelastic stability*. The emphasis is currently on vibration reduction, but with close attention being paid, nonetheless, to the fact that rotors are subject to a variety of potentially unstable response phenomena. The advent of new rotor concepts increases the probability of an otherwise benign characteristic rotor response becoming a major aeroelastic instability issue. In any practical design the instabilities must be well understood, and ways must be found to suppress them in all conceivable flight conditions. Last, it should be noted that one area of concern that closely relates to structural dynamics and aeroelasticity is that of noise. The high degree of noise characteristic of most rotorcraft has spawned the growing technical areas of rotor far-field noise *acoustics* and structure-borne interior noise *acoustoelasticity*. Although both these subjects are important and timely, this text is limited to considerations of only vibrations and aeromechanical and aeroelastic instabilities.

structural dynamics：结构动力学
structural integrity：结构完整性
vibratory enviorment：振动环境
rotary wing aircraft：旋转翼飞行器
aeroelastic stability：气动弹性稳定性
vibration reduction：振动削减
far-field noise：远场噪声
interior noise：内部噪声
acoustics：声学
acoustoelasticity：声弹性理论

1.1 Rotary Wing vs Fixed Wing

1.1 旋转翼与固定翼

The agenda for structural dynamics and aeroelasticity of rotorcraft is distinguished from that of fixed wings and/or airborne spacecraft in a variety of ways. First, vibration is a major problem with rotorcraft in all forward-flight conditions, both steady and maneuvering flight. However, this vibration is not broadband; rather, it occurs at discrete frequencies related to the main and tail rotor rotational frequencies. Indeed, the principal driver of the rotorcraft vibration problem is the essentially unsteady aerodynamics of the main rotor *even in steady forward flight*.

Second, not only is the aerodynamic environment of the rotor unsteady, but at the same time it is also nonlinear, governed by non-infinitesimal motion and periodicity in conditions traditionally held constant with fixed wing applications. Presently, the understanding of rotorcraft aerodynamics is a slowly growing technology, and despite its key role in analyzing rotorcraft vibration and aeroelastic stability, it still requires much more work for practical analysis.

In addition to the problems directly arising from unsteady aerodynamics are those related to the fact that the rotor blades of contemporary helicopter rotors

forward-flight condition：前飞状态
steady and maneuvering flight: 稳态飞行和机动飞行
main（tail）rotor：主（尾）旋翼
rotational frequency：旋转频率
unsteady aerodynamics：非定常空气动力学
nonlinear：非线性
periodicity：周期性
rotor blade：旋翼桨叶

rotational centrifugal force field：旋转离心力场
gyroscopic characteristic：陀螺特性
Coriolis force：科里奥利力
inertial loading：惯性载荷
linear operator：线性算子

are "structural lightweights" compared with their fixed wing counterparts. They achieve a large measure of their stiffening from the tension induced by the rotational centrifugal force field. The rotational environment of the rotor blades also gives rise to a host of rotation-related phenomena: gyroscopic characteristics, Coriolis forces, and a variety of nonlinear inertial loadings. While still governed principally by linear operators, the resulting aeroelastic description of rotor blade elastic responses is consequently fraught with nonlinearities that modify the results obtained using only linear analysis.

flexibility：柔性
interaction：交互作用
airframe：机身

The relatively high degree of rotor flexibility also drives the significant interaction occurring between the rotor and the also flexible rotorcraft airframe. At present, the stubbornness of typical modern-day helicopter airframes to yield to accurate dynamic analysis continues to pose a very important challenge to the rotorcraft structural dynamist. Furthermore, contrary to the case of fixed wing aircraft, where aeroelastic stability characteristics are the priority issue, the dynamic analysis of rotorcraft airframes must be relatively much more accurate to enable a reasonably accurate calculation of the vibration characteristics.

1.2 方法论

engineering solution：工程求解
acceptability：可接受性

1.2 Methodology

Stated in the context of an engineering solution, the structural dynamics problem of rotorcraft entails three general concurrent avenues of approach: 1) Knowledge of the vibratory environment of the rotorcraft must be acquired. Principally, this means knowing the essentially unsteady aerodynamic characteristics of rotorcraft. 2) The extent to which the structure responds to the environment must be calculated. 3) The resulting responses must be judged for acceptability; if they are not acceptable, ways must be found to make them so.

