

刘彦佩选集

(Selected Publications of Y.P.Liu)

第十九编

时代文献出版社

刘彦佩选集

(Selected Publications of Y.P.Liu)

第十九编



时代文献出版社

刘彦佩选集（第十九编）

作 者：刘彦佩

出版单位：时代文献出版社

编辑设计：北京时代弄潮文化发展公司

地 址：北京中关村海淀图书城25号家谱传记楼二层

电 话：010-62525116 13693651386

网 址：www.grcsw.com

印 刷：京冀印刷厂

开 本：880×1230 1/16

版 次：2016年3月第1版

书 号：ISBN 978-988-18772-5-3

定 价：全套 1978.00元（共计23编）

版权所有 翻印必究

第十九编 目录

Introductory Map Theory.....	8957
19.1 Abstract Embeddings	8971
19.2 Abstract Maps	9011
19.3 Duality.....	9035
19.4 Orientability	9073
19.5 Orientable Maps	9094
19.6 Nonorientable Maps	9115
19.7 Isomorphisms of Maps	9134
19.8 Asymmetrization	9160
19.9 Rooted Petal Bundles	9182
19.10 Asymmetrized Maps	9205
19.11 Maps with Symmetry	9238
19.12 Genus Polynomials.....	9252
19.13 Genus with Partitions.....	9267
19.14 Super Maps of a Graph	9296
19.15 Equations with Partitions	9315
19.16 Appendices	9344

Yanpei LIU

Institute of Mathematics
Beijing Jiaotong University
Beijing 100044, P.R.China
Email: ypliu@bjtu.edu.cn

Introductory Map Theory

**Kapa & Omega, Glendale, AZ
USA**

2010

This book can be ordered in a paper bound reprint from:

Books on Demand

ProQuest Information & Learning

(University of Microfilm International)

300 N.Zeeb Road

P.O.Box 1346, Ann Arbor

MI 48106-1346, USA

Tel:1-800-521-0600(Customer Service)

<http://wwwlib.umi.com/bod>

Peer Reviewers:

L.F.Mao, Chinese Academy of Mathematics and System Science, P.R.China.

J.L.Cai, Beijing Normal University, P.R.China.

H.Ren, East China Normal University, P.R.China.

R.X.Hao, Beijing Jiaotong University, P.R.China.

Copyright 2010 by Kapa & Omega, Glendale, AZ and Yanpei Liu

Many books can be downloaded from the following **Digital Library of Science**:

<http://www.gallup.unm.edu/~smarandache/eBooks-otherformats.htm>

ISBN: 978-1-59973-134-6

Printed in America

Preface

Maps as a mathematical topic arose probably from the four color problem [Bir1, Ore1] and the more general map coloring problem [HiC1, Rin1, Liu11] in the mid of nineteenth century although maps as polyhedra which go back to the Platonic age. I could not list references in detail on them because it is well known for a large range of readers and beyond the scope of this book. Here, I only intend to present a comprehensive theory of maps as a rigorous mathematical concept which has been developed mostly in the last half a century.

However, as described in the book [Liu15] maps can be seen as graphs in development from partition to permutation and as a basis extended to Smarandache geometry shown in [Mao3–4]. This is why maps are much concerned with abstraction in the present stage.

In the beginning, maps as polyhedra were as a topological, or geometric object even with geographical consideration [Kem1]. The first formal definition of a map was done by Heffter from [Hef1] in the 19th century. However, it was not paid an attention by mathematicians until 1960 when Edmonds published a note in the AMS Notices with the dual form of Heffter's in [Edm1, Liu3].

Although this concept was widely used in literature as [Liu1–2, Liu4–6, Rin1–3, Sta1–2, *et al*], its disadvantage for the nonorientable case involved does not bring with some convenience for clarifying some related mathematical thinking.

Since Tutte described the nonorientability in a new way [Tut1–3], a number of authors begin to develop it in combinatorization of continuous objects as in [Lit1, Liu7–10, Vin1–2, *et al*].

