Igor R. Shafarevich

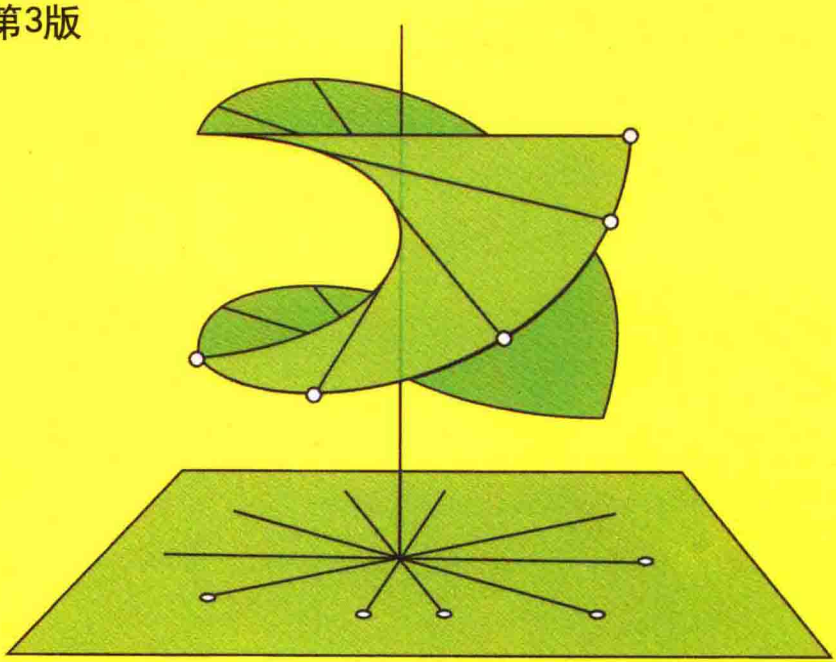
Basic Algebraic Geometry 1

Varieties in Projective Space

Third Edition

基础代数几何 第1卷

第3版



Springer

世界图书出版公司
www.wpcbj.com.cn

Igor R. Shafarevich

Basic Algebraic Geometry 1

Varieties in Projective Space

Third Edition

Springer

图书在版编目（CIP）数据

基础代数几何．第1卷：第3版 = Basic Algebraic Geometry 1 : Varieties in Projective Space Third Edition : 英文 /（俄罗斯）I. R. 沙法列维奇（I. R. Shafarevich）著．—影印本．—北京：世界图书出版公司北京公司，2016.11
ISBN 978-7-5192-2070-9

Ⅰ．①基… Ⅱ．①沙… Ⅲ．①代数几何—英文 Ⅳ．① O187

中国版本图书馆 CIP 数据核字（2016）第 271699 号

著　　者： Igor R. Shafarevich
责任编辑： 刘　慧　高　蓉
装帧设计： 任志远

出版发行： 世界图书出版公司北京公司
地　　址： 北京市东城区朝内大街 137 号
邮　　编： 100010
电　　话： 010-64038355（发行）　64015580（客服）　64033507（总编室）
网　　址： http://www.wpcbj.com.cn
邮　　箱： wpcbjst@vip.163.com
销　　售： 新华书店
印　　刷： 三河市国英印务有限公司
开　　本： 711mm × 1245mm　1/24
印　　张： 14
字　　数： 269 千
版　　次： 2017 年 1 月第 1 版　　2017 年 1 月第 1 次印刷
版权登记： 01-2016-6782
定　　价： 55.00 元

版权所有　翻印必究

（如发现印装质量问题，请与所购图书销售部门联系调换）

Basic Algebraic Geometry 1

Igor R. Shafarevich
Algebra Section
Steklov Mathematical Institute
of the Russian Academy of Sciences
Moscow, Russia

Translator
Miles Reid
Mathematics Institute
University of Warwick
Coventry, UK

ISBN 978-3-642-37955-0 ISBN 978-3-642-37956-7 (eBook)
DOI 10.1007/978-3-642-37956-7
Springer Heidelberg New York Dordrecht London

Library of Congress Control Number: 2013945284

Mathematics Subject Classification (2010): 14-01

Translation of the 3rd Russian edition entitled "Osnovy algebraicheskoj geometrii". MCCME, Moscow 2007, originally published in Russian in one volume

