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粘性/粘弹性流体流动和热迁移问题的微分求积法

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Shanghai University Doctoral Dissertation (2004)

Differential Quadrature Method for Viscous/Viscoelastic Fluid Flow and Heat Transfer Problems

Candidate: Abdul-Sattar Jaber Ali Al-Saif
Major: Fluid Mechanics
Supervisor: Professor Zhu Zheng-you

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上 海 大 学

本论文经答辩委员会全体委员审查，确认符合上海大学博士学位论文质量要求.

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答辩委员会对论文的评语

微分求积法(DQM)是求解非线性边值问题的一种有效的方法. 论文作者研究并发展了 DQM, 并应用于二维不可压缩流动和传热问题, 论文选题恰当, 具有重要的理论意义和应用价值, 论文的主要成果如下.

1. 为了求解二维不可压的 Navier-Stokes 方程和热传导的耦合问题, 提出了一种混合 DQM, 并进行了数值模拟. 与其他方法进行比较, 说明该方法具有适合于较大雷诺数时的流体流动问题的计算, 且具有精度高、收敛性好、工作量少等优点.

2. 提出了具有迎风机制的局部微分求积法, 使其能够用来求解不规则区域的流动问题, 拓宽了 DQM 的应用范围.

3. 将小参数展开得到了两个平行板中的不可压二阶粘弹性流体的二维稳态流问题的零阶和一阶近似方程, 并利用 DQM 成功地进行了数值模拟.

4. 对两个平行平板之间的二维不可压二阶粘弹性流体的热传导耦合的稳态流动, 提出了一种将 DQ 方法和分裂区域技巧相结合的方法, 对零阶和一阶近似方程成功地进行了的数值模拟, 得到了区域边界比较准确的结果. 使 DQM 能够比较灵活地用来求解问题.

该论文拓宽了 DQM 的应用范围, 具有一定的创新性. 论文构思严密, 理论正确, 结果可信, 表述清楚, 论文表明作者已掌握了扎实系统的基础理论和宽广深入的专门知识, 具备了较强的独立从事科学研究的能力. 该论文已达到博士学位论文的水平.

答辩委员会表决结果

答辩中表达清楚，回答问题正确，经答辩委员会讨论，全票(5 票)通过答辩，建议授予博士学位

答辩委员会主席：鲁传敬

2004 年 5 月 26 日

摘 要

一般讲，在流体力学中由于控制方程是非常复杂的非线性方程组，所以不可能得到问题的精确解. 因此为了得到非线性方程组的解，研究者们提出了各种数值计算方法，有限差分法(FD)和有限元(FE)是其中两类常用的方法. 实际上，在许多场合中我们只需要在少数点上求得适当精度的解就够了. 但是在采用 FD 和 FE 时，为了得到在少数点上适当精确的解往往需要使用大量的网格点. 因此当使用这些方法时需要很大的工作量和存储量. 但是，如果采用 1970 年 Bellman 提出的微分求积法(DQ)则只需要少量的节点就能得到较高精度的解. 此外，由于 DQ 还具有使用方便、节点间距选取任意等优点，因此近年来吸引了许多研究者的注意.

传统的 DQ 只适用于正规区域的问题，并且缺少迎风机制来处理流体流动的对流性质. 为了使 DQ 能适用于求解不规则区域中的流体流动问题，本文中提出了一种具有迎风机制的局部化的 DQ(称为 ULDQ). 利用 ULDQ 对一些不可压与热迁移耦合的粘性和粘弹性流体的二维流动问题求得了满意的数值结果. 本文的主要成果如下：

(1) 因为在传统的 DQM 中缺乏迎风机制来刻画流体流动中的对流特性，所以当雷诺数 Re 较大时，流体流动的数值试验常常失败. 在本文中提出了一种预估校正法，它在每一时间迭代步中，对对流项先用传统的迎风差分格式进行了预估，然后用 DQ 对全部空间变量进行校正，这种方法称为混合微分求积法. 利用

这种方法对二维不可压的 Navier-Stokes 方程和热迁移的耦合问题进行了数值模拟. 结果表明这种混合微分求积法对较大雷诺数时的流体的计算仍是适用的，和传统的有限差分方法相比，混合微分求积法具有较好的收敛性、较高的精度和较少工作量等优点.

