Series in Contemporary Applied Mathematics
CAM 17

Hyperbolic Problems

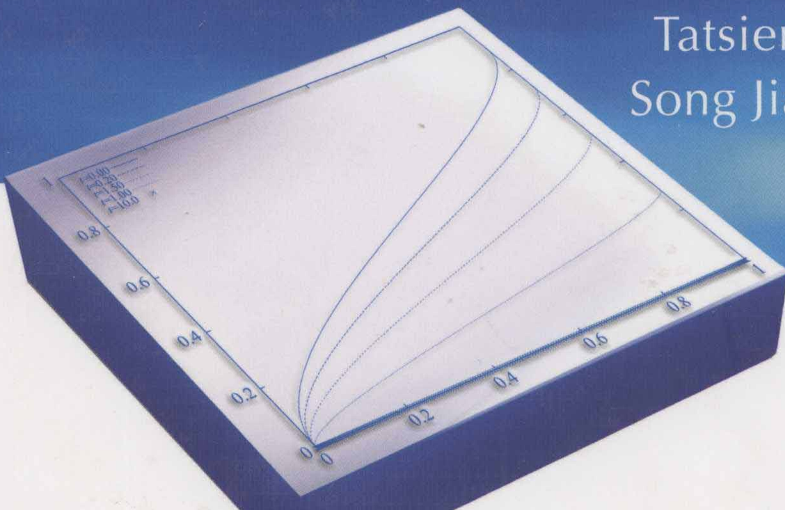
Theory, Numerics and Applications

Volume 1

双曲问题——理论、数值方法及应用

（第一卷）

Tatsien Li
Song Jiang
editors



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Preface

The 13th International Conference on Hyperbolic Problems: Theory, Numerics and Applications (HYP 2010) was held in Beijing, China, from June 15 to 19, 2010. Over 200 participants attended the conference and 162 among them came from abroad. There were 10 plenary lectures, 18 invited talks and around 140 contributed talks in parallel sessions.

The objective of the conference is to bring together experts, researchers and students with interest in theoretical analysis, numerical simulations, and applications of hyperbolic partial differential equations and related mathematical models appearing in applied sciences. The conference keeps the traditional balance of the HYP series, blending theory, numerics and applications, and an emphasis is put on nonlinear problems and applications in various fields such as fluid mechanics, elasticity, astrophysics, biomathematics, traffic flow, etc. As has been done in the past, a special effort has been made it possible for young scientists to attend this conference and to promote their interaction with the more senior researchers.

The biannual HYP series of international conferences on Hyperbolic Problems was initiated by C. Carasso, P.-A. Raviart and D. Serre with the first conference held in Saint-Étienne, France in 1986. Since then it has been organized in Aachen (Germany, 1988), Uppsala (Sweden, 1990), Taormina (Italy, 1992), Stony Brook (USA, 1994), Hong Kong (China, 1996), Zurich (Swizerland, 1998), Magdeburg (Germany, 2000), Pasadena (USA, 2002), Osaka (Japan, 2004), Lyon (France, 2006), and Maryland (USA, 2008). Throughout these years, it has become one of the highest quality and most successful conference series in applied mathematics.

Hyperbolic problems, which are probably originated from Euler's study on the acoustic wave in 1755, not only have a long history but also have extremely rich physical background. The development is highly stimulated by their applications to Physics, Biology, and Engineering Sciences, in particular by the design of effective numerical algorithms. Due to recent rapid development of computers, more and more scientists use hyperbolic partial differential equations and related evolutionary equations as basic tools when propose new mathematical models of various phenomena and related numerical algorithms. We believe that various fields in science and engineering will bring us further into future interests in hyperbolic problems.

The scientific committee of this conference consists of Yann Brenier (France), Alberto Bressan (USA), Constantine Dafermos (USA), Xiaqi Ding (China), Ling Hsiao (China), Rolf Jeltsch (Switzerland), Song Jiang (China), Kenneth Karlsen (Norway), Shuichi Kawashima (Japan), Dietmar Kroener (Germany), Randall LeVeque (USA), Tatsien Li (China), Tai-Ping Liu (USA), Helena J. Nussenzveig Lopes (Brazil), Pierangelo Marcati (Italy), Denis Serre (France), Chi-Wang Shu (USA), Eitan Tadmor (USA) and Zhouping Xin (China). We thank all members of the scientific committee for recommending plenary and invited speakers and promoting this conference.

