Joseph I. Kapusta, Charles Gale

Finite-Temperature Field Theory Principles and Applications

Second Edition

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Finite-Temperature Field Theory
Principles and Applications

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FINITE-TEMPERATURE FIELD THEORY
Principles and Applications

This book develops the basic formalism and theoretical techniques for studying relativistic quantum field theory at high temperature and density. Specific physical theories treated include QED, QCD, electroweak theory, and effective nuclear field theories of hadronic and nuclear matter. Topics include functional integral representation of the partition function, diagrammatic expansions, linear response theory, screening and plasma oscillations, spontaneous symmetry breaking, the Goldstone theorem, resummation and hard thermal loops, lattice gauge theory, phase transitions, nucleation theory, quark–gluon plasma, and color superconductivity. Applications to astrophysics and cosmology include white dwarf and neutron stars, neutrino emissivity, baryon number violation in the early universe, and cosmological phase transitions. Applications to relativistic nucleus–nucleus collisions are also included.

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Preface

What happens when ordinary matter is so greatly compressed that the electrons form a relativistic degenerate gas, as in a white dwarf star? What happens when the matter is compressed even further so that atomic nuclei overlap to form superdense nuclear matter, as in a neutron star? What happens when nuclear matter is heated to such great temperatures that the nucleons and pions melt into quarks and gluons, as in high-energy nuclear collisions? What happened in the spontaneous symmetry breaking of the unified theory of the weak and electromagnetic interactions during the big bang? Questions like these have fascinated us for a long time. The purpose of this book is to develop the fundamental principles and mathematical techniques that enable the formulation of answers to these mind-boggling questions. The study of matter under extreme conditions has blossomed into a field of intense interdisciplinary activity and global extent. The analysis of the collective behavior of interacting relativistic systems spans a rich palette of physical phenomena. One of the ultimate goals of the whole program is to map out the phase diagram of the standard model and its extensions.

This text assumes that the reader has completed graduate level courses in thermal and statistical physics and in relativistic quantum field theory. Our aims are to convey a coherent picture of the field and to prepare the reader to read and understand the original and current literature. The book is not, however, a compendium of all known results; this would have made it prohibitively long. We start from the basic principles of quantum field theory, thermodynamics, and statistical mechanics. This development is most elegantly accomplished by means of Feynman's functional integral formalism. Having a functional integral expression for the partition function allows a straightforward derivation of diagrammatic rules for interacting field theories. It also provides a framework for defining gauge theories on finite lattices, which then enables integration by Monte Carlo techniques. The formal aspects are illustrated with applications drawn from fields of research that are close to the authors' own experience. Each chapter carries its own exercises, reference list, and select bibliography.

The book is based on *Finite-Temperature Field Theory*, written by one of us (JK) and published in 1989. Although the fundamental principles have not changed, there have been many important developments since then, necessitating a new book.

We would like to acknowledge the assistance of Frithjof Karsch and Steven Gottlieb in transmitting some of their results of lattice computations, presented in Chapter 10, and Andrew Steiner for performing the numerical calculations used to prepare many of the figures in Chapter 11. We are grateful to a number of friends, colleagues, and students for their helpful comments and suggestions and for their careful reading of the manuscript, especially Peter Arnold, Eric Braaten, Paul Ellis, Philippe de Forcrand, Bengt Friman, Edmond Iancu, Sangyong Jeon, Keijo Kajantie, Frithjof Karsch, Mikko Laine, Stefan Leupold, Guy Moore, Ulrich Mosel, Robert Pisarski, Brian Serot, Andrew Steiner, and Laurence Yaffe.

