

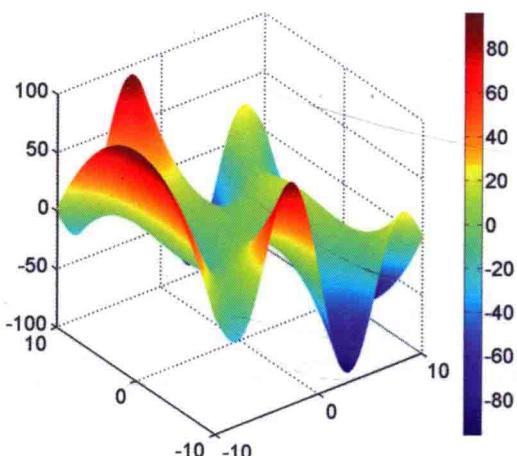
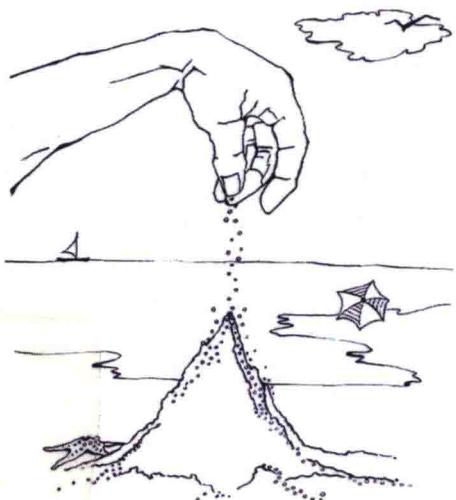


# Extremal Optimization

Fundamentals,  
Algorithms and Applications

极值优化：原理、算法和应用

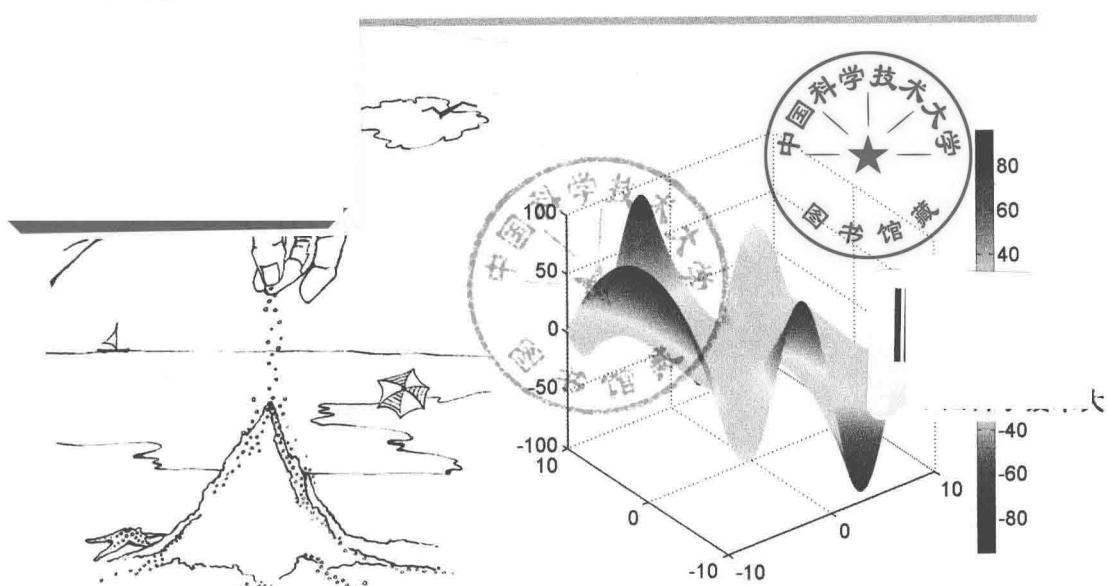
吕勇哉 陈玉旺 陈泯融 陈 鹏 曾国强 著



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## 图书在版编目(CIP)数据

极值优化：原理、算法和应用 = Extremal Optimization: Fundamentals, Algorithms and Applications:  
英文/吕勇哉等著. —北京：化学工业出版社，2016.6

