

严波 皮文丽 编

工程力学专业英语

清华大学出版社

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内 容 简 介

本书选编了工程力学专业涉及的科技英文文章,包括理论力学、材料力学、弹性力学、结构力学、振动力学、塑性力学、实验力学、计算力学、疲劳与断裂、复合材料力学和流体力学等内容,并注意纳入相应课程的新概念,覆盖面广。本书还分单元介绍了英文科技报告、学位论文、期刊论文写作技巧和英汉科技文章的翻译技巧,旨在培养学生科技论文写作和翻译的基本技能。各单元课文正文适用于教师精讲,第一篇阅读材料适用于本科生自学,或在教师指导下学习,第二篇阅读材料内容相对较深,主要适用于研究生学习参考。此外,每篇文章后面均列出了生词和短语,附录中列出了常用专业词汇和短语,便于读者查阅。

本书无论是在内容选材还是在内容编写上均具有特色,是一本适用于工程力学专业和力学类专业本科生的实用教材,也可供土木、机械、材料等相关工科专业的本科生、研究生及相关工程技术人员学习参考。

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前 言

《工程力学专业英语》是在编者编写的《工程力学专业英语阅读教材》(讲义)的基础上改编而成。该讲义最初编写于1997年,于1998年开始在重庆大学工程力学本科专业使用,2002年编者根据教学使用的情况进行了一次修改,至今已经使用了十余年时间。本次正式出版前,编者对原讲义进行了全面的修改。

本书共分14个单元,每一单元包括一篇课文和两篇阅读材料,选材广泛,针对性强,内容新。所有文章均选自英文原文著作,内容涉及理论力学、材料力学、弹性力学、结构力学、振动力学、塑性力学、实验力学、计算力学、疲劳与断裂、复合材料力学和流体力学等工程力学专业主干课程和相关课程的基本概念和内容,并注意纳入了相应课程的新概念和新内容。各单元课文部分适用于教师精讲,第一篇阅读材料适用于本科生自学,第二篇阅读材料适用于研究生学习参考。为便于学生学习掌握工程力学专业英语词汇和概念,每篇文章后均列出了专业词语、词组和惯用语。

为了提高学生科技英文应用能力,书中分单元讲解了英文科技报告和论文的写作以及科技文章英汉翻译的基本技巧。这两部分内容主要参考国内外同类教材编写而成,讲解中尽量给出与力学专业相关的例句。

本书的编写得到重庆大学资源及环境科学学院万玲教授的大力支持,重庆大学工程力学系的部分教师和同学对本书内容提出了很多宝贵的意见和建议,使得本教材得以顺利完成。重庆大学工程力学系研究生崔伟同学认真仔细地描绘了书中所有示图,在此一并表示感谢。

由于编者的水平有限,不当和错误在所难免,恳望读者批评指正。

编 者

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Unit One

Some Basic Concepts of Mechanics^[1,2]

Mechanics is a branch of physics concerned with motion or change in position of physical objects. It is sometimes further subdivided into:

- (1) Kinematics, which is concerned with the geometry of the motion;
- (2) Dynamics, which is concerned with the physical causes of the motion;
- (3) Statics, which is concerned with conditions under which no motion is apparent.

Some important basic concepts occurred in mechanics are described in the following sections.

Mathematical Models

A mathematical description of physical phenomena is often simplified by replacing actual physical objects by suitable mathematical models. For example, in describing the rotation of the earth about the sun we can for many practical purposes treat the earth and sun as points.

Space, Time and Matter

From everyday experience, we all have some idea as to the meaning of each of the following terms or concepts. However, we would certainly find it difficult to formulate completely satisfactory definitions. We take them as undefined concepts.

(1) Space. This is closely related to the concepts of point, position, direction and displacement. Measurement in space involves the concepts of length or distance, with which we assume familiarity. Units of length are feet, meters, miles, etc. Here we assume that space is Euclidean, i. e. the space of Euclid's geometry.

(2) Time. This concept is derived from our experience of having one event taking place after, before or simultaneous with another event. Measurement of time is achieved, for example, by use of clocks. Units of time are seconds, hours, years, etc.