Contents

1
Review of quantum statistical mechanics

Thermodynamics is used to describe the bulk properties of matter in or near equilibrium. Many scientists, notably Boyle, Carnot, Clausius, Gay-Lussac, Gibbs, Joule, Kelvin, and Rumford, contributed to the development of the field over three centuries. Quantities such as mass, pressure, energy, and so on are readily defined and measured. Classical statistical mechanics attempts to understand thermodynamics by the application of classical mechanics to the microscopic particles making up the system. Great progress in this field was made by physicists like Boltzmann and Maxwell. Temperature, entropy, particle number, and chemical potential are thus understandable in terms of the microscopic nature of matter. Classical mechanics is inadequate in many circumstances however, and ultimately must be replaced by quantum mechanics. In fact, the ultraviolet catastrophe encountered by the application of classical mechanics and electromagnetism to blackbody radiation was one of the problems that led to the development of quantum theory. The development of quantum statistical mechanics was achieved by a number of twentieth century physicists, most notably Planck, Einstein, Fermi, and Bose. The purpose of this chapter is to give a mini-review of the basic concepts of quantum statistical mechanics as applied to noninteracting systems of particles. This will set the stage for the functional integral representation of the partition function, which is a cornerstone of modern relativistic quantum field theory and the quantum statistical mechanics of interacting particles and fields.

1.1 Ensembles

One normally encounters three types of ensemble in equilibrium statistical mechanics. The *microcanonical* ensemble is used to describe an isolated system that has a fixed energy E, a fixed particle number N, and a fixed volume V. The *canonical* ensemble is used to describe a system in contact with a heat reservoir at temperature T. The system can freely exchange energy with the reservoir, but the particle number and volume are fixed. In the *grand canonical* ensemble the system can exchange particles as well as energy with a reservoir. In this ensemble the temperature, volume, and chemical potential μ are fixed quantities. The standard thermodynamic relations are summarized in appendix section A1.1.

In the canonical and grand canonical ensembles, $T^{-1} = \beta$ may be thought of as a Lagrange multiplier that determines the mean energy of the system. Similarly, μ may be thought of as a Lagrange multiplier that determines the mean number of particles in the system. In a relativistic quantum system, where particles can be created and destroyed, it is most straightforward to compute observables in the grand canonical ensemble. For that reason we use the grand canonical ensemble throughout this book. There is no loss of generality in doing so because one may pass over to either of the other ensembles by performing an inverse Laplace transform on the variable μ and/or the variable β. See appendix section A1.2.

Consider a system described by a Hamiltonian H and a set of conserved number operators $\hat{N}_i$. (A hat or caret is used to denote an operator for emphasis or whenever there is the possibility of an ambiguity.) In QED, for example, the number of electrons minus the number of positrons is a conserved quantity, not the number of electrons or positrons separately, because of reactions like $e^+e^- \to e^+e^+e^-e^-$. These number operators must be Hermitian and must commute with H as well as with each other. They must also be extensive (scale with the volume of the system) in order that the usual macroscopic thermodynamic limit can be taken. The statistical density matrix $\hat{\rho}$ is the fundamental object in equilibrium statistical mechanics:

$$\hat{\rho} = \exp\left[-\beta\left(H - \mu_i \hat{N}_i\right)\right] \tag{1.1}$$

Here and throughout the book a repeated index is assumed to be summed over. In QED the sum would run over two conserved number operators if one allowed for both electrons and muons. The statistical density matrix is used to compute the ensemble average of any desired observable, represented by the operator $\hat{A}$, via

$$A = \langle \hat{A} \rangle = \frac{\mathrm{Tr}\, \hat{A}\hat{\rho}}{\mathrm{Tr}\, \hat{\rho}} \tag{1.2}$$

where Tr denotes the trace operation.

The grand canonical partition function

$$Z = Z(V, T, \mu_1, \mu_2, \ldots) = \mathrm{Tr}\, \hat{\rho} \tag{1.3}$$

is the single most important function in thermodynamics. From it all the thermodynamic properties may be determined. For example, the pressure, particle number, entropy, and energy are, in the infinite-volume limit, given by

$$\begin{aligned} P &= \frac{\partial(T \ln Z)}{\partial V} \\ N_i &= \frac{\partial(T \ln Z)}{\partial \mu_i} \\ S &= \frac{\partial(T \ln Z)}{\partial T} \\ E &= -PV + TS + \mu_i N_i \end{aligned} \tag{1.4}$$

1.2 One bosonic degree of freedom

As a simple example consider a time-independent single-particle quantum mechanical mode that may be occupied by bosons. Each boson in that mode has the same energy ω. There may be 0, 1, 2, or any number of bosons occupying that mode. There are no interactions between the particles. This system may be thought of as a set of noninteracting quantized simple harmonic oscillators. It will serve as a prototype of the relativistic quantum field theory systems to be introduced in later chapters. We are interested in computing the mean particle number, energy, and entropy. Since the system has no volume there is no physical pressure.