1.2.1 对环境的认识

frequency-domain analysis：频域分析
time domain：时域
transient response：瞬态响应
elastomeric device：弹性元件（装置）
composite structure：复合材料结构
coupling：耦合

1.2.1 Knowledge of the Environment

To know the vibratory environment requires that many of the concepts relating to frequency-domain analysis be brought to bear. An understanding of unsteady aerodynamic loadings is required, as are ways of formulating these loadings in both the frequency domain (for both vibration and instability problems) and the time domain (for transient responses involving nonlinearity problems). Also, knowledge of the "real-world" characteristics of the dynamic components used in rotorcraft is a continuing challenge. Elastomeric devices and composites structures are seeing increased usage, and their characteristics are often unique and either deviate to a degree from completely linear descriptions or present new linear coupling relationships.

1.2.2 响应计算

differential equation：微分方程
resonant response：共振响应
unstable response：不稳定响应
once per rotor rev：旋翼每转一次

1.2.2 Calculating the Responses

Calculating the dynamic responses of structures to their environment requires the idealizations that linear differential equations of motion provide. In some cases linear systems are insufficient for an accurate formulation, and non-linear differential equations must be used. In general, two types of responses command our attention almost exclusively: resonant responses and unstable responses. Examples of resonant responses are the once per rotor rev accelerations imparted to a helicopter fuselage by an unbalanced rotor, and the (number of blades) per rev accelerations resulting in the fuselage from a periodic main rotor wake impingement on the horizontal stabilizer. An example of an unstable response is the flutter of a wing or rotor blade. Customarily, a resonant response is ideally thought of as the excitation of a structure at one or more of its natural frequencies by an external energy source, whereas an unstable response is thought of as a structure driving itself from an internal energy source. It is a complication with rotorcraft that, in practice, it is often difficult to ascertain which of these responses is in fact occurring. This is because both types of response can exhibit measured behavior that "blossoms" for periods of time, and furthermore, both types can exhibit limit amplitudes of sinusoidal motion.

fuselage：机体
rotor wake：旋翼尾迹
horizontal stabilizer：水平安定面
flutter：颤振
natural frequency：固有频率
energy source：能量源
amplitude：幅值
sinusoidal motion：正弦运动

1.2.3 Evaluation and Modification

1.2.3 评估和修改

To pass engineering judgment on any given structural configuration, acceptable limits of response must be defined in an appropriate quantitative sense. Such limits are typically defined by the primary considerations of structural integrity and passenger comfort. If the structure is stressed too high, static failure of the material will occur. If the structure is dynamically stressed too high, too often, it fails in a fatigue mode. Furthermore, failure by instability may be precipitated by a partial failure or weakening in a subsidiary structure. Ultimately the structural integrity problem is one of achieving a configuration that is both fail-safe and has an acceptable component life. Whereas the structural problem has an objectively measurable solution (structure either is or is not fail-safe, or has the required x number of hours of minimum useful life), the problem of achieving passenger comfort is measured only relatively. Certainly a reasonable goal is to achieve cabin acceleration levels that are less than those of the last ship built, or, better yet, that are "jet smooth." Present goals are to achieve vibratory levels that are generally less than $0.05\,g$ in all components.

Perhaps of most importance to the engineering judgment aspect of the broad problem is the ability to change the structure to achieve a desired improvement in the dynamic responses of the structure. As yet, this ability is still somewhat of a craft involving educated trial and error. However, this situation is rapidly changing, due to the emergence of a variety of powerful analysis tools and their increasingly more efficient and cost-effective implementations on computers (both large mainframe machines and personal computers).

configuration：构型
quantitative：定量的
passenger comfort：乘员舒适性
static failure：静力破坏
fatigue：疲劳
fail-safe：破损安全
useful life：使用寿命
vibratory level：振动水平
"jet smooth"："像喷气式客机一样平稳"
dynamic response：动响应
trial and error：反复试验