(3) Matter. Physical objects are composed of "small bits of matter" such as atoms and molecules. From this we arrive at the concept of a material object called a particle which can be considered as occupying a point in space and perhaps moving as time goes by. A

measure of the “quantity of matter” associated with a particle is called its mass. Units of mass are grams, kilograms, etc. Unless otherwise stated we shall assume that the mass of a particle does not change with time.

Length, mass and time are often called dimensions from which other physical quantities are constructed.

Scalars and Vectors

Various quantities of physics, such as length, mass and time, require for their specification a single real number (apart from units of measurement which are decided upon in advance). Such quantities are called scalars and the real number is called the magnitude of the quantity. A scalar is represented analytically by a letter such as t, m , etc.

Other quantities of physics, such as displacement, require for their specification a direction as well as magnitude. Such quantities are called vectors. A vector is represented analytically by a bold faced letter such as \mathbf{A} in Fig. 1-1. Geometrically it is represented by an arrow PQ where P is called the initial point and Q is called the terminal point. The magnitude or length of the vector is then denoted by $|\mathbf{A}|$ or A .

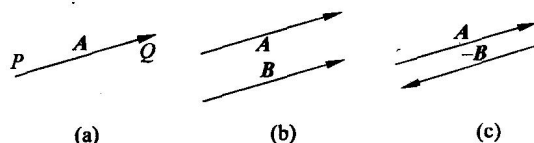


Fig. 1-1

Balancing Forces

A “static” situation is one where all the objects are motionless. If an object remains motionless, then $F=ma$ tells us that the total force acting on it must be zero. (The converse is not true, of course. The total force on an object is also zero if it moves with constant nonzero velocity. But we’ll deal only with statics problems here.) The whole goal in a statics problem is to find out what the various forces have to be so that there is zero net force acting on each object. Since a force is a vector, this goal involves breaking the force up into its components. You can pick Cartesian coordinates, polar coordinates, or another set. It is usually clear from the problem which system will make your calculations easiest. Once you pick a system, you simply have to demand that the total force in each direction is zero.

There are many different types of forces in the world, most of which are large-scale effects of complicated things going on at smaller scales. For example, the tension in a rope comes from the chemical bonds that hold the molecules in the rope together (and these chemical forces are just electrical forces). In doing a mechanics problem involving a rope, there is certainly no need to analyze all the details of the forces taking place at the molecular scale. You simply call the force in the rope a “tension” and get on with the

problem. Four types of forces come up repeatedly:

Tension. Tension is the general name for a force that a rope, stick, etc., exerts when it is pulled on. Every piece of the rope feels a tension force in both directions, except the end point, which feels a tension on one side and a force on the other side from whatever object is attached to the end.

In some cases, the tension may vary along the rope. The “Rope wrapped around a pole” example is a good illustration of this. In other cases, the tension must be the same everywhere. For example, in a hanging massless rope, or in a massless rope hanging over a frictionless pulley, the tension must be the same at all points, because otherwise there would be a net force on at least one tiny piece, and then $F = ma$ would yield an infinite acceleration for this tiny piece.

Normal force. This is the force perpendicular to a surface that the surface applies to an object. The total force applied by a surface is usually a combination of the normal force and the friction force. But for frictionless surface such as greasy ones or ice, only the normal force exists. The normal force comes about because the surface actually compresses a tiny bit and acts like a very rigid spring. The surface gets squashed until the restoring force equals the force the object applies.

Remarks: For the most part, the only difference between a “tension” and a “normal force” is the direction of the force. Both situations can be modeled by a spring. In the case of a tension, the spring (a rope, a stick, or whatever) is stretched, and the force on the given object is directed toward the spring. In the case of normal force, the spring is compressed, and the force on the given object is directed away from the spring. Things like sticks can provide both normal forces and tension. But a rope, for example, has a hard time providing a normal force.

In practice, in the case of elongated objects such as stick, a compressive force is usually called a “compressive tension”, or a “negative tension”, instead of a normal force. So by these definitions, a tension can point either way. At any rate, it’s just semantics. If you use any of these descriptions for a compressed stick, people will know what you mean.