(2) 虽然微分求积法已被用来求解许多流体力学中的问题，但它们仅局限于正规区域的解，同时缺乏迎风机制来刻画流体流动的对流性质. 本文中提出了一种方法，它直接把迎风机制引入到 DQ 中，同时使用一种局部化的技巧来处理不规则区域的流动问题. 我们称经过上述改进的 DQ 为迎风的局部微分求积法. 利用这种方法对一个在不规则区域中的二维不可压的流体和热迁移的耦合问题进行了数值模拟. 和差分法相比较，这种方法更为精确，并且具有较少的工作量.

(3) 本文讨论了在两个平行板中的不可压的二阶粘弹性流体的二维稳态流体问题. 利用对方程中的小参数进行展开的方法，求得了零阶和一阶近似方程. 利用 DQM 和本文提出的一种迭代方法成功地得到了问题的数值解. 数值结果表明在入口处，流体的弹性对流动有明显的影响；而在离入口较远处，流体弹性的影响很微弱.

(4) 进一步讨论了在两个平行板中的不可压的二阶粘弹性流体和热迁移耦合的二维稳态流动问题，其中，在能量方程中包含了一个粘性的耗散函数. 当流体的弹性性质微弱时，利用摄动方法得到了零阶和一阶近似方程. 为了在管壁附近得到较高精度的数值解，半区域被分成两个小区域，其一是靠近板壁的薄区域，称为内部区域；另一是厚的区域称为外部区域. 在内部区域和外部区域中的控制方程分别用 DQ 方法进行离散；并对得到的离散

方程用分界面处的匹配条件连接起来(这种技巧称为区域分裂技巧). 本文给出了一种迭代方法来求解所得到的离散方程组. 利用这种区域分裂技巧和 DQ 方法成功地完成了两个平行平板之间的二维不可压二阶粘弹性流体的热迁移耦合的稳态流动的数值模拟. 所得数值结果和已有结果比较在定性上是非常吻合的. 根据数值结果以看到，在中心线附近弹性的影响是微弱的，在交界面和平板附近，弹性的影响很强. 同时数值结果也表明，使用区域分裂技巧的 DQ 是一种非常有效的离散方法，它具有精度高、收敛性好以及工作量少等优点.

关键词 微分求积法, Navier-Stokes 方程, 热传导, 粘性/粘弹性流体流动, 不规则区域, 迎风格式

Abstract

In general, it is impossible to obtain the exact solutions of problems in fluid mechanics, due to their governing equations are a very complicated system of nonlinear equations. Hence various numerical methods are presented for solving these ones. The finite difference (FD) method and finite element (FE) method are two kinds of usual methods. In many cases it is enough to obtain moderately accurate solutions at a few points. But one has to use a large quantity of grid points in order to obtain moderately accurate solutions at a few points as the FD method and FE method are adopted. Thus, a large quantity of the computational workloads and storages are required when these methods are used. However, for the differential quadrature (DQ) method proposed by R. Bellman in 1970, it only needs applying a few grid points in order to get high-precise solutions. Furthermore the DQ method possesses advantages including easy to use and arbitrary to choose the grid spacing. Hence the DQ method has attracted many researchers attention in recent years.

But the traditional DQ method is only suitable for solving problems with regular domains and there is lack of upwind mechanism to treat with the convection in the fluid flow. In order to make the DQ method appropriate for solving problems in the fluid mechanics with irregular domains, a localized DQ method having upwind mechanism (ULDQ) is proposed in this dissertation. By using

the ULDQ method, some numerical results for incompressible two-dimensional flow problems of viscous or viscoelastic fluid coupled with heat transfer are obtained. The main results contain as follows:

(1) Because there is a lack of upwind mechanism to characterize the convection of the fluid flow in the traditional differential quadrature method, so the numerical experiments for fluid flow are usually defeated as the Reynolds number are larger. In Chapter 3, at the every temporal iterative step, a prediction-adjustment method in which at first the traditional upwind difference scheme is applied to predict for convective terms, then the DQ method are used to adjust all terms in spatial variables. This method is called the mixed differential quadrature method. By using this mixed method the numerical experiments for the coupled problems of two-dimensional incompressible Navier-Stokes equations with heat equation are made successfully. The results obtained show that the mixed differential quadrature method is appropriate to solve the fluid flow with higher Reynolds numbers. These results also point out that the mixed differential quadrature method owns advantages including the good convergence, high accuracy and less workloads comparing with the conventional differential quadrature method.

