

Selected Papers of Guo Han-ying

Editorial Committee of
“Selected Papers of Guo Han-ying”

郭汉英

论文 选集

郭汉英选集编委会
选编

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怀念郭汉英(代序)

我和汉英共事开始于1970年,当时汉英在参加相对论批判后认识到需要研究相对论。由于他的努力,相对论批判组调进了一批来自几个研究所的人员,都是在研究工作中有成绩的,陆启铿先生和我也在其中。这个组后来改名物理所十三室,分为两个小组,分别研究相对论和量子场论。汉英是第一个小组的成员,从那时开始一直活跃在理论物理研究的领域。

在“文化大革命”中这个研究室提供了一个难得的能够比较专心于基础研究的小环境,内部的学术气氛和同事关系很好。因为没有写批判相对论的文章,这个研究室受到来自于“四人帮”有关的压力,被派进工作组进行“整顿”,矛头是集中在汉英身上的,但是他和大家一起坚持下来了。这个研究室的一些成员在“文化大革命”后成为著名学者。我自己因此赢得本来不会有的五年宝贵的工作时间,为此,我应当感谢汉英。

1978年以后我和汉英在中国科学院理论物理研究所共事。汉英在研究工作中常找人讨论,我也是其中之一。他能够听取别人的意见,不介意别人的批评。汉英对研究工作是很执著的。最后的十余年中他患有比较重的心脏病,但他一直坚持工作,虽然年近七十,仍每天到办公室做研究,有时周末也是如此,工作的强度不逊于健康的年青人。这种精神令人敬佩。

他在四十年间发表了二百余篇论文,在时空性质、引力理论和理论物理以及数学物理的其他几个方面提出了不少新的见解,做出了重要贡献。这本选集收录了他的部分论文,从中可以了解他的学术成就。

汉英的突然离去,使他的许多朋友、同行和学生悲伤。这本选集的出版也是寄托大家对他的怀念。

戴元本

2010年12月24日

永久的纪念

郭汉英同志聪明,很有才智,知识广博,对事物的反应很快。他和我及其他人等合作写过多篇文章,从1970年到2010年去世前不久,他经常到我的办公室讨论学术问题。得悉他病重之后,我曾赶到中关村医院重症病房去看望,然为时已晚,未能最终一见。然而,在我办公室的黑板上,我们讨论学问时,他所写的整黑板公式还没有擦去,睹物思人,不胜唏嘘。黑板上的粉笔痕迹,日久终于消失,他的文集出版,将给人们一个永久的纪念。

陆启铿

2011年3月

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Part I

量子群

Quantum Group and Related

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$SU_{q, \hbar \rightarrow 0}(2)$ and $SU_{q, \hbar}(2)$, the classical and quantum q -deformations of the $SU(2)$ algebra

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Abstract. By means of classical harmonic oscillators and their canonical quantization, we show that the q -deformation of the $SU(2)$ Lie algebra can be realized at a classical level in the sense of Poisson brackets, denoted by $SU_{q, \hbar \rightarrow 0}(2)$, and that the canonical quantization of the system leads to the quantum q -deformation of the $SU(2)$ algebra, denoted by $SU_{q, \hbar}(2)$. This means that the q -deformation and the $\hbar$ -quantization are in general two different concepts.

1. Introduction

Recently there has been a great deal of interest in the study of the quantum groups or the quantized enveloping algebras [1–3]. The nomenclature of these beautiful algebraic structures reflects the fact that the q -deformations have so far always emerged together with quantization characterized by the Planck constant $\hbar$ or by some parameter playing a role loosely like $\hbar$. In the classical limit $\hbar \rightarrow 0$, the q -deformation of the Lie algebra disappears and the algebraic structure reduces to the usual Lie algebraic structure. We find, however, that this is not the case in general [4]. In fact, the q -deformation of Lie algebras can be realized at a classical level in the sense of Poisson brackets, while after quantization of the system these classical q -deformed structures give rise to their quantum counterparts. Namely, the q -deformation and the $\hbar$ -quantization are different concepts in principle.

In this paper, we present a concrete example, the case of the $SU(2)$ algebra, by means of harmonic oscillators. First, we realize the classical q -deformation of the $SU(2)$ algebra, denoted by $SU_{q, \hbar \rightarrow 0}(2)$, by a set of Poisson brackets of q -deformed generators $J'_\pm$ and J'_3 on the phase space V with symplectic form Ω of two undeformed one-dimensional linear oscillators. In the limit $q \rightarrow 1$, we find that $SU_{q, \hbar \rightarrow 0}(2)$ reduces to the usual $SU(2)$ algebra in the sense of Poisson brackets of the undeformed generator $J_\pm$, J_3 . It is of interest that in terms of complex coordinates in the phase space the q -deformation is then associated with Beltrami deformations with certain dilatation ratio. Second, we quantize the system canonically and recover the realization of quantum q -deformation of the $SU(2)$ algebra, denoted by $SU_{q, \hbar}(2)$ in our notation, in terms of the q -analogues of the quantum oscillators [5–8]. In our approach, it is easy to see that the classical counterpart of $SU_{q, \hbar}(2)$ is just $SU_{q, \hbar \rightarrow 0}(2)$ and the undeformed