Friction. Friction is the force parallel to a surface that a surface applies to an object. Some surfaces, such as sandpaper, have a great deal of friction. Some, such as greasy ones, have essentially no friction. There are two types of friction, called “kinetic” friction and “static” friction.

Kinetic friction deals with two objects moving relative to each other. It is usually a good approximation to say that the kinetic friction between two objects is proportional to the normal force between them. The constant of proportionality is called μ_k (the “coefficient of kinetic friction”), where μ_k depends on the two surfaces involved. Thus, $F = \mu_k N$, where N is the normal force. The direction of the force is opposite to the motion.

Static friction deals with two objects at rest relative to each other. In the static case, we have $F \leq \mu_s N$ (where μ_s is the “coefficient of static friction”). Note the inequality sign.

All we can say prior to solving a problem is that the static friction force has a maximum value equal to $F_{\max} = \mu_s N$. In a given problem, it is most likely less than this. For example, if a block of large mass M sits on a surface with coefficient of friction μ_s , and you give the block a tiny push to the right (tiny enough so that it doesn't move), then the friction force is of course not equal to $\mu_s N = \mu_s Mg$ to the left. Such a force would send the block sailing off to the left. The true friction force is simply equal and opposite to the tiny force you apply. What the coefficient μ_s tells us is that if you apply a force larger than $\mu_s Mg$ (the maximum friction force on a horizontal table), then the block will speed up moving to the right.

Gravity. Consider two point objects, with masses M and m , separated by a distance R . Newton's gravitational force law says that the force between these objects is attractive and has magnitude $F = GMm/R^2$, where $G = 6.67 \times 10^{-11} \text{ m}^3 / (\text{kg} \cdot \text{s}^2)$. A sphere may be treated like a point mass located at its center. Therefore, an object on the surface of the earth feels a gravitational force equal to

$$F = m(GM/R^2) \equiv mg$$

where M is the mass of the earth, and R is its radius. This equation defines g . Plugging in the numerical values, we obtain (as you can check) $g \approx 9.8 \text{ m/s}^2$. Every object on the surface of the earth feels a force of mg downward. If the object is not accelerating, then there must also be other forces present (normal forces, etc.) to make the total force equal to zero.

Words and Expressions

mechanics [mi'kæniks]	<i>n.</i>	力学
kinematics [ˌkinə'mætiks]	<i>n.</i>	运动学; 动力学
dynamics [dai'næmiks]	<i>n.</i>	动力学; 力学; 动态
statics ['stætiks]	<i>n.</i>	静力学
scalar ['skeilə]	<i>n.</i>	数量; 标量
vector ['vektə]	<i>n.</i>	矢量; 向量
force [fɔ:s]	<i>n.</i>	力; 力量
Cartesian coordinate		笛卡儿坐标
polar coordinate		极坐标
tension ['tenʃən]	<i>n.</i>	张量, 张力, 拉力
acceleration [æk,selə'reiʃən]	<i>n.</i>	加速度
normal force		法向力
friction force		摩擦力
stretch [stretʃ]	<i>vt.</i> & <i>vi.</i>	伸展, 拉紧, 延伸
elongate ['ilɔŋgeit]	<i>vt.</i> & <i>vi.</i>	拉长; 伸长; 延长
kinetic friction		动摩擦
static friction		静摩擦
sandpaper	<i>n.</i>	砂纸
gravitation force		引力; 重力

Writing Skill of Experimental Research Report: INTRODUCTION (1)^[32]

An *experimental research report* is a paper written by an investigator to describe a research study that he or she has completed. The purpose of the report is to explain to others in the field what the objectives, methods, and findings of the study were. The report may be published in a professional *journal*, it may appear as a *monograph* distributed by a research institution or publishing company, or it may be written in the form of a *thesis* or *dissertation* as part of the requirements for a university degree.