(2) Although the differential quadrature method has been applied to solve various problems in fluid mechanics successfully, it is limited with regular domains and an absence upwind mechanism to characterize the convection of the fluid flow. The upwind mechanism is directly introduced into the traditional DQ method and a

localization technique is applied to deal with the irregularity of flow regions in Chapter 4. The DQ method improved above is called the upwind local differential quadrature method. By using this method, the numerical simulations for the coupled problem of incompressible laminar flow with heat transfer in an irregular region are made successfully. Comparing with the low-order finite difference method, the upwind local differential quadrature method is more accurate and it only requires less computational workload.

(3) A problem of two-dimensional steady flow for an incompressible second-order viscoelastic fluid between two parallel plates is discussed by using the perturbation technique in Chapter 5. By expanding the governing equations with respect to a small parameter, the zero and first order approximation equations are obtained. By using the differential quadrature method and an iterative technique presented in the thesis the numerical solution is obtained successfully

The numerical results show that the effect of elasticity of fluid upon the flow is evident at the entrance near the wall and weak far from the entrance. At the same time, the results also show that the more accurate numerical solutions with a few grid points can be obtained when we use the DQ method.

(4) A P roblem of two-dimensional steady flow of an incompressible second-order viscoelastic fluid coupled with heat transfer between parallel plates is discussed. A viscous dissipation function is included in the energy equation. When the elastic property of the fluid is weaker, the zero-order and first-order governing

equations are gotten from the perturbation method. In order to obtain highly accurate numerical solutions near the tube wall, the half-domain is divided into two sub-domains, in which one is thin near the wall, it is the inner domain and another is thick, it is the outer domain. The governing equations in the inner and outer domain are discretized respectively by using DQ method. The matching conditions at the interface are presented to link obtained discrete governing equations and we call this technique the technique of splitting domain. An iterative method for solving these discretized equations is given in Chapter 6. By the DQ method and technique of splitting domain proposed here, the numerical simulations for the problem of two-dimensional steady flow of an incompressible second-order viscoelastic fluid coupled with heat transfer between parallel plates are carried out successfully. The numerical results obtained are in agreement with existing results qualitatively. From these numerical results we can see that the effect of elasticity on flow near the centerline is slight and the effects in the interface and near the wall are strong. At the same time, the numerical results also show that the differential quadrature method with the technique of splitting domain is a very efficient method and it owns advantages including the higher accuracy and good convergence as well as the less computation workload.

Key words differential quadrature method, Navier-Stokes equation, heat transfer, viscous/viscoelastic fluid flow, irregular domain, upwind scheme

Contents

Chapter One Introduction

1.1 Background

In the resent years, the research of fluid dynamics, that is a branch of applied mathematics, has given an impetus to develop the theory of partial differential equations, theory of complex variables, vector and tensor analysis and nonlinear methods. Although the existing numerical computational software has become a common and effective tool of design and planning, the research to develop efficient numerical methods for solving partial differential equations, especially problems in fluid dynamics, has become an important topic.

To study fluid flow problems it is necessary to know fluid properties, for example, density, specific weight, specific gravity, absolute or dynamic viscosity and so on. More details about these properties can be found in any textbook of fluid dynamics, such as Munson *et al*[1] and Franzini and Finnemone[2]. Furthermore, it is also necessary to set up a rational mathematical model describing a particular flow phenomenon. In fact, the problem of real fluid flow is of great complexity due to the many physical effects and a considerable set of non-linear partial differential equations involved.