Denote the state of the system by $|n\rangle$, which means that there are n bosons in the system. The state $|0\rangle$ is called the vacuum. The properties of these states are

$$\langle n|n'\rangle = \delta_{nn'} \quad \text{orthogonality} \tag{1.5}$$

$$\sum_{n=0}^{\infty} |n\rangle\langle n| = 1 \qquad \text{completeness} \tag{1.6}$$

One may think of the bras $\langle n|$ and kets $|n\rangle$ as row and column vectors, respectively, in an infinite-dimensional vector space. These vectors form a complete set. The operation in (1.5) is an inner product and the number 1 in (1.6) stands for the infinite-dimensional unit matrix.

It is convenient to introduce creation and annihilation operators, $a^\dagger$ and a, respectively. The creation operator creates one boson and puts it in the mode under consideration. Its action on a number eigenstate is

$$a^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle \tag{1.7}$$

Similarly, the annihilation operator annihilates or removes one boson,

$$a|n\rangle = \sqrt{n}|n-1\rangle \tag{1.8}$$

unless $n = 0$, in which case it annihilates the vacuum,

$$a|0\rangle = 0 \tag{1.9}$$

Apart from an irrelevant phase, the coefficients appearing in (1.7) and (1.8) follow from the requirements that $a^\dagger$ and a be Hermitian conjugates and that $a^\dagger a$ be the number operator $\hat{N}$. That is,

$$\hat{N}|n\rangle = a^\dagger a|n\rangle = n|n\rangle \tag{1.10}$$

As a consequence the commutator of a with $a^\dagger$ is

$$[a, a^\dagger] = aa^\dagger - a^\dagger a = 1 \tag{1.11}$$

We can build all states from the vacuum by repeated application of the creation operator:

$$|n\rangle = \frac{1}{\sqrt{n!}}(a^\dagger)^n|0\rangle \tag{1.12}$$

Next we need a Hamiltonian. Up to an additive constant, it must be ω times the number operator. Starting with a wave equation in nonrelativistic or relativistic quantum mechanics the additive constant emerges naturally. One finds that

$$H = \tfrac{1}{2}\omega\big(aa^\dagger + a^\dagger a\big) = \omega\left(a^\dagger a + \tfrac{1}{2}\right) = \omega\left(\hat{N} + \tfrac{1}{2}\right) \tag{1.13}$$

The additive term $\frac{1}{2}\omega$ is the zero-point energy. Usually this term can be ignored. Exceptions arise when the vacuum changes owing to a background field, such as the gravitational field or an electric field, as in the Casimir effect. We shall drop this term in the rest of the chapter and leave it as an exercise to repeat the following analysis with the inclusion of the zero-point energy.

The states $|n\rangle$ are simultaneous eigenstates of energy and particle number. We can assign a chemical potential to the particles. This is possible because there are no interactions to change the particle number. The partition function is easily computed:

$$\begin{aligned} Z &= \mathrm{Tr}\, \mathrm{e}^{-\beta(H-\mu\hat{N})} = \mathrm{Tr}\, \mathrm{e}^{-\beta(\omega-\mu)\hat{N}} \\ &= \sum_{n=0}^{\infty}\langle n|\mathrm{e}^{-\beta(\omega-\mu)\hat{N}}|n\rangle = \sum_{n=0}^{\infty}\mathrm{e}^{-\beta(\omega-\mu)n} \\ &= \frac{1}{1-\mathrm{e}^{-\beta(\omega-\mu)}} \end{aligned} \tag{1.14}$$

The mean number of particles is found from (1.4) to be

$$N = \frac{1}{\mathrm{e}^{\beta(\omega-\mu)} - 1} \tag{1.15}$$