The above representations are all with complication in constructing an embedding, or all distinct embeddings of a graph on a surface.

However, the joint tree model of an embedding completed in recent years and initiated from the early articles at the end of seventies in the last century by the present author as shown in [Liu1-2] enables us to make the complication much simpler.

Because of the generality that an asymmetric object can always be seen with some local symmetry in certain extent, the concepts of graphs and maps are just put in such a rule. In fact, the former is corresponding to that a group of two elements sticks on an edge and the later is that a group of four elements sticks on an edge such that a graph without symmetry at all is in company with local symmetry. This treatment will bring more advantages for observing the structure of a graph. Of course, the later is with restriction of the former because of the later as a permutation and the former as a partition.

The joint tree representation of an embedding of a graph on two dimensional manifolds, particularly surfaces(compact 2-manifolds without boundary in our case), is described in Chapter I for simplifying a number of results old and new.

This book contains the following chapters in company with related subjects.

In Chapter I, the embedding of a graph on surfaces are much concerned because they are motivated to building up the theory of abstract maps related with Smarandache geometry.

The second chapter is for the formal definition of abstract maps. One can see that this matter is a natural generalization of graph embedding on surfaces.

The third chapter is on the duality not only for maps themselves but also for operations on maps from one surface to another. One can see how the duality is naturally deduced from the abstract maps described in the second chapter.

The fourth chapter is on the orientability. One can see how the orientability is formally designed as a combinatorial invariant. The fifth chapter concentrates on the classification of orientable maps. The sixth chapter is for the classification of nonorientable maps.

From the two chapters: Chapter V and Chapter VI, one can see

how the procedure is simplified for these classifications.

The seventh chapter is on the isomorphisms of maps and provides an efficient algorithm for the justification and recognition of the isomorphism of two maps, which has been shown to be useful for determining the automorphism group of a map in the eighth chapter. Moreover, it enables us to access an automorphism of a graph.

The ninth and the tenth chapters observe the number of distinct asymmetric maps with the size as a parameter. In the former, only one vertex maps are counted by favorite formulas and in the latter, general maps are counted from differential equations. More progresses about this kind of counting are referred to read the recent book[Liu7] and many further articles[Bax1, BeG1, CaL1-2, ReL1-3, *etc*].

The next chapter, Chapter XI, only presents some ideas for accessing the symmetric census of maps and further, of graphs. This topic is being developed in some other directions[KwL1-2] and left as a subject written in the near future.

From Chapter XII through Chapter XV, extensions from basic theory are much concerned with further applications.

Chapter XII discusses in brief on genus polynomial of a graph and all its super maps rooted and unrooted on the basis of the joint tree model. Recent progresses on this aspect are referred to read the articles [Liu13-15, LiP1, WaL1-2, ZhL1-2, ZuL1, *etc*].

Chapter XIII is on the census of maps with vertex or face partitions. Although such census involves with much complication and difficulty, because of the recent progress on a basic topic about trees via an elementary method firstly used by the author himself we are able to do a number of types of such census in very simple way. This chapter reflects on such aspects around.

Chapter XIV is on graphs that their super maps are particularly considered for asymmetrical and symmetrical census via their semi-automorphism and automorphism groups or via embeddings of graphs given [Liu19, MaL1, MaT1, MaW1, *etc*].

Chapter XV, is on functional equations discovered in the census of a variety of maps on sphere and general surfaces. Although their

well definedness has been done, almost all of them have not yet been solved up to now.

Three appendices are compliment to the context. One provides the clarification of the concepts of polyhedra, surfaces, embeddings, and maps and their relationship. The other two are for exhaustively calculating numerical results and listing all rooted and unrooted maps for small graphs with more calculating results compared with those appearing in [Liu14], [Liu17] and [Liu19].