The organizational format for all experimental research reports is basically the same, regardless of the field of study in which the author is working. The major sections of a typical experimental research report in the order in which they are usually presented are

ABSTRACT

INTRODUCTION

METHOD

RESULTS

DISCUSSION

The *introduction* serves as an orientation for readers of the report, giving them the perspective they need to understand the detailed information coming in later sections.

Generally, the *introduction* of an experimental research report can be divided into the following five parts, or stages,

Stage I: General statement(s) about a field of research to provide the readers with a *setting* for the problem to be reported.

Stage II: More specific statements about the aspects of the problem *already studied* by other researchers.

Stage III: Statement(s) that indicate *the need for more investigation*.

Stage IV: Very specific statement(s) giving *the purpose/objectives* of the writer's study.

Stage V: Optimal statement(s) that give a *value or justification* for carrying out the study.

However, writers do not always arrange the stages of their *introduction* in this exact order. Sometimes a writer interrupts one stage with another, and then returns to the earlier stage. Sometimes Stage II, which is usually called "The Review of Literature", is completely separated from the rest of the *introduction*. In theses and dissertations, for example, it is often written as a separate chapter. Stage V is often omitted entirely. However, the general plan given here is very common and is the easiest for the beginning research writer to use.

INTRODUCTION: Establishing a Context

In stage I, the writer establishes a context, or frame of reference, to help readers understand how the research fits into a wider field of study.

(1) Information Conventions

(a) Inventing the Setting

You should write the setting (Stage I) of your *introduction* so that it provides your readers with the background necessary to see the particular topic of your research in relation to a general area of study. In order to do this, start with obvious, generally accepted statements about the area in which you are working. Then, step by step, move the readers to your specific topic. You may do this in just a few sentences or in several paragraphs.

You can think of this stage as a process of first, establishing a “universe” for your readers; then, isolating one “galaxy” within this universe; and finally, leading your readers to one “star” in the galaxy. That “star” is your specific topic.

- Begin with accepted statements of fact related to your *general area* (your “universe”).
- Within the general area, identify one *subarea* (your “galaxy” which includes your topic).
- Indicate your *topic* (your “star”).

(b) Linking Ideas through Old and New Information Order

To lead the readers smoothly through the ideas in Stage I, writers link sentences by making use of old and new information. This is done by placing old information—that is, information already known to the readers—at the beginning of sentences and placing new information at the end.

(2) Language Conventions

(a) General and Specific Noun Phrases

As we have seen, Stage I of the *introduction* usually begins with factual statements about the general area which includes your specific topic. When you write these kinds of general statements, it is conventional to use nouns that refer to objects or concepts at the highest possible level of generality. English offers several ways to construct these general nouns, which we examine in this section.

Statements in the setting of an *introduction* tend to be general in nature. Instead of referring to specific things, they often refer to *entire classes* of things. When you write sentences that contain nouns referring to an entire class of things, you should use *generic noun phrases* to carry this meaning. Generic noun phrases refer to all members of a particular class of living things.

In English there are different ways to write generic noun phrases. If the noun is *countable*, you can make it generic by adding the plural marker -s and omitting any article, or by using it in its singular form with the indefinite article *a* or *an*.

Example: *Composite materials* are widely used in engineering practices. (plural)

Example: The mechanical properties of *a new composite material* must be tested before its application in engineering. (singular, meaning “any new composite material”)

When the noun you want to use is *uncountable*, you can make it generic by omitting any article.

Example: Thirty years later, *composite production* had more than doubled. (meaning

“all composite production”)

In addition, English has a fourth way of forming generic nouns you should learn to recognize and use. A *countable noun in its singular form* sometimes carries the generic meaning when used with the definite article *the*. This kind of generic noun phrase is often used when referring to living creatures or familiar machinery and equipment.

Example: The United States has experienced the integration of *the computer* into society. (meaning “computers in general”)

We have seen that the first part of Stage I, the setting of the *introduction*, usually contains a large proportion of generic noun phrases. Later in the setting, you will probably find it necessary to refer to specific items and concepts in order to move the reader from the general area toward your specific topic. This requires the use of specific noun phrase—that is, nouns refer to particular, individual members of a class rather than to the class as a whole. In English, nouns with this meaning can be written in several ways.

