

Graduate Texts in Physics

Norbert Straumann

General Relativity

Second Edition

广义相对论 第2版

Springer

世界图书出版公司
www.wpcbj.com.cn

Norbert Straumann

General Relativity

Second Edition



图书在版编目 (CIP) 数据

广义相对论 = General Relativity Second Edition : 英文 / (瑞士) N.
斯特劳曼 (N. Straumann) 著 . — 影印本 . — 北京 : 世界图书出版
公司北京公司, 2016
ISBN 978-7-5192-2076-1

I . ① 广… II . ① 斯… III . ① 广义相对论—教材—英文
IV . ① O412.1

中国版本图书馆 CIP 数据核字 (2016) 第 271810 号

著 者: Norbert Straumann
责任编辑: 刘 慧 高 蓉
装帧设计: 任志远

出版发行: 世界图书出版公司北京公司
地 址: 北京市东城区朝内大街 137 号
邮 编: 100010
电 话: 010-64038355 (发行) 64015580 (客服) 64033507 (总编室)
网 址: <http://www.wpcbj.com.cn>
邮 箱: wpcbjst@vip.163.com
销 售: 新华书店
印 刷: 三河市国英印务有限公司
开 本: 711mm × 1245mm 1/24
印 张: 31.5
字 数: 605 千
版 次: 2017 年 1 月第 1 版 2017 年 1 月第 1 次印刷
版权登记: 01-2016-7823
定 价: 99.00 元

版权所有 翻印必究

(如发现印装质量问题, 请与所购图书销售部门联系调换)

Graduate Texts in Physics

Graduate Texts in Physics

Graduate Texts in Physics publishes core learning/teaching material for graduate- and advanced-level undergraduate courses on topics of current and emerging fields within physics, both pure and applied. These textbooks serve students at the MS- or PhD-level and their instructors as comprehensive sources of principles, definitions, derivations, experiments and applications (as relevant) for their mastery and teaching, respectively. International in scope and relevance, the textbooks correspond to course syllabi sufficiently to serve as required reading. Their didactic style, comprehensiveness and coverage of fundamental material also make them suitable as introductions or references for scientists entering, or requiring timely knowledge of, a research field.

Series Editors

Professor William T. Rhodes

Department of Computer and Electrical Engineering and Computer Science
Imaging Science and Technology Center
Florida Atlantic University
777 Glades Road SE, Room 456
Boca Raton, FL 33431
USA
wrhodes@fau.edu

Professor H. Eugene Stanley

Center for Polymer Studies Department of Physics
Boston University
590 Commonwealth Avenue, Room 204B
Boston, MA 02215
USA
hes@bu.edu

Professor Richard Needs

Cavendish Laboratory
JJ Thomson Avenue
Cambridge CB3 0HE
UK
rn11@cam.ac.uk

Norbert Straumann
Mathematisch-Naturwiss. Fakultät
Institut für Theoretische Physik
Universität Zürich
Zürich, Switzerland

ISSN 1868-4513
Graduate Texts in Physics

ISBN 978-94-007-5409-6

DOI 10.1007/978-94-007-5410-2

Springer Dordrecht Heidelberg New York London

ISSN 1868-4521 (electronic)

ISBN 978-94-007-5410-2 (eBook)

Library of Congress Control Number: 2012950312

© Springer Science+Business Media Dordrecht 2004, 2013

This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed. Exempted from this legal reservation are brief excerpts in connection with reviews or scholarly analysis or material supplied specifically for the purpose of being entered and executed on a computer system, for exclusive use by the purchaser of the work. Duplication of this publication or parts thereof is permitted only under the provisions of the Copyright Law of the Publisher's location, in its current version, and permission for use must always be obtained from Springer. Permissions for use may be obtained through RightsLink at the Copyright Clearance Center. Violations are liable to prosecution under the respective Copyright Law.

The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

While the advice and information in this book are believed to be true and accurate at the date of publication, neither the authors nor the editors nor the publisher can accept any legal responsibility for any errors or omissions that may be made. The publisher makes no warranty, express or implied, with respect to the material contained herein.

Reprint from English language edition:

General Relativity Second Edition

by Norbert Straumann

Copyright © Springer Science+Business Media Dordrecht 2004, 2013

Springer is a part of Springer Science+Business Media

All Rights Reserved

This reprint has been authorized by Springer Science & Business Media for distribution in China Mainland only and not for export therefrom.