1.2.4 Organization of the Text

1.2.4 本书内容安排

After an initial three chapters covering basic concepts needed for all aspects of rotorcraft structural dynamics, this text is divided into three main sections. The first two sections reflect the emphasis on the two main problem areas: vibration and aeroelastic stability. Chapters 5 through 8 are specifically devoted to vibration issues, with an emphasis on methods for structural modification for achieving vibration reduction. Chapters 9 through 14 are devoted to mechanical, aeromechanical, and aeroelastic stability issues. In addition to presenting expositions of well-established instability phenomena of rotors, techniques are presented for analyzing the stability of general multiple degree-of-freedom systems. For both of

structural modification：结构修改
mechanical：机械的
aeromechanical：气动机械的
multiple degress-of-freedom system：多自由度系统

these problem areas, i.e., vibration and aeroelastic stability, chapters are presented dealing with appropriate experimental procedures.

blade section：桨叶剖面

The third section of the text consists of Chapters 15 through 18, which deal with material that has special applications to both of the two main areas. Chapter 15 presents material relating to elastomeric devices, as these have found a permanent niche in rotorcraft structural dynamics applications. Chapter 16 presents specifics for characterizing the blade section with an emphasis on the application of composite materials. Chapter 17 describes instances of cross-over interactions of "outside" dynamics-related material with the more standard material of rotorcraft structural dynamics covered in the previous chapters. Finally, Chapter 18 presents some concluding thoughts together with an abbreviated description of key milestones in the development of rotary wing structural dynamics and aeroelasticity.

It is to be hoped that this text will provide an in-depth introduction to the subject material as well as a useful reference resource both for actual formulations and bibliographies. The theory of linear differential equations is the primary requisite mathematical discipline employed in this text; most of the analytical formulations involve only the integrating of a handful of basic, relatively simple concepts.

3

Rotating Beams 旋转梁

Because of the aerodynamic performance gains achievable, current helicopter rotor blades have evolved into relatively high-aspect-ratio structures. From a structural dynamics standpoint such a structure is a simplification in that rotor blades can then be treated as one dimensional, that is, their elastic characteristics can be specified as functions of only the radius. A further consequence of this one dimensionality is that we need define the elastic deflections of rotor blades only in terms of bending deformations transverse to the blade, torsion, and, in some special cases, axial elongation and cross-sectional warping.

In this chapter the basic concepts relating to blade vibration characteristics are developed. These concepts include theory for and practical schemes for obtaining natural frequencies and mode shapes in both in-plane and out-of-plane bending and in torsion for relatively simple blades, that is, blades that have no pitch, twist, or preconing. Such simplified blade configurations produce what we will refer to as *uncoupled mode* characteristics. The uncoupled modal responses are characterized as having, for any one mode, *all* of the motion in either the in-plane, out-of-plane, or torsional degrees of freedom. In reality, rotor blades do have all of these complicating characteristics (pitch, twist, and precone), and material is therefore presented explaining how the basic uncoupled characteristics for the simplified blade are modified or coupled by these additional considerations.

high-aspect-ratio structure：高展弦比结构

从结构动力学的观点来看，旋翼桨叶可以简化为一维结构来处理。

桨叶的弹性变形可以只用弯曲、扭转变形来表达，在某些特殊情况下，也包括拉伸变形和剖面的翘曲变形。

mode shape：振型
in-plane/out-of-plane bending：面内 / 面外弯曲
pitch：变距

twist：扭转
precone：预锥
uncoupled mode：不耦合模态

3.1 Basic Equations for Bending

3.1 弯曲基本方程

The principal characteristics of rotating beams that distinguish them from conventional beams are that they function in a tension field that is variable with span and that they have additional dynamic loadings accruing from the rotational field. Because of the one dimensionality of the beam, the appropriate starting point is to take an infinitesimal spanwise element of the beam and then form the equilibrium conditions governing it.