- *Referring to assumed or shared information.* Use the definite article *the* if you assume your readers share knowledge of the specific thing you are referring to.

Example: In recent years the growth of application of the finite element software has been accelerating in *the world*.

- *Pointing back to old information.* Use the definite article *the* when referring to a specific thing which you have already mentioned (the first mentioned usually uses the indefinite article *a/an*).

Example: Professor Belytschko proposed a mesh-free method recently. *The new method* is very efficient to simulate the process of crack propagation.

- *Pointing forward to specific information.* Use the definite article *the* when the specific meaning is made clear in a following phrase or clause.

Example: *The load* which is carried by the beam must be less than 100N.

(b) Guidelines for Marking Generic and Specific Noun Phrases

If you are having difficulty determining which, if any, article to use before a noun or a noun phrase, ask yourself the following sequence of questions:

- Is the noun meant in a *general* or a *specific* sense? If it is *specific*, use “the” before the noun. If it is *general*, ask yourself a follow-up question:
- Is the noun *countable* or *uncountable*? If it is countable, use *a* or *an* (singular) or *s* on the end (plural). If it is uncountable, use no article or *-s* ending.

(c) Expressing Old Information

There are several ways you can state old information to connect back to the information in a previous sentence. One way is to simply repeat a word or to use a derived form of the word.

Example: The idea of the finite element method (FEM) was originally proposed at the inception of the 20th century. It is a common knowledge that the FEM has to be carried out by coding a program.

Another way you can indicate old information is to use pronouns and pointing words.

Example: The concept of stress is the heart of our subject. *It* is the unique way continuum mechanics has for specifying the interaction between one part of a material body and another.

Sometimes you can assume the reader knows the old information without your having to state it explicitly.

Example: Buckling of pressure vessel may take place if the structure does not have enough stability to sustain internal pressure. *The accidents* [of buckling] usually give rise to big economic loss.

(3) A Sample

The following paragraph is adopted from the *introduction* of an academic paper [33], which may help the readers understand how to establish a context at the beginning of an *introduction*.

Repairs of cracked components in aerospace structures are becoming more and more important due to the requirement of operation safety. The repair methods based on adhesively bonded fiber-reinforced polymer (FRP) composite patches have been demonstrated to be very promising to these cracked structures. FRP composite patches have the advantages of high ratios of stiffness and strength to weight, and are more structurally efficient and much less damaging to the repaired structures than fastened metallic patches. Although double-sided repair with FRP composite patches is more effective in reinforcement, single-sided repair plays a more important role because, in the most of practical repairs, it is difficult or even impossible to access both sides of the cracked structures that needed to be repaired. Fatigue crack growth behavior of cracked panels after being repaired decides the extension of fatigue life or service life of the repaired structures. Therefore, the evaluation of fatigue crack growth behavior of cracked panels repaired with a FRP composite patch becomes a focus in this research area.

Reading Material(1): Kinetic Energy and Work^[3]

As I have said, there are many different kinds of energy. Perhaps the most basic is kinetic energy (or KE), which for a single particle of mass m traveling with speed v is defined to be

$$T = \frac{1}{2}mv^2 \quad (1)$$

Let us imagine the particle moving through space and examine the change in its kinetic energy as it moves between two neighboring points r_1 and $r_1 + dr$ on its path as shown in Figure 1. The time derivative of T is easily evaluated if we note that $v^2 = v \cdot v$, so that

$$\frac{dT}{dt} = \frac{1}{2}m \frac{d}{dt}(v \cdot v) = \frac{1}{2}m(\dot{v} \cdot v + v \cdot \dot{v}) = m \dot{v} \cdot v \quad (2)$$

By the second law, the factor $m \dot{v}$ is equal to the net force F on the particle, so that

$$\frac{dT}{dt} = \mathbf{F} \cdot \mathbf{v} \tag{3}$$

If we multiply both sides by dt , then since $\mathbf{v}dt$ is the displacement $d\mathbf{r}$, we find

$$dT = \mathbf{F} \cdot d\mathbf{r} \tag{4}$$

The expression on the right, $\mathbf{F} \cdot d\mathbf{r}$, is defined to be the **work done by the force \mathbf{F}** in the displacement $d\mathbf{r}$. Thus we have proved the **Work-KE theorem**, that the change in the particle's kinetic energy between two neighboring points on its path is equal to the work done by the net force as it moves between the two points.