Preface

Physics and mathematics students are as eager as ever to become acquainted with the foundations of general relativity and some of its major applications in astrophysics and cosmology. I hope that this textbook gives a comprehensive and timely introduction to both aspects of this fascinating field, and will turn out to be useful for undergraduate and graduate students.

This book is a complete revision and extension of my previous volume '*General Relativity and Relativistic Astrophysics*' that appeared about twenty years ago in the Springer Series '*Texts and Monographs in Physics*'; however, it cannot be regarded just as a new edition.

In Part I the foundations of general relativity are thoroughly developed. Some of the more advanced topics, such as the section on the initial value problem, can be skipped in a first reading.

Part II is devoted to tests of general relativity and many of its applications. Binary pulsars—our best laboratories for general relativity—are studied in considerable detail. I have included an introduction to gravitational lensing theory, to the extent that the current literature on the subject should become accessible. Much space is devoted to the study of compact objects, especially to black holes. This includes a detailed derivation of the Kerr solution, Israel's proof of his uniqueness theorem, and a derivation of the basic laws of black hole physics. Part II ends with Witten's proof of the positive energy theorem.

All the required differential geometric tools are developed in Part III. Readers who have not yet studied a modern mathematical text on differential geometry should not read this part in linear order. I always indicate in the physics parts which mathematical sections are going to be used at a given point of the discussion. For example, Cartan's powerful calculus of differential forms is not heavily used in the foundational chapters. The mathematical part should also be useful for other fields of physics. Differential geometric and topological tools play an increasingly important role, not only in quantum field theory and string theory, but also in classical disciplines (mechanics, field theory).

A textbook on a field as developed and extensive as general relativity must make painful omissions. When the book was approaching seven hundred pages, I had to

give up the original plan to also include cosmology. This is really deplorable, since cosmology is going through a fruitful and exciting period. On the other hand, this omission may not be too bad, since several really good texts on various aspects of cosmology have recently appeared. Some of them are listed in the references. Quantum field theory on curved spacetime backgrounds is not treated at all. Other topics have often been left out because my emphasis is on direct physical applications of the theory. For this reason nothing is said, for instance, about black holes with hair (although I worked on this for several years).

The list of references is not very extensive. Beside some useful general sources on mathematical, physical, and historical subjects, I often quoted relatively recent reviews and articles that may be most convenient for the reader to penetrate deeper into various topics. Pedagogical reasons often have priority in my restrictive selection of references.

The present textbook would presumably not have been written without the enduring help of Thiemo Kessel. He not only typed very carefully the entire manuscript, but also urged me at many places to give further explanations or provide additional information. Since—as an undergraduate student—he was able to understand all the gory details, I am now confident that the book is readable. I thank him for all his help.

I also thank the many students I had over the years from the University of Zurich and the ETH for their interest and questions. This feedback was essential in writing the present book. Many of the exercises posed in the text have been solved in my classes.

Finally, I would like to thank the Tomalla Foundation and the Institute for Theoretical Physics of the University of Zurich for financial support.

Post Script I would be grateful for suggestions of all kind and lists of mistakes. Since I adopted—contrary to my habits—the majority convention for the signature of the metric, I had to change thousands of signs. It is unlikely that no sign errors remained, especially in spinorial equations.

Comments can be sent to me at <norbert.straumann@gmail.com>.

Zurich, Switzerland

Norbert Straumann

Preface to the Second Edition

The present edition is a thorough revision of my book on general relativity that appeared in 2004 in the Springer Series “Text and Monographs in Physics”. I hope that almost no errors have survived.

A lot of effort went into refining and improving the text. Furthermore, new material has been added. Beside some updates, for instance on the marvellous double pulsar system, and many smaller additions, the following sections have been added:

Sect. 2.11 General Relativistic Ideal Magnetohydrodynamics; Sect. 3.10 Domain of Dependence and Propagation of Matter Disturbances; Sect. 3.11 Boltzmann Equation in GR; most of Sect. 8.6.4: The First Law of Black Hole Dynamics; Sect. 15.11 Covariant Derivatives of Tensor Densities.

As in the previous edition, space did not allow me to give an adequate introduction to the vast field of modern cosmology. But I have now at least added a concise treatment of the Friedmann–Lemaître models, together with some crucial observations which can be described within this idealized class of cosmological models.

Solutions to some of the more difficult exercises have been included, as well as new exercises (often with solutions).

In preparing this new edition, I have benefited from suggestions, criticism, and advice by students and colleagues. Eric Sheldon read the entire first edition and suggested lots of improvements of my Swiss influenced use of English. Particular thanks go to Domenico Giulini for detailed constructive criticism and help. Positive reactions by highly estimated colleagues—especially by the late Juergen Ehlers—encouraged me to invest the energy for this new edition.