This conference is supported by the Academy of Mathematics and Systems Science, the Institute of Applied Physics and Computational Mathematics, Capital Normal University, Central China Normal University, Fudan University, Shanghai Jiaotong University, and Wuhan University. It is also sponsored by Shanghai Key Laboratory for Contemporary Applied Mathematics, Wuhan Institute of Physics and Mathematics, Chinese Academy of Sciences, National Natural Science Foundation of China, and Tianyuan Foundation for Mathematics.

These two volumes contain 80 original research and review papers which are written by leading researchers and promising young scientists, and cover a diverse range of multi-disciplinary topics addressing theoretical, modeling and computational issues arising under the umbrella of "Hyperbolic Partial Differential Equations". All the articles are peer-reviewed. We would like to thank all the referees for their efforts done for reading and judging the submitted articles. Special thanks go to Dr. Tao Wang for his laborious job arranging the final layout of articles.

Finally, we are extremely thankful to our colleagues Professors Feimin Huang, Hailiang Li, Yaguang Wang, Huijiang Zhao and Changjiang Zhu, and Dr. Xiaoyun Zhai, Dr. Yi Wang, and many graduate students from the Academy of Mathematics and Systems Science, the Institute of Applied Physics and Computational Mathematics, Capital Normal University, Central China Normal University, Fudan University, Shanghai Jiaotong University and Wuhan University, for their assistance in making this conference successful and comfortable.

June 2011
Tatsien Li, Song Jiang

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SBV Regularity for Scalar Conservation Laws*

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Abstract

We outline a short proof of SBV regularity which can be extended to systems of conservation laws. The fundamental idea is the construction of an interaction measure which controls the creation of jumps.

1 Introduction

In recent years regularity estimates for nonlinear equation have received a lot of attention. A natural regularity question in hyperbolic conservation laws is the following: being the solution u in general BV, under which conditions it belongs to the smaller space Special BV (SBV)? We recall that the space SBV is the space of all BV functions for which the measure derivative u_x does not contain a Cantor part.

Since solution u to Hamilton-Jacobi equations with uniformly convex Hamiltonian are known to be semiconcave, then a similar question can be stated in this cases: we would like to prove that the measure second derivative of u does not have a Cantor part.

In this introduction we first review the most important results concerning SBV regularity.

The first positive result has been given in [1], where it is shown that the solution $u(t)$ of a genuinely nonlinear scalar conservation law in one space dimension is SBV up to countably many times. In that paper, the authors considers the characteristic lines

$$\dot{x} = f'(u(t,x)), \quad u(0,x) = y,$$

*This work is supported by ERC Grant CONSLAW.

and prove the following: every time a Cantor part in $u_x(t)$ appears, then there is a set of positive measure A such that all the characteristics starting from $y \in A$ are defined in the interval $[0,t]$ but cannot be prolonged more than t. By the σ-finiteness of $\mathcal{L}^1$, one can apply the same observation used to prove that the positive part of $u_x(t)$ is absolutely continuous up to countably many times, and deduce that up to countably many times the solution $u(t)$ is SBV.

The use of the measure of the set $A(t)$ of initial points for characteristics which can be prolonged up to time t has been applied to obtain extension of the above result: in [10] the SBV estimate is used for scalar balance laws, later extended to Temple systems in [3] and in [5] to the case of Hamilton-Jacobi equation in several space dimension with uniformly convex Hamiltonian.

In [4] a different approach is introduced. The authors show that there is a bounded measure μ^{jump} which controls the creation of jumps in the entropic solution u to a genuinely nonlinear hyperbolic system of conservation laws. The idea is particularly simple to understand: the genuinely nonlinearity makes easy to create jumps, but only by interaction and cancellation one can remove them. This measure allows to make balances concerning only the continuous part of the measure u_x, and thus to recover SBV regularity as a consequence of decay properties of the solution: in fact, for Burgers equation, the fundamental estimate

$$-\frac{\mathcal{L}^1(B)}{t-T} + \mu^{\mathrm{jump}}(T,t) \leq u_x(T,B) \leq \frac{\mathcal{L}^1(B)}{T} \tag{1.1}$$

resembles the decay estimate for positive waves. In (1.1), B is a Borel set and $0 \leq T \leq t$.