© Springer-Verlag Berlin Heidelberg 1977, 1994, 2013
This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed. Exempted from this legal reservation are brief excerpts in connection with reviews or scholarly analysis or material supplied specifically for the purpose of being entered and executed on a computer system, for exclusive use by the purchaser of the work. Duplication of this publication or parts thereof is permitted only under the provisions of the Copyright Law of the Publisher's location, in its current version, and permission for use must always be obtained from Springer. Permissions for use may be obtained through RightsLink at the Copyright Clearance Center. Violations are liable to prosecution under the respective Copyright Law.
The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.
While the advice and information in this book are believed to be true and accurate at the date of publication, neither the authors nor the editors nor the publisher can accept any legal responsibility for any errors or omissions that may be made. The publisher makes no warranty, express or implied, with respect to the material contained herein.

Reprint from English language edition:
Basic Algebraic Geometry 1: Varieties in Projective Space Third Edition
by Igor R. Shafarevich
Copyright © Springer Berlin Heidelberg 1997, 1994, 2013
Springer Berlin Heidelberg is a part of Springer Science+Business Media
All Rights Reserved

This reprint has been authorized by Springer Science & Business Media for distribution in China Mainland only and not for export therefrom.

试读结束：需要全本请在线购买： www.ertongbook.com

Preface

The third edition differs from the previous two in some fairly minor corrections and a number of additions. Both of these are based on remarks and advice from readers of the earlier editions. The late B.G. Moishezon worked as editor on the first edition, and the text reflects his advice and a number of his suggestions. I was equally fortunate with the editor of the second edition, V.L. Popov, to whom I am grateful for a careful and thoughtful reading of the text. In addition to this, both the first and the second edition were translated into English, and the publisher Springer-Verlag provided me with a number of remarks from Western mathematicians on the translation of the first edition. In particular the translator of the second edition, M. Reid, contributed some improvements with his careful reading of the text. Other mathematicians who helped me in writing the book are mentioned in the preface to the first two editions. I could add a few more names, especially V.G. Drinfeld and A.N. Parshin.

The most substantial addition in the third edition is the proof of the Riemann–Roch theorem for curves, which was merely stated in previous editions. This is a fundamental result of the theory of algebraic curves, having many applications; however, none of the known proofs are entirely straightforward. Following Parshin's suggestion, I have based myself on the proof contained in Tate's work; as Tate wrote in the preface, this proof is a result of his and Mumford's efforts to adapt the general theory of Grothendieck residues to the one dimensional case. An attractive feature of this approach is that all the required properties of residues of differential follow from unified considerations.

This book is a general introduction to algebraic geometry. Its aim is a treatment of the subject as a whole, including the widest possible spectrum of topics. To judge by comments from readers, this is how the previous editions were received. The reader wishing to get into more specialised areas may benefit from the books and articles listed in the bibliography at the end. A number of publications reflecting the most recent achievements in the subject are mentioned in this edition.

From the Preface to the Second Edition (1988)

The first edition of this book came out just as the apparatus of algebraic geometry was reaching a stage that permitted a lucid and concise account of the foundations of the subject. The author was no longer forced into the painful choice between sacrificing rigour of exposition or overloading the clear geometrical picture with cumbersome algebraic apparatus.

The 15 years that have elapsed since the first edition have seen the appearance of many beautiful books treating various branches of algebraic geometry. However, as far as I know, no other author has been attracted to the aim which this book set itself: to give an overall view of the many varied aspects of algebraic geometry, without going too far afield into the different theories. There is thus scope for a second edition. In preparing this, I have included some additional material, rather varied in nature, and have made some small cuts, but the general character of the book remains unchanged.