旋转梁区别于一般梁的主要特征是其工作在沿展向变化的离心力场中。

取一无限小的展向微段，建立平衡。

3.1.1 Equilibrium of a Spanwise Element

3.1.1 展向微段平衡方程

Figure 3.1 shows a (rotating) cantilevered beam with a distributed axial loading $f_x(x)$ [arising from centrifugal force and resulting in distributed tension $T(x)$] and a distributed transverse loading $f_z(x,t)$.

cantilevered beam：悬臂梁
a distributed axial loading：轴向分布载荷

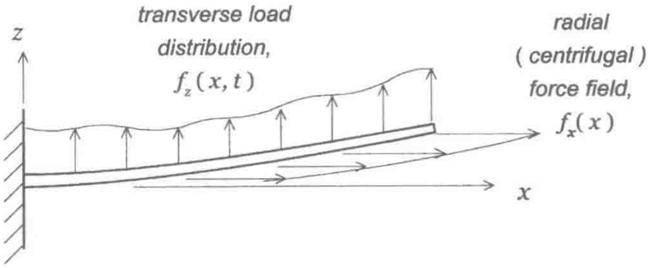

Fig. 3.1 Loading distributions of a rotating cantilevered beam.

a free-body diagram：分离体图（见图 3.2）
分离体关于轴向和横向取力和力矩平衡，得式（3.1）。

Let us then consider an arbitrarily located spanwise element and construct a free-body diagram (see Fig. 3.2); note that we include the internal elastic loads and the external loadings, as defined in Fig. 3.1. The element is then equilibrated with respect to the axial and transverse forces and the moments:

$$\Sigma F_x = 0; \quad \Sigma F_z = 0; \quad \Sigma M = 0 \tag{3.1}$$

3.1.2 横向弯曲微分方程

3.1.2 Basic Differential Equation for Transverse Bending

The aforementioned conditions of equilibrium, when taken together with the standard beam bending equation

$$M(x,t) = EI \frac{\partial^2}{\partial x^2} z(x,t) \tag{3.2}$$

受拉梁的横向弯曲微分方程，式（3.3）。

yield the following basic differential equation for the beam in tension:

$$\frac{\partial^2}{\partial x^2}\left[EI \frac{\partial^2 z}{\partial x^2}\right] - \frac{\partial}{\partial x}\left[T \frac{\partial z}{\partial x}\right] = f_z(x,t) \tag{3.3}$$

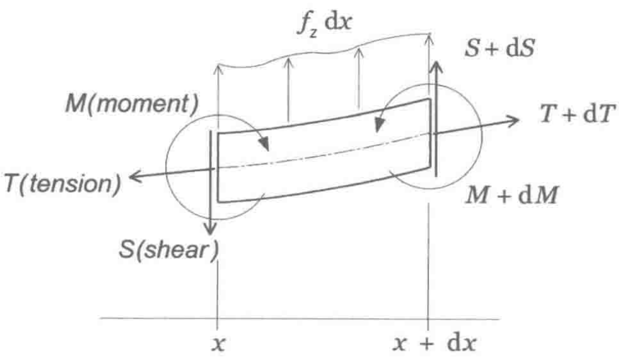

Fig. 3.2 Free-body diagram for a spanwise element of a rotating beam.

Boundary conditions. The appropriate boundary conditions for all rotating beams are that they have zero deflection at the root and zero bending moment and shear at the tip. These conditions are stated mathematically as

旋转悬臂梁的边界条件。
根部变形为零，尖部剪力和弯矩为零。

$$z(0,t) = \frac{\partial^2 z}{\partial x^2}(L,t) = \frac{\partial^3 z}{\partial x^3}(L,t) = 0 \tag{3.4}$$

Additionally, a fourth boundary condition must be imposed at the beam root based on the type of fixity at that point. If the beam represents an *articulated rotor blade*, then the beam has the boundary condition appropriate to a hinge, and the bending moment is taken to be zero:

根据桨叶根部的连接类型，需要附加第四个边界条件。
articulated rotor blade：铰接式旋翼桨叶
根部弯矩为零，式（3.5）

$$\frac{\partial^2 z}{\partial x^2}(0,t) = 0 \tag{3.5}$$

If the beam represents a *hingeless rotor blade*, then the beam root has the cantilevered boundary condition, and the root slope is taken to be a constant equal to some built-in coning value β_B:

hingeless rotor blade：无铰式旋翼桨叶
built-in coning：预锥角
根部转角等于预锥角，式（3.6）

$$\frac{\partial z}{\partial x}(0,t) = \beta_B \tag{3.6}$$

Distributed tension. The tension in the beam arising from the centrifugal force field can be obtained by integrating the axial load distribution $f_x (= m\Omega^2 x)$:

离心力场引起的分布拉力。

$$T(x,t) = \Omega^2 \int_x^L m x_1 \, \mathrm{d}x_1 \tag{3.7}$$

3.1.3 Transverse Bending Motion

3.1.3 横向弯曲运动

At this point none of the development given earlier takes account of the fact that the beam is in (transverse) motion. The dynamics of this motion can then be simulated using D'Alembert's principle; the transverse inertial load distribution resulting from beam transverse motion is included explicitly in f_z.

