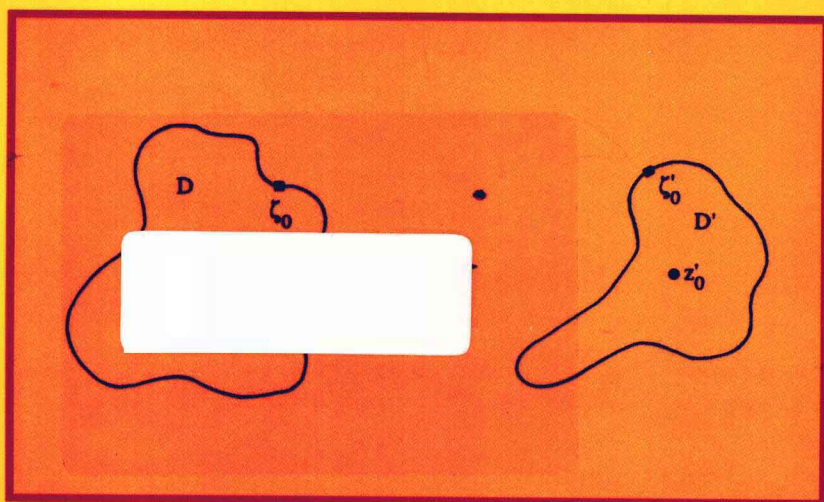


Undergraduate Texts in Mathematics

Bruce P. Palka

An Introduction to Complex Function Theory

复函数论导论



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Bruce P. Palka

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To my parents,
Leonard Palka and Charlotte Fogarty Palka

Preface

The book at hand has its origins in and reflects the structure of a course that I have given regularly over the years at the University of Texas. The course in question is an undergraduate honors course in complex analysis. Its subscribers are for the most part math and physics majors, but a smattering of engineering students, those interested in a more substantial and more theoretically oriented introduction to the subject than our normal undergraduate complex variables course offers, can usually be found in the class. My approach to the course has been from its inception to teach it in everything save scope like a beginning graduate course in complex function theory. (To be honest, I have included some material in the book that I do not ordinarily cover in the course, this with the admitted purpose of making the book a suitable text for a first course in complex analysis at the graduate level.) Thus, the tone of the course is quite rigorous, while its pace is rather deliberate. Faced with a clientele that is bright, but mathematically less sophisticated than, say, a class of mathematics graduate students would be, I considered it imperative to give students access to a complete written record of the goings-on in my lectures, one containing full details of proofs that I might only sketch in class, the accent there being on the central idea involved in an argument rather than on the nitty-gritty technicalities of the proof. I also deemed it wise to provide the students with a generous supply of worked-out examples appropriate to the lecture material. Since none of the textbooks available when I started teaching the course had exactly the emphasis I was looking for, I began to compile my own set of lecture notes. It is these notes that have evolved into the present book.

In rough terms the course I have been describing comprises Chapters I, III, IV, V, VII, and VIII of the book, together with the first three sections of Chapter IX. Chapter II, a resume of information from plane topology, is a reference chapter. It would never occur to me — nor would I recommend to anyone else — to go systematically through this chapter in teaching a complex variables course. Instead, the ideas from Chapter II get dispersed

throughout my lectures, each topological notion being brought up as it becomes germane to the development of complex function theory. While this system works fine in the setting of a lecture, I find it disruptive to the ongoing narrative of a book. Therefore, just as other authors have done before me, I have chosen to assemble all the background material from elementary topology in a single place for ease of reference. Chapters VI and X, and with them the last two sections of Chapter IX, furnish “enrichment topics” to those who wish to proceed slightly beyond the essential core of basic complex analysis. The subject matter in Chapters VI and X would, I think, be regarded as standard in most beginning graduate courses.

Located at the end of each chapter is a collection of exercises. Though some of these are intended to foster the development of the computational skills pertinent to complex analysis, most have a pronounced theoretical flavor to them, in keeping with the course for which they were designed. Many of the “classic” exercises in function theory turn up among these problems. Quite a few of the exercises, on the other hand, are original to this book (or they are, at least, to the best of my knowledge).

It is high time that I expressed my gratitude to everyone who has had a hand in the creation of this book. These individuals include a number of graduate students at Texas — Michael Pearson, Michael Westmoreland, and Edward Burger are three that spring immediately to mind — who carefully read through early versions of the manuscript, helped rid it of numerous errors, and, most importantly, identified places that from a student’s perspective were badly in need of change. I am grateful to colleagues (in particular, to Barbara Flinn and Jean McKemie) who agreed to “field test” portions of the manuscript in their own classes. Their input has greatly improved the finished product. My special thanks go to Aimo Hinkkanen, with whom I’ve had many useful conversations during the final stages of preparation of the book and who has been an invaluable source of suggestions for problems. This book would have remained a pipedream were it not for the diligent efforts of Suzy Crumley, who typed it, and Buff Miner, who did the graphics and generally oversaw the production of the manuscript. Both patiently bore the brunt of my revisionist tendencies. Needless to say, they share none of the blame for the inevitable errors that have crept into the text and managed to escape detection under my proofreading. The editorial staff at Springer-Verlag (notably, Rob Torop and his successor, Ulrike Schmickler-Hirzebruch) have been extremely helpful and understanding. Above all, I appreciate the fact that they did not pressure me with deadlines during my stint as graduate advisor, when my literary output slowed to a trickle. A “tusen tack” goes to the Mittag-Leffler Institute in Djursholm, Sweden, where some finishing touches were applied to the manuscript in the course of my stay there during the academic year 1989-90 (and where Kari Hag and David Herron obliged with some greatly valued proofreading). My teacher, Fred Gehring, has been a source of both inspiration and encouragement for the undertaking. Finally, I would like to

acknowledge the support of my wife, Mary Ann, and my sons, Kevin and Sean. Despite being innocent bystanders, they were often in perfect position to catch the flak of my frustration when things did not go as planned with this project. To them I say: the struggle is over and dad is a happy camper again.

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