ISBN 978-7-122-27001-6

I. ①极… II. ①吕… III. ①极值 (数学)-英文  
IV. ①0172

中国版本图书馆 CIP 数据核字 (2016) 第 095422 号

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责任编辑：宋 辉 吴 刚  
责任校对：宋 夏

装帧设计：张 辉

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出版发行：化学工业出版社（北京市东城区青年湖南街 13 号 邮政编码 100011）  
印 装：北京京华彩印刷有限公司  
710mm×1000mm 1/16 印张 21 1/4 字数 410 千字 2016 年 8 月北京第 1 版第 1 次印刷

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购书咨询：010-64518888（传真：010-64519686） 售后服务：010-64518899  
网 址：<http://www.cip.com.cn>  
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定 价：128.00 元

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# Extremal Optimization

Fundamentals, Algorithms, and Applications

Yong-Zai Lu • Yu-Wang Chen  
Min-Rong Chen • Peng Chen  
Guo-Qiang Zeng



Chemical Industry Press



CRC Press

Taylor & Francis Group  
Boca Raton London New York

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CRC Press is an imprint of the  
Taylor & Francis Group, an **Informa** business

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# Preface

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With the high demand and the critical situation of solving hard optimization problems we are facing in social, environment, bioinformatics, traffic, and industrial systems, the development of more efficient novel optimization solutions has been a serious challenge to academic and practical societies in an information-rich era. In addition to the traditional math-programming-inspired optimization solutions, computational intelligence has been playing an important role in developing novel optimization solutions for practical applications. On the basis of the features of system complexity, a new general-purpose heuristic for finding high-quality solutions to NP-hard (nondeterministic polynomial-time) optimization problems, the so-called “extremal optimization (EO),” was proposed by Boettcher and Percus. In principle, this method is inspired by the Bak–Sneppen model of self-organized criticality describing “far-from-equilibrium phenomena,” from statistical physics, a key concept describing the complexity in physical systems. In comparison with other modern heuristics, such as simulated annealing, genetic algorithm (GA), through testing on some popular benchmarks (TSP [traveling salesman problem], coloring, K-SAT, spin glass, etc.) of large-scale combinatorial-constrained optimization problems, EO shows superior performance in the convergence and capability of dealing with computational complexity, for example, the phase transition in search dynamics and having much fewer tuning parameters.

The aim of this book is to introduce the state-of-the-art EO solutions from fundamentals, methodologies, and algorithms to applications based on numerous classic publications and the authors’ recent original research results, and to make EO more popular with multidisciplinary aspects, such as operations research, software, systems control, and manufacturing. Hopefully, this book will promote the movement of EO from academic study to practical applications. It should be noted that EO has a strong basic science foundation in statistical physics and bioevolution, but from the application point of view, compared with many other metaheuristics, the application of EO is much simpler, easier, and straightforward. With more studies in EO search dynamics, the hybrid solutions with the marriage of EO and other metaheuristics, and the real-world application, EO will be an additional weapon to

deal with hard optimization problems. The contents of this book cover the following four aspects:

1. General review for real-world optimization problems and popular solutions with a focus on computational complexity, such as “NP-hard” and the “phase transitions” occurring on the search landscape.
2. General introduction to computational extremal dynamics and its applications in EO from principles, mechanisms, and algorithms to the experiments on some benchmark problems such as TSP, spin glass, Max-SAT (maximum satisfiability), and graph partition. In addition, the comparisons of EO with some popular heuristics, for example, simulated annealing and GA, are given through analytical and simulation studies.
3. The studies on the fundamental features of search dynamics and mechanisms in EO with a focus on self-organized optimization, evolutionary probability distribution, and structure features (e.g., backbones) are based on the authors’ recent research results. Moreover, modified extremal optimization (MEO) solutions and memetic algorithms are presented.
4. On the basis of the authors’ research results, the applications of EO and MEO in multiobjective optimization, systems modeling, intelligent control, and production scheduling are presented.

The authors have made great efforts to focus on the development of MEO and its applications, and also present the advanced features of EO in solving NP-hard problems through problem formulation, algorithms, and simulation studies on popular benchmarks and industrial applications. This book can be used as a reference for graduate students, research developers, and practical engineers when they work on developing optimization solutions for those complex systems with hardness that cannot be solved with mathematical optimization or other computational intelligence, such as evolutionary computations. This book is divided into the following three parts.