Zurich, Switzerland

Norbert Straumann

Contents

Part I The General Theory of Relativity

1	Introduction	3
2	Physics in External Gravitational Fields	7
2.1	Characteristic Properties of Gravitation	7
2.1.1	Strength of the Gravitational Interaction	7
2.1.2	Universality of Free Fall	8
2.1.3	Equivalence Principle	9
2.1.4	Gravitational Red- and Blueshifts	10
2.2	Special Relativity and Gravitation	12
2.2.1	Gravitational Redshift and Special Relativity	12
2.2.2	Global Inertial Systems Cannot Be Realized in the Presence of Gravitational Fields	13
2.2.3	Gravitational Deflection of Light Rays	14
2.2.4	Theories of Gravity in Flat Spacetime	14
2.2.5	Exercises	18
2.3	Spacetime as a Lorentzian Manifold	19
2.4	Non-gravitational Laws in External Gravitational Fields	21
2.4.1	Motion of a Test Body in a Gravitational Field	22
2.4.2	World Lines of Light Rays	23
2.4.3	Exercises	23
2.4.4	Energy and Momentum “Conservation” in the Presence of an External Gravitational Field	25
2.4.5	Exercises	27
2.4.6	Electrodynamics	28
2.4.7	Exercises	31
2.5	The Newtonian Limit	32
2.5.1	Exercises	33
2.6	The Redshift in a Stationary Gravitational Field	34
2.7	Fermat’s Principle for Static Gravitational Fields	35

2.8	Geometric Optics in Gravitational Fields	38
2.8.1	Exercises	42
2.9	Stationary and Static Spacetimes	42
2.9.1	Killing Equation	44
2.9.2	The Redshift Revisited	45
2.10	Spin Precession and Fermi Transport	48
2.10.1	Spin Precession in a Gravitational Field	49
2.10.2	Thomas Precession	50
2.10.3	Fermi Transport	51
2.10.4	The Physical Difference Between Static and Stationary Fields	53
2.10.5	Spin Rotation in a Stationary Field	55
2.10.6	Adapted Coordinate Systems for Accelerated Observers . .	56
2.10.7	Motion of a Test Body	58
2.10.8	Exercises	60
2.11	General Relativistic Ideal Magnetohydrodynamics	62
2.11.1	Exercises	63
3	Einstein's Field Equations	65
3.1	Physical Meaning of the Curvature Tensor	65
3.1.1	Comparison with Newtonian Theory	69
3.1.2	Exercises	69
3.2	The Gravitational Field Equations	72
3.2.1	Heuristic "Derivation" of the Field Equations	73
3.2.2	The Question of Uniqueness	74
3.2.3	Newtonian Limit, Interpretation of the Constants Λ and κ .	78
3.2.4	On the Cosmological Constant Λ	79
3.2.5	The Einstein-Fokker Theory	82
3.2.6	Exercises	82
3.3	Lagrangian Formalism	84
3.3.1	Canonical Measure on a Pseudo-Riemannian Manifold . .	84
3.3.2	The Einstein-Hilbert Action	85
3.3.3	Reduced Bianchi Identity and General Invariance	87
3.3.4	Energy-Momentum Tensor in a Lagrangian Field Theory .	89
3.3.5	Analogy with Electrodynamics	92
3.3.6	Meaning of the Equation $\nabla \cdot T = 0$	94
3.3.7	The Equations of Motion and $\nabla \cdot T = 0$	94
3.3.8	Variational Principle for the Coupled System	95
3.3.9	Exercises	95
3.4	Non-localizability of the Gravitational Energy	97
3.5	On Covariance and Invariance	98
3.5.1	Note on Unimodular Gravity	101
3.6	The Tetrad Formalism	102
3.6.1	Variation of Tetrad Fields	103
3.6.2	The Einstein-Hilbert Action	104