In this short paper we give the fundamental ideas in the particularly simple case of a scalar conservation law with convex flux, and we show how the quantity $f'(u)$ is in SBV, i.e. $\sigma := D_x f(u)$ does not have a Cantor part. The paper is organized as follows. After recalling some basic fact of wavefront tracking approximation, we introduce the wave balances of the quantities $\sigma = D_x f'(u)$ and it positive, negative, continuous and jump part. These balances allows to define the new interaction measure μ^{jump}, and to prove that it is a finite measure on $\mathbb{R}^+ \times \mathbb{R}$. Once this measure has been introduced, we use this to compute the wave balance in regions bounded by characteristics, i.e. solution to the differential inclusion $\dot{x} \in f'(u(t,x))$. The argument at this point is standard, the new ingredient is the use of the measure μ^{jump} to control the continuous part of σ.

2 Preliminaries

We consider the scalar conservation law

$$u_t + f(u)_x = 0, \qquad u(0,x) = u_0(x), \tag{2.1}$$

with $u_0(x) \in \mathrm{BV}(\mathbb{R})$: for simplicity we will assume $|u_0(x)| = 1$. The flux f is assumed smooth, at least $C^2(\mathbb{R})$, and we will denote with f', f'' its first and second derivatives.

A standard method for constructing a sequence of approximate solutions converging to the unique entropy solution of (2.1) is given in [8]: let $\nu \in \mathbb{N}$ and consider the points $\{z2^{-\nu}\}_{z\in\mathbb{Z}}$. Let f^ν be the piecewise linear function whose nodes are at the points $\{(z2^{-\nu}, f(z2^{-\nu}))\}_{z\in\mathbb{Z}}$:

$$f^\nu(u) := (1-\alpha)f\big(z2^{-\nu}\big)+\alpha f\big((z+1)2^{-\nu}\big), \quad u = (1-\alpha)z2^{-\nu}+\alpha(z+1)2^{-\nu}.$$

The standard Riemann solver for f^ν takes values only in $\{z2^{-\nu}\}_{z\in\mathbb{Z}}$ if u^-, u^+ belongs to the same set $\{z2^{-\nu}\}_{z\in\mathbb{Z}}$, so that a solution can be constructed by starting with $u_0^\nu \in BV(\mathbb{R}, \{z2^{-\nu}\}_{z\in\mathbb{Z}})$. In fact, one solves the initial Riemann problem (finitely many!), and then each time two or more waves collide (an interaction point) the procedure starts again by solving this newly generated Riemann problem.

The key argument is that at each interaction either the Glimm interaction functional decreases (or equivalently the number of waves), or the total variation decreases, and in both cases of a fixed positive quantity depending only on ν. Hence the number of interaction is finite, and one has not to worry about the cases of infinite interaction points in finite time.

The convergence to the entropy solution follows because the approximated solutions generate a 1-Lipschitz semigroup in L^1, and the set $BV(\mathbb{R}, \{z2^{-\nu}\}_{z\in\mathbb{Z}})$ is dense in $L^\infty(\mathbb{R}, [0,1])$ as $\nu \to +\infty$ w.r.t. the L^1-norm.

Since the number of interactions is countable, we can perturb a little bit the speed of the waves in order to require that no multiple interaction occurs: only two waves at a time interact. This will simplify the computations in the next sections.

The fundamental assumption on the flux f is the following.

Assumption 2.1. *The flux f is a convex function.*

This implies that the entropy admissible jumps $[u^-, u^+]$ satisfy $u^+ < u^-$. As for notation, by C we will denote a sufficiently large constant.

3 Wave balances

In this section we develop some balances for particular waves measures. These are nonlinear functions of the derivative Du, which is a signed