The three parts of the book now appear as two separate volumes. Book 1 corresponds to Part I, Chapters 1–4, of the first edition. Here quite a lot of material of a rather concrete geometric nature has been added: the first section, forming a bridge between coordinate geometry and the theory of algebraic curves in the plane, has been substantially expanded. More space has been given over to concrete algebraic varieties: Grassmannian varieties, plane cubic curves and the cubic surface. The main role that singularities played in the first edition was in giving rigorous definition to situations we wished to avoid. The present edition treats a number of questions related to degenerate fibres in families: degenerations of quadrics and of elliptic curves, the Bertini theorems. We discuss the notion of infinitely near points of algebraic curves on surfaces and normal surface singularities. Finally, some applications to number theory have been added: the zeta function of algebraic varieties over a finite field and the analogue of the Riemann hypothesis for elliptic curves.

Books 2 and 3 corresponds to Parts II and III, Chapters 5–9 of the first edition. They treat the foundations of the theory of schemes, abstract algebraic varieties and algebraic manifolds over the complex number field. As in the Book 1 there are a number of additions to the text. Of these, the following are the two most important. The first is a discussion of the notion of moduli spaces, that is, algebraic varieties that classify algebraic or geometric objects of some type; as an example we work out the theory of the Hilbert polynomial and the Hilbert scheme. I am very grateful to V.I. Danilov for a series of recommendations on this subject. In particular the proof of Theorem 6.7 of Section 4.3, Chapter 6, is due to him. The second addition is the definition and basic properties of a Kähler metric and a description (without proof) of Hodge's theorem.

For the most part, this material is taken from my old lectures and seminars, from notes provided by members of the audience. A number of improvements of proofs have been borrowed from the books of Mumford and Fulton. A whole series of misprints and inaccuracies in the first edition were pointed out by readers, and by readers of the English translation. Especially valuable was the advice of Andrei Tyurin and Viktor Kulikov; in particular, the proof of Theorem 4.13 was provided by Kulikov. I offer sincere thanks to all these.

Many substantial improvements are due to V.L. Popov, who edited the second edition, and I am very grateful to him for all the work and thought he has put into the book. I have the pleasure, not for the first time, of expressing my deep gratitude to the translator of this book, Miles Reid. His thoughtful work has made it possible to patch up many uneven places and inaccuracies, and to correct a few mathematical errors.

From the Preface to the First Edition (1972)

Algebraic geometry played a central role in 19th century math. The deepest results of Abel, Riemann, Weierstrass, and many of the most important works of Klein and Poincaré were part of this subject.

The turn of the 20th century saw a sharp change in attitude to algebraic geometry. In the 1910s Klein[1] writes as follows: "In my student days, under the influence of the Jacobi tradition, Abelian functions were considered as the unarguable pinnacle of math. Every one of us felt the natural ambition to make some independent progress in this field. And now? The younger generation scarcely knows what Abelian functions are." (From the modern viewpoint, the theory of Abelian functions is an analytic aspect of the theory of Abelian varieties, that is, projective algebraic group varieties; compare the historical sketch.)

Algebraic geometry had become set in a way of thinking too far removed from the set-theoretic and axiomatic spirit that determined the development of math at the time. It was to take several decades, during which the theories of topological, differentiable and complex manifolds, of general fields, and of ideals in sufficiently general rings were developed, before it became possible to construct algebraic geometry on the basis of the principles of set-theoretic math.

Towards the middle of the 20th century algebraic geometry had to a large extent been through such a reconstruction. Because of this, it could again claim the place it had once occupied in math. The domain of application of its ideas had grown tremendously, both in the direction of algebraic varieties over arbitrary fields and of more general complex manifolds. Many of the best achievements of algebraic geometry could be cleared of the accusation of incomprehensibility or lack of rigour.

The foundation for this reconstruction was algebra. In its first versions, the use of precise algebraic apparatus often led to a loss of the brilliant geometric style characteristic of the preceding period. However, the 1950s and 60s have brought substantial simplifications to the foundation of algebraic geometry, which have allowed us to come significantly closer to the ideal combination of logical transparency and geometric intuition.

The purpose of this book is to treat the foundations of algebraic geometry across a fairly wide front, giving an overall account of the subject, and preparing the ground

---

[1] Klein, F.: Vorlesungen über die Entwicklung der Mathematik im 19. Jahrhundert, Grundlehren Math. Wiss. 24, Springer-Verlag, Berlin 1926. Jrb. 52, 22, p. 312.

for a study of the more specialised literature. No prior knowledge of algebraic geometry is assumed on the part of the reader, neither general theorems, nor concrete examples. Therefore along with development of the general theory, a lot of space is devoted to applications and particular cases, intended to motivate new ideas or new ways of formulating questions.