3.6.3	Consequences of the Invariance Properties of the Lagrangian $\mathcal{L}$	107
3.6.4	Lovelock's Theorem in Higher Dimensions	110
3.6.5	Exercises	111
3.7	Energy, Momentum, and Angular Momentum for Isolated Systems	112
3.7.1	Interpretation	116
3.7.2	ADM Expressions for Energy and Momentum	119
3.7.3	Positive Energy Theorem	121
3.7.4	Exercises	122
3.8	The Initial Value Problem of General Relativity	124
3.8.1	Nature of the Problem	124
3.8.2	Constraint Equations	125
3.8.3	Analogy with Electrodynamics	126
3.8.4	Propagation of Constraints	127
3.8.5	Local Existence and Uniqueness Theorems	128
3.8.6	Analogy with Electrodynamics	128
3.8.7	Harmonic Gauge Condition	130
3.8.8	Field Equations in Harmonic Gauge	130
3.8.9	Characteristics of Einstein's Field Equations	135
3.8.10	Exercises	136
3.9	General Relativity in 3 + 1 Formulation	137
3.9.1	Generalities	138
3.9.2	Connection Forms	139
3.9.3	Curvature Forms, Einstein and Ricci Tensors	142
3.9.4	Gaussian Normal Coordinates	145
3.9.5	Maximal Slicing	146
3.9.6	Exercises	147
3.10	Domain of Dependence and Propagation of Matter Disturbances	147
3.11	Boltzmann Equation in GR	149
3.11.1	One-Particle Phase Space, Liouville Operator for Geodesic Spray	149
3.11.2	The General Relativistic Boltzmann Equation	153

Part II Applications of General Relativity

4	The Schwarzschild Solution and Classical Tests of General Relativity	157
4.1	Derivation of the Schwarzschild Solution	157
4.1.1	The Birkhoff Theorem	161
4.1.2	Geometric Meaning of the Spatial Part of the Schwarzschild Metric	164
4.1.3	Exercises	164
4.2	Equation of Motion in a Schwarzschild Field	166
4.2.1	Exercises	169
4.3	Perihelion Advance	170
4.4	Deflection of Light	174
4.4.1	Exercises	178

4.5	Time Delay of Radar Echoes	180
4.6	Geodetic Precession	184
4.7	Schwarzschild Black Holes	187
4.7.1	The Kruskal Continuation of the Schwarzschild Solution	188
4.7.2	Discussion	194
4.7.3	Eddington–Finkelstein Coordinates	197
4.7.4	Spherically Symmetric Collapse to a Black Hole	199
4.7.5	Redshift for a Distant Observer	201
4.7.6	Fate of an Observer on the Surface of the Star	204
4.7.7	Stability of the Schwarzschild Black Hole	207
4.8	Penrose Diagram for Kruskal Spacetime	207
4.8.1	Conformal Compactification of Minkowski Spacetime	208
4.8.2	Penrose Diagram for Schwarzschild–Kruskal Spacetime	210
4.9	Charged Spherically Symmetric Black Holes	211
4.9.1	Resolution of the Apparent Singularity	211
4.9.2	Timelike Radial Geodesics	214
4.9.3	Maximal Extension of the Reissner–Nordstrøm Solution	216
	Appendix: Spherically Symmetric Gravitational Fields	220
4.10.1	General Form of the Metric	220
4.10.2	The Generalized Birkhoff Theorem	224
4.10.3	Spherically Symmetric Metrics for Fluids	225
4.10.4	Exercises	226
5	Weak Gravitational Fields	227
5.1	The Linearized Theory of Gravity	227
5.1.1	Generalization	230
5.1.2	Exercises	233
5.2	Nearly Newtonian Gravitational Fields	234
5.2.1	Gravitomagnetic Field and Lense–Thirring Precession	235
5.2.2	Exercises	236
5.3	Gravitational Waves in the Linearized Theory	237
5.3.1	Plane Waves	238
5.3.2	Transverse and Traceless Gauge	239
5.3.3	Geodesic Deviation in the Metric Field of a Gravitational Wave	240
5.3.4	A Simple Mechanical Detector	242
5.4	Energy Carried by a Gravitational Wave	245
5.4.1	The Short Wave Approximation	246
5.4.2	Discussion of the Linearized Equation $R_{\mu\nu}^{(1)}[h] = 0$	248
5.4.3	Averaged Energy-Momentum Tensor for Gravitational Waves	250
5.4.4	Effective Energy-Momentum Tensor for a Plane Wave	252
5.4.5	Exercises	253
5.5	Emission of Gravitational Radiation	256
5.5.1	Slow Motion Approximation	256

