Undergraduate Texts in Mathematics

Serge Lang

Calculus of Several Variables

Third Edition

多元微积分 第3版

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Foreword

The present course on calculus of several variables is meant as a text, either for one semester following *A First Course in Calculus*, or for a year if the calculus sequence is so structured.

For a one-semester course, no matter what, one should cover the first four chapters, up to the law of conservation of energy, which provides a beautiful application of the chain rule in a physical context, and ties up the mathematics of this course with standard material from courses on physics. Then there are roughly two possibilities:

> *One* is to cover Chapters V and VI on maxima and minima, quadratic forms, critical points, and Taylor's formula. One can then finish with Chapter IX on double integration to round off the one-term course.
> *The other* is to go into curve integrals, double integration, and Green's theorem, that is Chapters VII, VIII, IX, and X, §1. This forms a coherent whole.

Both paths have been followed at Yale, and they depend on the fashion of the moment, or the emphasis given to connections with other fields (physics or economics, for instance). I have no preference for either. Either way has considerable unity of style. Many of the results are immediate corollaries of the chain rule. The main idea is that given a function of several variables, if we want to look at its values at two points P and Q, we join these points by a curve (often a straight line segment), and then look at the values of the function on that curve. By this device, we are able to reduce a large number of problems in several variables to problems and techniques in one variable. For instance, the tangent plane, the directional derivative, the law of conservation of energy, and Taylor's formula are all handled in this manner.

One advantage of covering Green's theorem is that it provides a very elegant mixture of integration and differentiation techniques in one and two variables. This mixing is used frequently in applications to physics, and also serves to fix these techniques in the mind because of the way they are used. On the other hand, maxima-minima, critical points, and Taylor's formula find applications in linear programming, economics, and optimization problems. The only clear fact is that there is not enough time to cover both paths in one semester.

For a year's course, the rest of the book provides an adequate amount of material to be covered during the second semester. It consists of three topics, which are logically independent of each other and could be covered in any order. Some order must be chosen because it is necessary to project the course in a totally ordered way on the page axis (and the time axis), but logically, the choice is arbitrary. Pedagogically, the order chosen here seemed the one best suited for most people. These three topics are:

(a) Whichever curve integrals—Green's theorem, or maxima-minima-Taylor's formula were omitted from the first semester.
(b) Triple integration and surface integrals, which continue ideas of Chapters IX and X.
(c) Inverse mappings and the change of variables formula, including as much of matrices and determinants as are needed, and which may have been covered in another course about linear algebra.

Different instructors will cover these three topics in whatever order they prefer. For applications to economics, it would make sense to cover the chapters on maxima-minima and the quadratic form in Taylor's formula before doing triple integration and surface integrals. The methods used depend only on the techniques developed as corollaries of the chain rule.

I think it is important that even at this early stage, students acquire the idea that one can operate with differentiation just as with polynomials. Thus §4 of Chapter VI could be covered early.

I have included only that part of linear algebra which is immediately useful for the applications to calculus. My *Introduction to Linear Algebra* provides an appropriate text when a whole semester is devoted to the subject. Many courses are still structured to give primary emphasis to the analytic aspects, and only a few notions involving matrices and linear maps are needed to cover, say, the chain rule for mappings of one space into another, and to emphasize the importance of linear approximations. These, it seems to me, are the essential ingredients of a second semester of calculus for students who want to become acquainted rapidly with the most important basic notions and how they are used in practice. Many years ago, there was no linear algebra introduced in calculus courses. Intermediate years have probably seen an excessive amount—more than was needed. I try to strike a proper balance here.

Some proofs have been included. On the whole, our policy has been to include those proofs which illustrate fundamental principles and are free of technicalities. Such proofs, which are also short, should be learned by students without difficulty. Examples are the uniqueness of the potential function, the law of conservation of energy, the independence of an integral on the path if a potential function exists, Green's theorem in the simplest cases, etc.

Other proofs, like those of the chain rule, or the local existence of a potential function, can be given in class or omitted, depending on the level of interest of a class and the taste of the instructor. For convenience, such proofs have usually been placed at the end of each section.

Many worked-out examples have been added since previous editions, and answers to some exercises have been expanded to include more comprehensive solutions. I have done this to lighten the text on occasion. Such expanded solutions can also be viewed as worked-out examples simply placed differently, allowing students to think before they look up the answer if they have troubles with the problem.

I include an appendix on Fourier series, for the convenience of courses structured so that it is desirable to give an inkling of this topic some time during the second-year calculus, without waiting for a course in advanced calculus. It fits in nicely with scalar products.