It seems to me that, in the spirit of the biogenetic law, the student who repeats in miniature the evolution of algebraic geometry will grasp the logic of the subject more clearly. Thus, for example, the first section is concerned with very simple properties of algebraic plane curves. Similarly, Part I of the book considers only algebraic varieties in an ambient projective space, and the reader only meets schemes and the general notion of a variety in Part II.

Part III treats algebraic varieties over the complex number field, and their relation to complex analytic manifolds. This section assumes some acquaintance with basic topology and the theory of analytic functions.

I am extremely grateful to everyone whose advice helped me with this book. It is based on lecture notes from several courses I gave in Moscow University. Many participants in the lectures or readers of the notes have provided me with useful remarks. I am especially indebted to the editor B.G. Moishezon for a large number of discussions which were very useful to me. A series of proofs contained in the book are based on his advice.

Prerequisites

The nature of the book requires the algebraic apparatus to be kept to a minimum. In addition to an undergraduate algebra course, we assume known basic material from field theory: finite and transcendental extensions (but not Galois theory), and from ring theory: ideals and quotient rings. In a number of isolated instances we refer to the literature on algebra; these references are chosen so that the reader can understand the relevant point, independently of the preceding parts of the book being referred to. Somewhat more specialised algebraic questions are collected together in the Algebraic Appendix at the end of Book 1.

Recommendations for Further Reading

For the reader wishing to go further in the study of algebraic geometry, we can recommend the following references.

For the cohomology of algebraic coherent sheaves and their applications: see Hartshorne [37].

An elementary proof of the Riemann–Roch theorem for curves is given in W. Fulton, Algebraic curves. An introduction to algebraic geometry, W.A. Benjamin, Inc., New York–Amsterdam, 1969. This book is available as a free download from http://www.math.lsa.umich.edu/~wfulton/CurveBook.pdf.

For the general case of Riemann–Roch, see A. Borel and J.-P. Serre, Le théorème de Riemann–Roch, Bull. Soc. Math. France **86** (1958) 97–136,

Yu.I. Manin, Lectures on the K-functor in algebraic geometry, Uspehi Mat. Nauk **24**:5 (149) (1969) 3–86, English translation: Russian Math. Surveys **24**:5 (1969) 1–89,

W. Fulton and S. Lang, Riemann–Roch algebra, Grundlehren der mathematischen Wissenschaften **277**, Springer-Verlag, New York, 1985.

Moscow, Russia I.R. Shafarevich

Translator's Note

Shafarevich's book is the fruit of lecture courses at Moscow State University in the 1960s and early 1970s. The style of Russian mathematical writing of the period is very much in evidence. The book does not aim to cover a huge volume of material in the maximal generality and rigour, but gives instead a well-considered choice of topics, with a human-oriented discussion of the motivation and the ideas, and some sample results (including a good number of hard theorems with complete proofs). In view of the difficulty of keeping up with developments in algebraic geometry during the 1960s, and the extraordinary difficulties faced by Soviet mathematicians of that period, the book is a tremendous achievement.

The student who wants to get through the technical material of algebraic geometry quickly and at full strength should perhaps turn to Hartshorne's book [37]; however, my experience is that some graduate students (by no means all) can work hard for a year or two on Chapters 2–3 of Hartshorne, and still know more-or-less nothing at the end of it. For many students, it's just not feasible both to do the research for a Ph. D. thesis and to master all the technical foundations of algebraic geometry at the same time. In any case, even if you have mastered everything in scheme theory, your research may well take you into number theory or differential geometry or representation theory or math physics, and you'll have just as many new technical things to learn there. For all such students, and for the many specialists in other branches of math who need a liberal education in algebraic geometry, Shafarevich's book is a must.