From a large amount of materials, more than hundred observations for beginners probably senior undergraduates, more than hundred exercises for mainly graduates of master degree and more than hundred research problems for mainly graduates of doctoral degree are carefully designed at the end of each chapter in adapting the needs of such a wide range of readers for mastering, extending and investigating a number of ways to get further development on the basic theory of abstract maps.

Although I have been trying to design this book self contained as much as possible, some books such as [DiM1], [Mss1] and [GaJ1] might be helpful to those not familiar with basic knowledge of permutation groups, topology and computing complexity as background.

Since early nineties of the last century, a number of my former and present graduates were or are engaged with topics related to this book. Among them, I have to mention Dr. Ying Liu[LpL1], Dr. Yuanqiu Huang[HuL1], Dr. Junliang Cai[CaL1-2], Dr. Deming Li[LiL1], Dr. Han Ren[ReL1-3], Dr. Rongxia Hao[HaC1, HaL1], Dr. Zhaoxiang Li[LiQ1-2], Dr. Linfan Mao[MaL1, MaT1, MaW1], Dr. Erling Wei[WiL1-2], Dr. Weili He[HeL1], Dr. Liangxia Wan[WaL1-2], Dr. Yichao Chen[CnL1, CnR1], Dr. Yan Xu[XuL1-2], Dr. Wenzhong Liu[LwL1-2], Dr. Zeling Shao[ShL1], Dr. Yan Yang[YaL1-2], Dr. Guanghua Dong[DoL1], Ms. Ximei Zhao[ZhL1-2], Mr. Lifeng Li[LiP1], Ms. Huiyan Wang[WgL1], Ms. Zhao Chai[CiL1], Mr. Zilong Zhu[ZuL1], *et al* for their successful work related to this book.

On this occasion, I should express my heartiest appreciation of the financial support by KOSEF of Korea from the Com²MaC (Com-

binatorial and Computational Mathematics Research Center) of the Pohang University of Science and Technology in the summer of 2001. In that period, the intention of this book was established. Moreover, I should be also appreciated to the Natural Science Foundation of China for the research development reflected in this book under its Grants(60373030, 10571013, 10871021).

Y.P. Liu
Beijing, China
Jan., 2010

Contents

Preface	iii
Chapter I Abstract Embeddings	1
I.1 Graphs and networks	1
I.2 Surfaces	9
I.3 Embeddings	16
I.4 Abstract representation	22
I.5 Smarandache 2-manifolds with map geometry	27
Activities on Chapter I	32
I.6 Observations	32
I.7 Exercises	34
I.8 Researches	37
Chapter II Abstract Maps	41
II.1 Ground sets	41
II.2 Basic permutations	43
II.3 Conjugate axiom	46
II.4 Transitive axiom	49
II.5 Included angles	55
Activities on Chapter II	57
II.6 Observations	57
II.7 Exercises	58
II.8 Researches	59
Chapter III Duality	65
III.1 Dual maps	65

III.2 Deletion of an edge	72
III.3 Addition of an edge	85
III.4 Basic transformation	96
Activities on Chapter III	98
III.5 Observations	98
III.6 Exercises	99
III.7 Researches	101
Chapter IV Orientability	103
IV.1 Orientation	103
IV.2 Basic equivalence	107
IV.3 Euler characteristic	113
IV.4 Pattern examples	116
Activities on Chapter IV	119
IV.5 Observations	119
IV.6 Exercises	120
IV.7 Researches	122
Chapter V Orientable Maps	124
V.1 Butterflies	124
V.2 Simplified butterflies	126
V.3 Reduced rules	130
V.4 Orientable principles	134
V.5 Orientable genus	137
Activities on Chapter V	139
V.6 Observations	139
V.7 Exercises	140
V.8 Researches	142
Chapter VI Nonorientable Maps	145
VI.1 Barflies	145