I would like to express my appreciation for the helpful guidance provided by previous reviewers: M. B. Abrahamse (University of Virginia), Sherwood F. Ebey (University of the South), and William F. Keigher (Rutgers University).

I thank Anthony Petrello for working out many answers. I thank Mr. Gimli Khazad, William Scott and Gerhard Kroiss for communicating to me a number of misprints and corrections. I thank Ron Infante for helping with the proofreading.

New Haven, Connecticut S. LANG

Contents

Note. *Chapter IX on Double Integrals is self contained, and could be covered here.*

Part One

Basic Material

In the first chapter of this part, we consider vectors, which form the basic algebraic tool in investigating functions of several variables. The differentiation aspects of them which we take up are those which can be handled up to a point by "one variable" methods. The reason for this is that in higher dimensional space, we can join two points by a curve, and study a function by looking at its values only on this curve. This reduces many higher dimensional problems to problems of a one-dimensional situation.

CHAPTER I

Vectors

The concept of a vector is basic for the study of functions of several variables. It provides geometric motivation for everything that follows. Hence the properties of vectors, both algebraic and geometric, will be discussed in full.

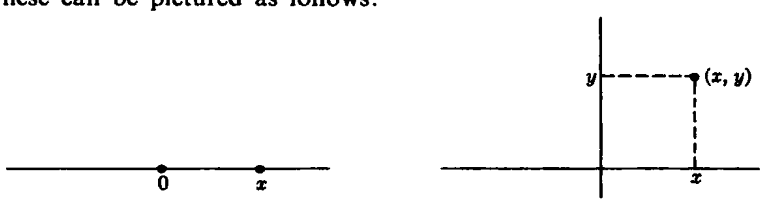
One significant feature of all the statements and proofs of this part is that they are neither easier nor harder to prove in 3-space than they are in 2-space.

I, §1. DEFINITION OF POINTS IN SPACE

We know that a number can be used to represent a point on a line, once a unit length is selected.

A pair of numbers (i.e. a couple of numbers) (x, y) can be used to represent a point in the plane.

These can be pictured as follows:



(a) Point on a line (b) Point in a plane

Figure 1

We now observe that a triple of numbers (x, y, z) can be used to represent a point in space, that is 3-dimensional space, or 3-space. We simply introduce one more axis. Figure 2 illustrates this.

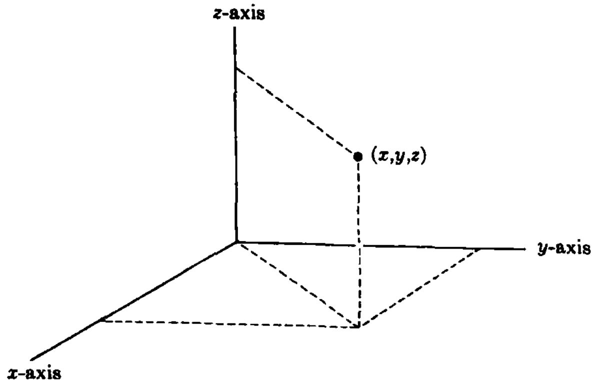


Figure 2

Instead of using x, y, z we could also use (x_1, x_2, x_3). The line could be called 1-space, and the plane could be called 2-space.

Thus we can say that a single number represents a point in 1-space. A couple represents a point in 2-space. A triple represents a point in 3-space.

Although we cannot draw a picture to go further, there is nothing to prevent us from considering a quadruple of numbers.

$$(x_1, x_2, x_3, x_4)$$

and decreeing that this is a point in 4-space. A quintuple would be a point in 5-space, then would come a sextuple, septuple, octuple, ...

We let ourselves be carried away and **define a point in n-space** to be an n-tuple of numbers

$$(x_1, x_2, \dots, x_n),$$

if n is a positive integer. We shall denote such an n-tuple by a capital letter X, and try to keep small letters for numbers and capital letters for points. We call the numbers $x_1, \dots, x_n$ the **coordinates** of the point X. For example, in 3-space, 2 is the first coordinate of the point $(2, 3, -4)$, and -4 is its third coordinate. We denote n-space by $\mathbf{R}^n$.

Most of our examples will take place when $n = 2$ or $n = 3$. Thus the reader may visualize either of these two cases throughout the book. However, three comments must be made.

First, we have to handle $n = 2$ and $n = 3$, so that in order to avoid a lot of repetitions, it is useful to have a notation which covers both these cases simultaneously, even if we often repeat the formulation of certain results separately for both cases.