The previous English translation by the late Prof. Kurt Hirsch has been used with great profit by many students over the last two decades. In preparing the new translation of the revised edition, in addition to correcting a few typographical errors and putting the references into English alphabetical order, I have attempted to put Shafarevich's text into the language used by the present generation of English-speaking algebraic geometers. I have in a few cases corrected the Russian text, or even made some fairly arbitrary changes when the original was already perfectly all right, in most case with the author's explicit or implicit approval. The footnotes are all mine: they are mainly pedantic in nature, either concerned with minor points of terminology, or giving references for proofs not found in the main text; my references do not necessarily follow Shafarevich's ground-rule of being a few pages

accessible to the general reader, without obliging him or her to read a whole book, and so may not be very useful to the beginning graduate student. It's actually quite demoralising to realise just how difficult or obscure the literature can be on some of these points, at the same time as many of the easier points are covered in any number of textbooks. For example: (1) the "principle of conservation of number" (algebraic equivalence implies numerical equivalence); (2) the Néron–Severi theorem (stated as Theorem D); (3) a punctured neighbourhood of a singular point of a normal variety over $\mathbb{C}$ is connected; (4) Chevalley's theorem that every algebraic group is an extension of an Abelian variety by an affine (linear) group. A practical solution for the reader is to take the statements on trust for the time being.

The two volumes have a common index and list of references, but only the second volume has the references for the historical sketch.

试读结束：需要全本请在线购买： www.ertongbook.com

Contents

Book 1: Varieties in Projective Space

1 Basic Notions 3

1 Algebraic Curves in the Plane 3

1.1 Plane Curves 3

1.2 Rational Curves 6

1.3 Relation with Field Theory 9

1.4 Rational Maps 11

1.5 Singular and Nonsingular Points 13

1.6 The Projective Plane 17

1.7 Exercises to Section 1 22

2 Closed Subsets of Affine Space 23

2.1 Definition of Closed Subsets 23

2.2 Regular Functions on a Closed Subset 25

2.3 Regular Maps 27

2.4 Exercises to Section 2 32

3 Rational Functions 34

3.1 Irreducible Algebraic Subsets 34

3.2 Rational Functions 36

3.3 Rational Maps 37

3.4 Exercises to Section 3 40

4 Quasiprojective Varieties 41

4.1 Closed Subsets of Projective Space 41

4.2 Regular Functions 46

4.3 Rational Functions 50

4.4 Examples of Regular Maps 52

4.5 Exercises to Section 4 53

5 Products and Maps of Quasiprojective Varieties 54

5.1 Products 54

5.2 The Image of a Projective Variety is Closed 57

5.3 Finite Maps 60

5.4 Noether Normalisation . . . 65
5.5 Exercises to Section 5 . . . 65
6 Dimension . . . 66
6.1 Definition of Dimension . . . 66
6.2 Dimension of Intersection with a Hypersurface . . . 69
6.3 The Theorem on the Dimension of Fibres . . . 75
6.4 Lines on Surfaces . . . 77
6.5 Exercises to Section 6 . . . 80

2 Local Properties . . . 83
1 Singular and Nonsingular Points . . . 83
1.1 The Local Ring of a Point . . . 83
1.2 The Tangent Space . . . 85
1.3 Intrinsic Nature of the Tangent Space . . . 86
1.4 Singular Points . . . 92
1.5 The Tangent Cone . . . 94
1.6 Exercises to Section 1 . . . 95
2 Power Series Expansions . . . 98
2.1 Local Parameters at a Point . . . 98
2.2 Power Series Expansions . . . 100
2.3 Varieties over the Reals and the Complexes . . . 104
2.4 Exercises to Section 2 . . . 106
3 Properties of Nonsingular Points . . . 106
3.1 Codimension 1 Subvarieties . . . 106
3.2 Nonsingular Subvarieties . . . 110
3.3 Exercises to Section 3 . . . 111
4 The Structure of Birational Maps . . . 113
4.1 Blowup in Projective Space . . . 113
4.2 Local Blowup . . . 115
4.3 Behaviour of a Subvariety Under a Blowup . . . 117
4.4 Exceptional Subvarieties . . . 119
4.5 Isomorphism and Birational Equivalence . . . 120
4.6 Exercises to Section 4 . . . 123
5 Normal Varieties . . . 124
5.1 Normal Varieties . . . 124
5.2 Normalisation of an Affine Variety . . . 128
5.3 Normalisation of a Curve . . . 130
5.4 Projective Embedding of Nonsingular Varieties . . . 134
5.5 Exercises to Section 5 . . . 136
6 Singularities of a Map . . . 137
6.1 Irreducibility . . . 137
6.2 Nonsingularity . . . 139
6.3 Ramification . . . 140
6.4 Examples . . . 143
6.5 Exercises to Section 6 . . . 146