VI.2 Simplified barflies	149
VI.3 Nonorientable rules	151
VI.4 Nonorientable principles	156
VI.5 Nonorientable genus	157
Activities on Chapter VI	159
VI.5 Observations	159
VI.6 Exercises	160
VI.7 Researches	162
Chapter VII Isomorphisms of Maps	164
VII.1 Commutativity	164
VII.2 Isomorphism theorem	168
VII.3 Recognition	172
VII.4 Justification	177
VII.5 Pattern examples	180
Activities on Chapter VII	185
VII.6 Observations	185
VII.7 Exercises	186
VII.8 Researches	188
Chapter VIII Asymmetrization	190
VIII.1 Automorphisms	190
VIII.2 Upper bound of group order	193
VIII.3 Determination of the group	196
VIII.4 Rootings	201
Activities on Chapter VIII	206
VIII.5 Observations	206
VIII.6 Exercises	207
VIII.7 Researches	209

Chapter IX	Rooted Petal Bundles	212
IX.1	Orientable petal bundles	212
IX.2	Planar petal bundles	217
IX.3	Nonorientable petal bundles	220
IX.4	The number of petal bundles	226
Activities on Chapter IX		230
IX.5	Observations	230
IX.6	Exercises	231
IX.7	Researches	232
Chapter X	Asymmetrized Maps	235
X.1	Orientable equation	235
X.2	Planar rooted maps	243
X.3	Nonorientable equation	250
X.4	Gross equation	255
X.5	The number of rooted maps	258
Activities on Chapter X		261
X.6	Observations	261
X.7	Exercises	262
X.8	Researches	265
Chapter XI	Maps with Symmetry	268
XI.1	Symmetric relation	268
XI.2	An application	270
XI.3	Symmetric principle	272
XI.4	General examples	274
Activities on Chapter XI		278
XI.5	Observations	278
XI.6	Exercises	279
XI.7	Researches	280

Chapter XII	Genus Polynomials	282
XII.1	Associate surfaces	282
XII.2	Layer division of a surface	285
XII.3	Handle polynomials	289
XII.4	Crosscap polynomials	290
	Activities on Chapter XII	292
XII.5	Observations	292
XII.6	Exercises	293
XII.7	Researches	294
Chapter XIII	Census with Partitions	297
XIII.1	Planted trees	297
XIII.2	Hamiltonian cubic maps	305
XIII.3	Halin maps	307
XIII.4	Biboundary inner rooted maps	310
XIII.5	General maps	315
XIII.6	Pan-flowers	317
	Activities on Chapter XIII	323
XIII.7	Observations	323
XIII.8	Exercises	324
XIII.9	Researches	325
Chapter XIV	Super Maps of a Graph	326
XIV.1	Semi-automorphisms on a graph	326
XIV.2	Automorphisms on a graph	329
XIV.3	Relationships	332
XIV.4	Nonisomorphic super maps	334
XIV.5	Via rooted super maps	336
	Activities on Chapter XIV	341
XIV.6	Observations	341

XIV.7 Exercises	342
XIV.8 Researches	334
Chapter XV Equations with Partitions	345
XV.1 The meson functional	345
XV.2 General maps on the sphere	350
XV.3 Nonseparable maps on the sphere	353
XV.4 Maps without cut-edge on surfaces	357
XV.5 Eulerian maps on the sphere	361
XV.6 Eulerian maps on surfaces	365
Activities on Chapter XV	370
XV.7 Observations	370
XV.8 Exercises	371
XV.9 Researches	372
Appendix I Concepts of Polyhedra, Surfaces, Embeddings and Maps	374
Ax.I.1 Polyhedra	374
Ax.I.2 Surfaces	377
Ax.I.3 Embeddings	381
Ax.I.4 Maps	384
Appendix II Table of Genus Polynomials for Embeddings and Maps of Small Size	389
Ax.II.1 Triconnected cubic graphs	389
Ax.II.2 Bouquets	398
Ax.II.3 Wheels	401
Ax.II.4 Link bundles	403
Ax.II.5 Complete bipartite graphs	405
Ax.II.6 Complete graphs	407