So far we have proved the Work-KE theorem only for an infinitesimal displacement $d\mathbf{r}$, but it generalizes easily to larger displacements. Consider the two points shown as r_1 and r_2 in Figure 1. We can divide the path between these points 1 and 2 into a large number of very small segments, to each of which we can apply the infinitesimal result (4).

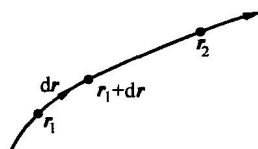


Figure 1 Three points on the path of a particle: r_1 and $r_1 + dr$ (with dr infinitesimal) and r_2

Adding all of these results, we find that the total change in T going from 1 to 2 is the sum $\sum \mathbf{F} \cdot d\mathbf{r}$ of all the infinitesimal

works done in all the infinitesimal displacements between points 1 and 2:

$$\Delta T \equiv T_2 - T_1 = \sum \mathbf{F} \cdot d\mathbf{r} \tag{5}$$

In the limit that all the displacements $d\mathbf{r}$ go to zero, this sum becomes an integral:

$$\sum \mathbf{F} \cdot d\mathbf{r} \rightarrow \int_1^2 \mathbf{F} \cdot d\mathbf{r} \tag{6}$$

This integral, called a **line integral**, is a generalization of the integral $\int f(x)dx$ over a single variable x , and its definition as the limit of the sum of many small pieces is closely analogous. If you feel any doubt about the symbol $\int_1^2 \mathbf{F} \cdot d\mathbf{r}$ on the right of (6), think of it as being just the sum on the left (with all the displacements infinitesimally small). In evaluating a line integral, it is usually possible to convert it into an ordinary integral over a single variable, as the following examples show. Notice that, as the name implies, the line integral depends (in general) on the path that the particle followed from point 1 to point 2.

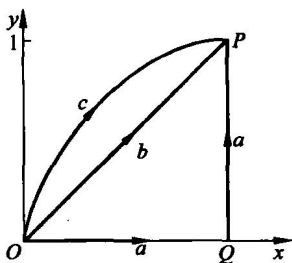


Figure 2 Three different paths, a , b , and c , from the origin to point $P=(1,1)$

The particular line integral on the right of (6) is called the work done by the force \mathbf{F} moving between points 1 and 2 along the path concerned.

Example: Three Line Integrals

Evaluate the line integral for the work done by the two-dimensional force $\mathbf{F} = (y, 2x)$ going from the origin O to the point $P = (1, 1)$ along each of the three paths shown in Figure 2. Path a goes from O to $Q = (1, 0)$ along the x axis and then from Q straight up to P , path b

goes straight from O to P along the line $y = x$, and path c goes round a quarter circle centered on Q .

The integral along path a is easily evaluated in two parts, if we note that on OQ the displacements have the form $d\mathbf{r} = (dx, 0)$, while on QP they are $d\mathbf{r} = (0, dy)$. Thus

$$\begin{aligned} W_a &= \int_a \mathbf{F} \cdot d\mathbf{r} = \int_0^Q \mathbf{F} \cdot d\mathbf{r} + \int_Q^P \mathbf{F} \cdot d\mathbf{r} = \int_0^1 F_x(x, 0) dx + \int_0^1 F_y(1, y) dy \\ &= 0 + 2 \int_0^1 dy = 2 \end{aligned}$$

On the path b , $x = y$, so that $dx = dy$, and

$$W_b = \int_b \mathbf{F} \cdot d\mathbf{r} = \int_b (F_x dx + F_y dy) = \int_0^1 (x + 2x) dx = 1.5$$

Path c is conveniently expressed parametrically as

$$\mathbf{r} = (x, y) = (1 - \cos\theta, \sin\theta)$$

where θ is the angle between OQ and the line from Q to the point (x, y) , with $0 < \theta < \pi/2$.

