刘彦佩选集

（Selected Publications of Y.P.Liu）

第十九编

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第十九编 目录

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Preface

Maps as a mathematical topic arose probably from the four color problem[Bir1, Ore1] and the more general map coloring problem[HiC1, Rin1, Liu11] in the mid of nineteenth century although maps as polyhedra which go back to the Platonic age. I could not list references in detail on them because it is well known for a large range of readers and beyond the scope of this book. Here, I only intend to present a comprehensive theory of maps as a rigorous mathematical concept which has been developed mostly in the last half a century.

However, as described in the book[Liu15] maps can be seen as graphs in development from partition to permutation and as a basis extended to Smarandache geometry shown in [Mao3–4]. This is why maps are much concerned with abstraction in the present stage.

In the beginning, maps as polyhedra were as a topological, or geometric object even with geographical consideration[Kem1]. The first formal definition of a map was done by Heffter from [Hef1] in the 19th century. However, it was not paid an attention by mathematicians until 1960 when Edmonds published a note in the AMS Notices with the dual form of Heffter's in [Edm1,Liu3].

Although this concept was widely used in literature as [Liu1–2, Liu4–6, Rin1–3, Sta1–2, *et al*], its disadvantage for the nonorientable case involved does not bring with some convenience for clarifying some related mathematical thinking.

Since Tutte described the nonorientability in a new way [Tut1–3], a number of authors begin to develop it in combinatorization of continuous objects as in [Lit1, Liu7–10, Vin1–2, *et al*].

The above representations are all with complication in constructing an embedding, or all distinct embeddings of a graph on a surface.

However, the joint tree model of an embedding completed in recent years and initiated from the early articles at the end of seventies in the last century by the present author as shown in [Liu1–2] enables us to make the complication much simpler.

Because of the generality that an asymmetric object can always be seen with some local symmetry in certain extent, the concepts of graphs and maps are just put in such a rule. In fact, the former is corresponding to that a group of two elements sticks on an edge and the later is that a group of four elements sticks on an edge such that a graph without symmetry at all is in company with local symmetry. This treatment will bring more advantages for observing the structure of a graph. Of course, the later is with restriction of the former because of the later as a permutation and the former as a partition.

The joint tree representation of an embedding of a graph on two dimensional manifolds, particularly surfaces(compact 2-manifolds without boundary in our case), is described in Chapter I for simplifying a number of results old and new.

This book contains the following chapters in company with related subjects.

In Chapter I, the embedding of a graph on surfaces are much concerned because they are motivated to building up the theory of abstract maps related with Smarandache geometry.

The second chapter is for the formal definition of abstract maps. One can see that this matter is a natural generalization of graph embedding on surfaces.

The third chapter is on the duality not only for maps themselves but also for operations on maps from one surface to another. One can see how the duality is naturally deduced from the abstract maps described in the second chapter.

The fourth chapter is on the orientability. One can see how the orientability is formally designed as a combinatorial invariant. The fifth chapter concentrates on the classification of orientable maps. The sixth chapter is for the classification of nonorientable maps.

From the two chapters: Chapter V and Chapter VI, one can see how the procedure is simplified for these classifications.

The seventh chapter is on the isomorphisms of maps and provides an efficient algorithm for the justification and recognition of the isomorphism of two maps, which has been shown to be useful for determining the automorphism group of a map in the eighth chapter. Moreover, it enables us to access an automorphism of a graph.

The ninth and the tenth chapters observe the number of distinct asymmetric maps with the size as a parameter. In the former, only one vertex maps are counted by favorite formulas and in the latter, general maps are counted from differential equations. More progresses about this kind of counting are referred to read the recent book[Liu7] and many further articles[Bax1, BeG1, CaL1–2, ReL1–3, *etc*].

The next chapter, Chapter XI, only presents some ideas for accessing the symmetric census of maps and further, of graphs. This topic is being developed in some other directions[KwL1–2] and left as a subject written in the near future.

From Chapter XII through Chapter XV, extensions from basic theory are much concerned with further applications.

Chapter XII discusses in brief on genus polynomial of a graph and all its super maps rooted and unrooted on the basis of the joint tree model. Recent progresses on this aspect are referred to read the articles [Liu13–15, LiP1, WaL1–2, ZhL1–2, ZuL1, *etc*].

Chapter XIII is on the census of maps with vertex or face partitions. Although such census involves with much complication and difficulty, because of the recent progress on a basic topic about trees via an elementary method firstly used by the author himself we are able to do a number of types of such census in very simple way. This chapter reflects on such aspects around.

Chapter XIV is on graphs that their super maps are particularly considered for asymmetrical and symmetrical census via their semi-automorphism and automorphism groups or via embeddings of graphs given [Liu19, MaL1, MaT1, MaW1, *etc*].

Chapter XV, is on functional equations discovered in the census of a variety of maps on sphere and general surfaces. Although their well definedness has been done, almost all of them have not yet been solved up to now.

Three appendices are compliment to the context. One provides the clarification of the concepts of polyhedra, surfaces, embeddings, and maps and their relationship. The other two are for exhaustively calculating numerical results and listing all rooted and unrooted maps for small graphs with more calculating results compared with those appearing in [Liu14], [Liu17] and [Liu19].

From a large amount of materials, more than hundred observations for beginners probably senior undergraduates, more than hundred exercises for mainly graduates of master degree and more than hundred research problems for mainly graduates of doctoral degree are carefully designed at the end of each chapter in adapting the needs of such a wide range of readers for mastering, extending and investigating a number of ways to get further development on the basic theory of abstract maps.

Although I have been trying to design this book self contained as much as possible, some books such as [DiM1], [Mss1] and [GaJ1] might be helpful to those not familiar with basic knowledge of permutation groups, topology and computing complexity as background.

Since early nineties of the last century, a number of my former and present graduates were or are engaged with topics related to this book. Among them, I have to mention Dr. Ying Liu[LpL1], Dr. Yuanqiu Huang[HuL1], Dr. Junliang Cai[CaL1–2], Dr. Deming Li[LiL1], Dr. Han Ren[ReL1–3], Dr. Rongxia Hao[HaC1, HaL1], Dr. Zhaoxiang Li[LiQ1–2], Dr. Linfan Mao[MaL1, MaT1, MaW1], Dr. Erling Wei[WiL1–2], Dr. Weili He[HeL1], Dr. Liangxia Wan[WaL1–2], Dr. Yichao Chen[CnL1, CnR1], Dr. Yan Xu[XuL1–2], Dr. Wenzhong Liu[LwL1–2], Dr. Zeling Shao[ShL1], Dr. Yan Yang[YaL1–2], Dr. Guanghua Dong[DoL1], Ms. Ximei Zhao[ZhL1–2], Mr. Lifeng Li[LiP1], Ms. Huiyan Wang[WgL1], Ms. Zhao Chai[CiL1], Mr. Zilong Zhu[ZuL1], *et al* for their successful work related to this book.

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Y.P. Liu
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