D'Alembert's principle：达朗贝尔原理

Transverse load distribution. Regarding the out-of-plane z motion, the transverse load distribution is also in the z direction:

$$f_z(x,t) = f_z(x,t)_{\substack{\text{applied}\\ \text{external}}} + f_z(x,t)_{\text{inertial}} \tag{3.8}$$

面外 z 方向的横向分布载荷，式（3.8）

Correspondingly, the transverse load distribution for the in-plane motion case is in the y direction and is similarly interpreted. Thus, for out-of-plane and in-plane motions z and y, respectively, the transverse loadings are given as follows:

Out-of-plane bending:

$$f_z(x,t)_{\text{inertial}} = -m\frac{\partial^2 z}{\partial t^2} \tag{3.9a}$$

面外弯曲运动的惯性载荷，式（3.9a）

In-plane bending:

面内弯曲运动的惯性载荷，式（3.9b）

$$f_y(x,t)_{\text{inertial}} = -m\frac{\partial^2 y}{\partial t^2} + m\Omega^2 y \tag{3.9b}$$

离心力在面内 y 方向的分量引起式（3.9b）中的第二项。

The second term in the expression for the in-plane inertial load distribution arises from the fact that the centrifugal force field is radial; hence, a component of this force field will be in the direction of the in-plane deformation y.

3.1.4 面外弯曲运动的微分方程

请读者自己验证如何从前述公式得到式（3.10）。

3.1.4 Differential Equation for Out-of-Plane Bending

The reader can verify that all of the preceding formulations can then be combined to yield the following differential equation for out-of-plane (transverse) bending:

$$m\ddot{z} + [EIz'']'' - \Omega^2\left[z'\int_x^L mx_1\,\mathrm{d}x_1\right]' = f_{\text{app}}(x,t) \tag{3.10}$$

面外弯曲运动方程具有两个特性：
(1) 带展向变化系数是线性方程，关于空间变量 x 四阶，关于时间 t 二阶；
(2) 该方程可以采用分离变量法求解。

This equation can be seen to have the following properties:

1) The equation is linear, of fourth order in the spatial variable x, second order in time t, and *with spanwise variable coefficients*.
2) The linearity of the equation allows a separation of variables solution scheme combined with superposition:

$$z(x,t) = \sum_{j=1}^{\infty} \gamma_j(x)q_j(t) \tag{3.11}$$

natural mode shape：固有模态振型
generalized coordinate：广义坐标

where $\gamma_j(x)$ is the jth *natural mode shape* (which satisfies the boundary conditions) and $q_j(t)$ is the jth *generalized coordinate* or modal response variable.

One mathematical significance of the $\gamma_j(x)$ function is that it satisfies the following *orthogonality condition*:

正交性条件，式（3.12）

$$\int_0^L m\gamma_j\gamma_k\,\mathrm{d}x = \begin{cases} M_j, & \text{for } k = j \\ 0, & \text{for } k \neq j \end{cases} \tag{3.12}$$

normal mode shape (eigenvector)：正则模态振型（特征矢量）
即使是带常系数的线性方程，也需要采用数值方法求解其固有频率和振型。
free-vibration (eigenvalue) problem：自由振动（特征值）问题

Calculation of the normal mode shapes γ_j (eigenvectors) and the corresponding natural frequencies ω_j (eigenvalues) defines the major computational task in analyzing rotating beams. Unfortunately, although the basic differential equation is linear, general solutions do not exist even for spanwise constant properties, and some form of numerical method must be employed. Two basic numerical methods are presented in this chapter for obtaining these calculations, but first the basic properties of the resulting solution for the free-vibration (eigenvalue) problem must be addressed.