5.5.2	Rapidly Varying Sources	259
5.5.3	Radiation Reaction (Preliminary Remarks)	261
5.5.4	Simple Examples and Rough Estimates	261
5.5.5	Rigidly Rotating Body	261
5.5.6	Radiation from Binary Star Systems in Elliptic Orbits	266
5.5.7	Exercises	269
5.6	Laser Interferometers	270
5.7	Gravitational Field at Large Distances from a Stationary Source	272
5.7.1	The Komar Formula	278
5.7.2	Exercises	280
5.8	Gravitational Lensing	280
5.8.1	Three Derivations of the Effective Refraction Index	281
5.8.2	Deflection by an Arbitrary Mass Concentration	283
5.8.3	The General Lens Map	286
5.8.4	Alternative Derivation of the Lens Equation	288
5.8.5	Magnification, Critical Curves and Caustics	290
5.8.6	Simple Lens Models	292
5.8.7	Axially Symmetric Lenses: Generalities	292
5.8.8	The Schwarzschild Lens: Microlensing	295
5.8.9	Singular Isothermal Sphere	298
5.8.10	Isothermal Sphere with Finite Core Radius	300
5.8.11	Relation Between Shear and Observable Distortions	300
5.8.12	Mass Reconstruction from Weak Lensing	301
6	The Post-Newtonian Approximation	307
6.1	Motion and Gravitational Radiation (Generalities)	307
6.1.1	Asymptotic Flatness	308
6.1.2	Bondi–Sachs Energy and Momentum	309
6.1.3	The Effacement Property	311
6.2	Field Equations in Post-Newtonian Approximation	312
6.2.1	Equations of Motion for a Test Particle	319
6.3	Stationary Asymptotic Fields in Post-Newtonian Approximation	320
6.4	Point-Particle Limit	322
6.5	The Einstein–Infeld–Hoffmann Equations	326
6.5.1	The Two-Body Problem in the Post-Newtonian Approximation	329
6.6	Precession of a Gyroscope in the Post-Newtonian Approximation	335
6.6.1	Gyroscope in Orbit Around the Earth	338
6.6.2	Precession of Binary Pulsars	339
6.7	General Strategies of Approximation Methods	340
6.7.1	Radiation Damping	345
6.8	Binary Pulsars	346
6.8.1	Discovery and Gross Features	346
6.8.2	Timing Measurements and Data Reduction	351
6.8.3	Arrival Time	351

6.8.4	Solar System Corrections	352
6.8.5	Theoretical Analysis of the Arrival Times	354
6.8.6	Einstein Time Delay	355
6.8.7	Roemer and Shapiro Time Delays	356
6.8.8	Explicit Expression for the Roemer Delay	359
6.8.9	Aberration Correction	361
6.8.10	The Timing Formula	363
6.8.11	Results for Keplerian and Post-Keplerian Parameters	366
6.8.12	Masses of the Two Neutron Stars	366
6.8.13	Confirmation of the Gravitational Radiation Damping	367
6.8.14	Results for the Binary PSR B 1534+12	369
6.8.15	Double-Pulsar	372
7	White Dwarfs and Neutron Stars	375
7.1	Introduction	375
7.2	White Dwarfs	377
7.2.1	The Free Relativistic Electron Gas	378
7.2.2	Thomas–Fermi Approximation for White Dwarfs	379
7.2.3	Historical Remarks	383
7.2.4	Exercises	384
7.3	Formation of Neutron Stars	385
7.4	General Relativistic Stellar Structure Equations	387
7.4.1	Interpretation of M	390
7.4.2	General Relativistic Virial Theorem	391
7.4.3	Exercises	392
7.5	Linear Stability	392
7.6	The Interior of Neutron Stars	395
7.6.1	Qualitative Overview	395
7.6.2	Ideal Mixture of Neutrons, Protons and Electrons	397
7.6.3	Oppenheimer–Volkoff Model	399
7.6.4	Pion Condensation	400
7.7	Equation of State at High Densities	401
7.7.1	Effective Nuclear Field Theories	401
7.7.2	Many-Body Theory of Nucleon Matter	401
7.8	Gross Structure of Neutron Stars	402
7.8.1	Measurements of Neutron Star Masses Using Shapiro Time Delay	404
7.9	Bounds for the Mass of Non-rotating Neutron Stars	405
7.9.1	Basic Assumptions	405
7.9.2	Simple Bounds for Allowed Cores	408
7.9.3	Allowed Core Region	408
7.9.4	Upper Limit for the Total Gravitational Mass	410
7.10	Rotating Neutron Stars	412
7.11	Cooling of Neutron Stars	415
7.12	Neutron Stars in Binaries	416