Thus on path c

$$d\mathbf{r} = (dx, dy) = (\sin\theta, \cos\theta) d\theta$$

and

$$\begin{aligned} W_c &= \int_c \mathbf{F} \cdot d\mathbf{r} = \int_c (F_x dx + F_y dy) \\ &= \int_0^{\pi/2} [\sin^2\theta + 2(1 - \cos\theta)\cos\theta] d\theta = 2 - \pi/4 = 1.21 \end{aligned}$$

With the notation of the line integral, we can rewrite the result (5) as

$$\Delta T \equiv T_2 - T_1 = \int_1^2 \mathbf{F} \cdot d\mathbf{r} \equiv W(1 \rightarrow 2) \quad (7)$$

where I have introduced the notation $W(1 \rightarrow 2)$ for the work done by \mathbf{F} moving from point 1 to point 2. The result is the Work-KE theorem for arbitrary displacements, large or small: The change in a particle's KE as it moves between points 1 and 2 is the work done by the net force.

It is important to remember that the work that appears on the right of (7) is the work done by the net force \mathbf{F} on the particle. In general, \mathbf{F} is the vector sum of various separate forces

$$\mathbf{F} = \mathbf{F}_1 + \cdots + \mathbf{F}_n \equiv \sum_{i=1}^n \mathbf{F}_i$$

(For example, the net force on a projectile is the sum of two forces, the weight and air resistance.) It is a most convenient fact that to evaluate the work done by the net force \mathbf{F} , we can simply add up the works done by the separate forces $\mathbf{F}_1, \mathbf{F}_2, \dots, \mathbf{F}_n$. This claim is easily proved as follows:

$$\begin{aligned} W(1 \rightarrow 2) &= \int_1^2 \mathbf{F} \cdot d\mathbf{r} = \int_1^2 \sum_i \mathbf{F}_i \cdot d\mathbf{r} \\ &= \sum_i \int_1^2 \mathbf{F}_i \cdot d\mathbf{r} = \sum_i W_i(1 \rightarrow 2) \end{aligned} \quad (8)$$

The crucial step, from the first line to the second, is justified because the integral of a sum of n terms is the same as the sum of the n individual integrals. The Work-KE theorem can therefore be rewritten as

$$T_2 - T_1 = \sum_{i=1}^n W_i(1 \rightarrow 2) \quad (9)$$

In practice, one almost always uses the theorem in this way: Calculate the work W_i done by each of the n separate forces on the particle and then set ΔT equal to the sum of all the W_i .

If the net force on a particle is zero, then the Work-KE theorem tells us that the particle's kinetic energy is constant. This simply says that the speed v is constant, which, though true, is not very interesting, since it already follows from Newton's first law.

Words and Expressions

kinetic energy		动能
work [wɜ:k]	<i>n.</i>	功
derivative [di'rivətiv]	<i>n.</i>	导数
time derivative		时间导数
displacement [dis'pleismənt]	<i>n.</i>	位移
integral ['intigrəl]	<i>n.</i>	积分; 整数
Newton's first law		牛顿第一定律

Reading Material(2): The Lagrangian Method^[2]

Consider the problem of a mass on the end of a spring. We can solve this, of course, by using $F=ma$ to write down $m\ddot{x} = -kx$. The solutions to this equation are sinusoidal functions, as we well know. We can, however, solve this problem by using another method which doesn't explicitly use $F=ma$. In many (in fact, probably most) physical situations, this new method is far superior to using $F=ma$.

We will present our new method by first stating its rules (without any justification) and showing that they somehow end up magically giving the correct answer. We will then give the method proper justification.

Here is the procedure. Form the following seemingly silly combination of the kinetic and potential energies (T and V , respectively),

$$L \equiv T - V \quad (1)$$

This is called the Lagrangian. Yes, there is a minus sign in the definition (a plus sign would simply give the total energy). In the problem of a mass on the end of a spring, $T = m\dot{x}^2/2$ and $V = kx^2/2$, so we have

$$L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2 \quad (2)$$