3 Divisors and Differential Forms 147
1 Divisors 147
1.1 The Divisor of a Function 147
1.2 Locally Principal Divisors 151
1.3 Moving the Support of a Divisor away from a Point 153
1.4 Divisors and Rational Maps 155
1.5 The Linear System of a Divisor 156
1.6 Pencil of Conics over $\mathbb{P}^1$ 159
1.7 Exercises to Section 1 161
2 Divisors on Curves 163
2.1 The Degree of a Divisor on a Curve 163
2.2 Bézout's Theorem on a Curve 167
2.3 The Dimension of a Divisor 168
2.4 Exercises to Section 2 169
3 The Plane Cubic 170
3.1 The Class Group 170
3.2 The Group Law 173
3.3 Maps 177
3.4 Applications 179
3.5 Algebraically Nonclosed Field 181
3.6 Exercises to Section 3 183
4 Algebraic Groups 184
4.1 Algebraic Groups 184
4.2 Quotient Groups and Chevalley's Theorem 185
4.3 Abelian Varieties 186
4.4 The Picard Variety 188
4.5 Exercises to Section 4 189
5 Differential Forms 190
5.1 Regular Differential 1-Forms 190
5.2 Algebraic Definition of the Module of Differentials 193
5.3 Differential p-Forms 195
5.4 Rational Differential Forms 197
5.5 Exercises to Section 5 199
6 Examples and Applications of Differential Forms 200
6.1 Behaviour Under Maps 200
6.2 Invariant Differential Forms on a Group 202
6.3 The Canonical Class 204
6.4 Hypersurfaces 206
6.5 Hyperelliptic Curves 209
7 The Riemann–Roch Theorem on Curves 210
7.1 Statement of the Theorem 210
7.2 Preliminary Form of the Riemann–Roch Theorem 213
7.3 The Residue of a 1-Form 217
7.4 Linear Algebra in Infinite Dimensional Vector Spaces . . . 219
7.5 The Residue Theorem 224

7.6 The Duality Theorem . . . 225
7.7 Exercises to Sections 6–7 . . . 227
8 Higher Dimensional Generalisations . . . 229

4 Intersection Numbers . . . 233
1 Definition and Basic Properties . . . 233
1.1 Definition of Intersection Number . . . 233
1.2 Additivity . . . 236
1.3 Invariance Under Linear Equivalence . . . 238
1.4 The General Definition of Intersection Number . . . 242
1.5 Exercises to Section 1 . . . 245
2 Applications of Intersection Numbers . . . 246
2.1 Bézout's Theorem in Projective and Multiprojective Space . 246
2.2 Varieties over the Reals . . . 248
2.3 The Genus of a Nonsingular Curve on a Surface . . . 251
2.4 The Riemann–Roch Inequality on a Surface . . . 253
2.5 The Nonsingular Cubic Surface . . . 255
2.6 The Ring of Cycle Classes . . . 258
2.7 Exercises to Section 2 . . . 259
3 Birational Maps of Surfaces . . . 260
3.1 Blowups of Surfaces . . . 260
3.2 Some Intersection Numbers . . . 261
3.3 Resolution of Indeterminacy . . . 263
3.4 Factorisation as a Chain of Blowups . . . 264
3.5 Remarks and Examples . . . 267
3.6 Exercises to Section 3 . . . 269
4 Singularities . . . 270
4.1 Singular Points of a Curve . . . 270
4.2 Surface Singularities . . . 273
4.3 Du Val Singularities . . . 274
4.4 Degeneration of Curves . . . 278
4.5 Exercises to Section 4 . . . 281

Algebraic Appendix . . . 283
1 Linear and Bilinear Algebra . . . 283
2 Polynomials . . . 285
3 Quasilinear Maps . . . 285
4 Invariants . . . 287
5 Fields . . . 288
6 Commutative Rings . . . 289
7 Unique Factorisation . . . 292
8 Integral Elements . . . 293
9 Length of a Module . . . 294