7.12.1	Some Mechanics in Binary Systems	416
7.12.2	Some History of X-Ray Astronomy	419
7.12.3	X-Ray Pulsars	420
7.12.4	The Eddington Limit	422
7.12.5	X-Ray Bursters	423
7.12.6	Formation and Evolution of Binary Systems	425
7.12.7	Millisecond Pulsars	427
8	Black Holes	429
8.1	Introduction	429
8.2	Proof of Israel's Theorem	430
8.2.1	Foliation of Σ , Ricci Tensor, etc.	431
8.2.2	The Invariant ${}^{(4)}R_{\alpha\beta\gamma\delta} {}^{(4)}R^{\alpha\beta\gamma\delta}$	434
8.2.3	The Proof (W. Israel, 1967)	435
8.3	Derivation of the Kerr Solution	442
8.3.1	Axisymmetric Stationary Spacetimes	443
8.3.2	Ricci Circularity	444
8.3.3	Footnote: Derivation of Two Identities	446
8.3.4	The Ernst Equation	447
8.3.5	Footnote: Derivation of Eq. (8.90)	449
8.3.6	Ricci Curvature	450
8.3.7	Intermediate Summary	455
8.3.8	Weyl Coordinates	455
8.3.9	Conjugate Solutions	458
8.3.10	Basic Equations in Elliptic Coordinates	459
8.3.11	The Kerr Solution	462
8.3.12	Kerr Solution in Boyer–Lindquist Coordinates	464
8.3.13	Interpretation of the Parameters a and m	465
8.3.14	Exercises	467
8.4	Discussion of the Kerr–Newman Family	467
8.4.1	Gyromagnetic Factor of a Charged Black Hole	468
8.4.2	Symmetries of the Metric	470
8.4.3	Static Limit and Stationary Observers	470
8.4.4	Killing Horizon and Ergosphere	471
8.4.5	Coordinate Singularity at the Horizon and Kerr Coordinates	475
8.4.6	Singularities of the Kerr–Newman Metric	476
8.4.7	Structure of the Light Cones and Event Horizon	476
8.4.8	Penrose Mechanism	477
8.4.9	Geodesics of a Kerr Black Hole	478
8.4.10	The Hamilton–Jacobi Method	478
8.4.11	The Fourth Integral of Motion	480
8.4.12	Equatorial Circular Geodesics	482
8.5	Accretion Tori Around Kerr Black Holes	485
8.5.1	Newtonian Approximation	486
8.5.2	General Relativistic Treatment	488

8.5.3	Footnote: Derivation of Eq. (8.276)	490
8.6	The Four Laws of Black Hole Dynamics	491
8.6.1	General Definition of Black Holes	491
8.6.2	The Zeroth Law of Black Hole Dynamics	492
8.6.3	Surface Gravity	492
8.6.4	The First Law	495
8.6.5	Surface Area of Kerr–Newman Horizon	495
8.6.6	The First Law for the Kerr–Newman Family	496
8.6.7	The First Law for Circular Spacetimes	497
8.6.8	The Second Law of Black Hole Dynamics	502
8.6.9	Applications	503
8.7	Evidence for Black Holes	505
8.7.1	Black Hole Formation	505
8.7.2	Black Hole Candidates in X-Ray Binaries	507
8.7.3	The X-Ray Nova XTE J118+480	508
8.7.4	Super-Massive Black Holes	509
	Appendix: Mathematical Appendix on Black Holes	511
8.8.1	Proof of the Weak Rigidity Theorem	511
8.8.2	The Zeroth Law for Circular Spacetimes	512
8.8.3	Geodesic Null Congruences	514
8.8.4	Optical Scalars	515
8.8.5	Transport Equation	516
8.8.6	The Sachs Equations	519
8.8.7	Applications	520
8.8.8	Change of Area	522
8.8.9	Area Law for Black Holes	525
9	The Positive Mass Theorem	527
9.1	Total Energy and Momentum for Isolated Systems	528
9.2	Witten's Proof of the Positive Energy Theorem	531
9.2.1	Remarks on the Witten Equation	534
9.2.2	Application	534
9.3	Generalization to Black Holes	535
9.4	Penrose Inequality	537
9.4.1	Exercises	538
	Appendix: Spin Structures and Spinor Analysis in General Relativity	538
9.5.1	Spinor Algebra	538
9.5.2	Spinor Analysis in GR	542
9.5.3	Exercises	545
10	Essentials of Friedmann–Lemaître Models	547
10.1	Introduction	547
10.2	Friedmann–Lemaître Spacetimes	549
10.2.1	Spaces of Constant Curvature	550
10.2.2	Curvature of Friedmann Spacetimes	551