Section I: Chapter 1 provides the general introduction to optimization with a focus on computational complexity, computational intelligence, the highlights of EO, and the organization of the book; Chapter 2 introduces the fundamental and numerical examples of extremal dynamics-inspired EO; and Chapter 3 presents the extremal dynamics-inspired self-organizing optimization.

Section II: Chapter 4 covers the development of modified EO, such as population-based EO, multistage EO, and modified EO with an extended evolutionary probability distribution. Chapter 5 presents the development of memetic algorithms, the integration of EO with other computational intelligence, such as GA, particle swarm optimization (PSO), and artificial bee colony (ABC). Chapter 6 presents the development of multiobjective optimization with extremal dynamics.

Section III includes the applications of EO in nonlinear modeling and predictive control, and production planning and scheduling, covered in Chapters 7 and 8, respectively.

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# Acknowledgments

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This book was written based on the published pioneering research results on extremal dynamics-inspired optimization, and the authors' recent research and development results on the fundamentals, algorithms, and applications of EO during the last decade at both Shanghai Jiao Tong University (SJTU) and Zhejiang University, China. We wish to thank the Department of Automation, SJTU, the Research Institute of Cyber Systems and Control, Zhejiang University, and the Research Institute of Supcon Co. for their funding of PhD programs and research projects. We are most grateful to Professor J. Chu at Zhejiang University, Professors Y. G. Xi and G. K. Yang at SJTU, and Directors Y. M. Shi and Z. S. Pan of Supcon Co. for their strong support and encouragement.

In particular, we are deeply indebted to the members of Chinese Academy of Engineering: Professor C. Wu and Professor T. Y. Chai; and SJTU Professor G. K. Yang who have freely given of their time to review the book proposal and write suggestions and recommendations.

We are grateful to Chemical Industrial Press for providing the funding to publish this book, and also to Ms. H. Song (commissioning editor, Chemical Industrial Press) and the staff of CRC Press for their patience, understanding, and effort in publishing this book.

This book was also supported by the National Natural Science Foundation of China (Nos. 61005049, 51207112, 61373158, 61472165, and 61272413), Zhejiang Province Science and Technology Planning Project (No. 2014C31074), National High Technology Research and Development Program of China (No. 2012AA041700), National Major Scientific and Technological Project (No. 2011ZX02601-005), National Science and Technology Enterprises Technological Innovation Fund (No. 11C26213304701), Zhejiang Province Major Scientific and Technological Project (No. 2013C01043), and the State Scholarship Fund of China.

Finally, we thank the relevant organizations for their permission to reproduce some figures, tables, and math formulas in this book. See specific figures for applicable source details.

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# **FUNDAMENTALS, METHODOLOGY, AND ALGORITHMS**

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I



# *Chapter 1*

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# General Introduction

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## 1.1 Introduction

With the revolutionary advances in science and technology during the last few decades, *optimization* has been playing a more and more important role in solving a variety of real-world systems for modeling, optimization, and decision problems. The major functions of *optimization* are to provide one or multiple solutions that are able to optimize (e.g., minimize or maximize) the desired objectives subject to the given constraints in the relevant search space. The optimization techniques have been popularly applied in business, social, environmental, biological, medical, man-made physical and engineering systems, etc. Due to the increase in computational complexity, traditional mathematics-inspired optimization solutions, such as mathematical programming (e.g., linear programming [LP], nonlinear programming [NLP], mix-integer programming [MIP]), can hardly be applied to solving some real-world complex optimization problems, such as the NP-hard (nondeterministic polynomial-time hard) problems (Korte and Vygen, 2012) defined in computational complexity theory.

To make optimization solutions applicable, workable, and realistic for those complex systems with roughing search landscape, and limited or no mathematical understanding (i.e., knowledge) between decision variables, desired criteria, and constraints, a number of alternative multidisciplinary optimization approaches have been developed. Instead of traditional optimization methodologies and algorithms, computer science and computational intelligence (CI)-inspired *meta-heuristics search* optimization solutions (Patrick and Michalewicz, 2008) have been developed. The CI (Engelbrecht, 2007) applied in optimization is to stimulate a set of natural ways to deal with complex computational problems in optimization solutions. The major features of CI are to model and stimulate the behaviors and features of