References . . . 297

Index . . . 301

Book 2: Schemes and Varieties

5 Schemes . . . 3
1 The Spec of a Ring . . . 5
1.1 Definition of Spec A . . . 5
1.2 Properties of Points of Spec A . . . 7
1.3 The Zariski Topology of Spec A . . . 9
1.4 Irreducibility, Dimension . . . 11
1.5 Exercises to Section 1 . . . 14
2 Sheaves . . . 15
2.1 Presheaves . . . 15
2.2 The Structure Presheaf . . . 17
2.3 Sheaves . . . 19
2.4 Stalks of a Sheaf . . . 23
2.5 Exercises to Section 2 . . . 24
3 Schemes . . . 25
3.1 Definition of a Scheme . . . 25
3.2 Glueing Schemes . . . 30
3.3 Closed Subschemes . . . 32
3.4 Reduced Schemes and Nilpotents . . . 35
3.5 Finiteness Conditions . . . 36
3.6 Exercises to Section 3 . . . 38
4 Products of Schemes . . . 40
4.1 Definition of Product . . . 40
4.2 Group Schemes . . . 42
4.3 Separatedness . . . 43
4.4 Exercises to Section 4 . . . 46

6 Varieties . . . 49
1 Definitions and Examples . . . 49
1.1 Definitions . . . 49
1.2 Vector Bundles . . . 53
1.3 Vector Bundles and Sheaves . . . 56
1.4 Divisors and Line Bundles . . . 63
1.5 Exercises to Section 1 . . . 67
2 Abstract and Quasiprojective Varieties . . . 68
2.1 Chow's Lemma . . . 68
2.2 Blowup Along a Subvariety . . . 70
2.3 Example of Non-quasiprojective Variety . . . 74
2.4 Criterions for Projectivity . . . 79
2.5 Exercises to Section 2 . . . 81
3 Coherent Sheaves . . . 81
3.1 Sheaves of $\mathcal{O}_X$-Modules . . . 81
3.2 Coherent Sheaves . . . 85
3.3 Dévissage of Coherent Sheaves . . . 88
3.4 The Finiteness Theorem . . . 92

3.5 Exercises to Section 3 93
4 Classification of Geometric Objects and Universal Schemes 94
4.1 Schemes and Functors 94
4.2 The Hilbert Polynomial 100
4.3 Flat Families 103
4.4 The Hilbert Scheme 107
4.5 Exercises to Section 4 110

Book 3: Complex Algebraic Varieties and Complex Manifolds

7 The Topology of Algebraic Varieties 115
1 The Complex Topology 115
1.1 Definitions 115
1.2 Algebraic Varieties as Differentiable Manifolds; Orientation 117
1.3 Homology of Nonsingular Projective Varieties 118
1.4 Exercises to Section 1 121
2 Connectedness 121
2.1 Preliminary Lemmas 121
2.2 The First Proof of the Main Theorem 122
2.3 The Second Proof 124
2.4 Analytic Lemmas 126
2.5 Connectedness of Fibres 127
2.6 Exercises to Section 2 128
3 The Topology of Algebraic Curves 129
3.1 Local Structure of Morphisms 129
3.2 Triangulation of Curves 131
3.3 Topological Classification of Curves 133
3.4 Combinatorial Classification of Surfaces 137
3.5 The Topology of Singularities of Plane Curves 140
3.6 Exercises to Section 3 142
4 Real Algebraic Curves 142
4.1 Complex Conjugation 143
4.2 Proof of Harnack's Theorem 144
4.3 Ovals of Real Curves 146
4.4 Exercises to Section 4 147

8 Complex Manifolds 149
1 Definitions and Examples 149
1.1 Definition 149
1.2 Quotient Spaces 152
1.3 Commutative Algebraic Groups as Quotient Spaces 155
1.4 Examples of Compact Complex Manifolds not Isomorphic to Algebraic Varieties 157
1.5 Complex Spaces 163
1.6 Exercises to Section 1 165

试读结束：需要全本请在线购买： www.ertongbook.com