高等学校数学双语教学推荐教材

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Calculus

威廉・布里格斯 (William Briggs)

莱尔・科克伦 (Lyle Cochran)

伯纳德・吉勒特 (Bernard Gillett)

**四**中国人民大学出版社

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# 微抚场

(上册 Calculus

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## Calculus

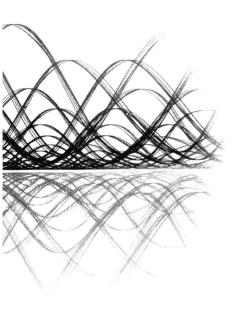
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## Preface

This textbook supports a three-semester or four-quarter calculus sequence typically taken by students in mathematics, engineering, and the natural sciences. Our approach is based on many years of teaching calculus at diverse institutions using the best teaching practices we know.

Throughout the book, a concise and lively narrative motivates the ideas of calculus. Reviewers and class testers have consistently told us that the book mirrors the course they teach. Equally important, we believe that students will actually read the book. Topics are introduced through concrete examples, applications, and analogies rather than through abstract arguments. We appeal to students' intuition and geometric instincts to make calculus natural and believable. Once this intuitive foundation is established, generalizations and abstractions follow. We include informative proofs in the text, but less transparent proofs appear at the end of the sections or in Appendix B.

## Pedagogical Features

#### **Exercises**

The exercises at the end of each section are one of the strongest features of the text. They are graded, varied, and original. In addition, they are labeled and carefully organized into groups.

- Each exercise set begins with *Review Questions* that check students' conceptual understanding of the essential ideas from the section.
- Basic Skills exercises are confidence-building problems that provide a solid foundation for the more challenging exercises to follow. Each example in the narrative is linked directly to a block of Basic Skills exercises via Related Exercises references at the end of the example solution.
- Further Explorations exercises expand on the Basic Skills exercises by challenging students to think creatively and to generalize newly acquired skills.
- Applications exercises connect skills developed in previous exercises to applications and modeling problems that demonstrate the power and utility of calculus.
- Additional Exercises are generally the most difficult and challenging problems; they include proofs of results cited in the narrative.

Each chapter concludes with a comprehensive set of Review Exercises.

## **Figures**

Given the power of graphics software and the ease with which many students assimilate visual images, we devoted considerable time and deliberation to the figures in this book. Whenever possible, we let the figures communicate essential ideas using annotations

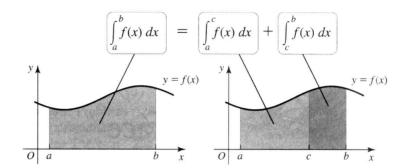


FIGURE 5.29

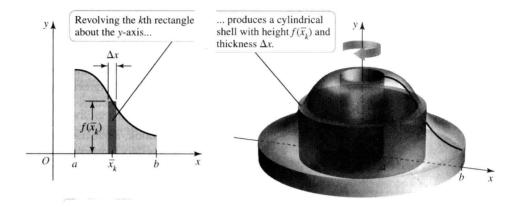


FIGURE 6.31

reminiscent of an instructor's voice at the board. Readers will quickly find that the figures facilitate learning in new ways.

## Quick Check and Margin Notes

The narrative is interspersed with *Quick Check* questions that encourage students to read with pencil in hand and that resemble the kinds of questions instructors pose in class. Answers to the *Quick Check* questions are found at the end of the section in which they occur. *Margin Notes* offer reminders, provide insight, and clarify technical points.

## **Guided Projects**

The *Instructor's Resource Guide and Test Bank* contains 78 *Guided Projects*. These projects allow students to work in a directed, step-by-step fashion, with various objectives: to carry out extended calculations, to derive physical models, to explore related theoretical topics, or to investigate new applications of calculus. The *Guided Projects* vividly demonstrate the breadth of calculus and provide a wealth of mathematical excursions that go beyond the typical classroom experience. A list of suggested *Guided Projects* is included at the end of each chapter.

## **Technology**

We believe that a calculus text should help students strengthen their analytical skills and demonstrate how technology can extend (not replace) those skills. The exercises and examples in this text emphasize this balance. Calculators and graphing utilities are additional tools in the kit, and students must learn when and when not to use them. Our goal is to accommodate the different policies about technology that various instructors may use.

Throughout the book, exercises marked with **III** indicate that the use of technology—ranging from plotting a function with a graphing utility to carrying out a calculation using a computer algebra system—may be needed.

## **Interactive Figures**

The textbook is supported by a groundbreaking electronic book, created by Eric Schulz of Walla Walla Community College. This "live book" contains the complete text of the print book plus interactive animated versions of approximately 700 figures. Instructors can use these animations in the classroom to illustrate the important ideas of calculus, and students can explore the interactive animations while they are reading the textbook. In each case, these animations will help build students' geometric intuition of calculus. Available only within MyMathLab, the eBook provides instructors with powerful new teaching tools that expand and enrich the learning experience for teachers.

## Content Highlights

In writing this text, we identified content in the calculus curriculum that consistently presents challenges to our students. We made organizational changes to the standard presentation of these topics or slowed the pace of the narrative to facilitate students' comprehension of material that is traditionally difficult. Two noteworthy modifications appear in the material for Calculus II and Calculus III, as outlined below.

Often appearing near the end of the term, the topics of sequences and series are the most challenging in Calculus II. By splitting this material into two chapters, we have given these topics a more deliberate pace and made them more accessible without adding significantly to the length of the narrative.

There *is* a clear and logical path through multivariate calculus, which is not apparent in many textbooks. We have carefully separated functions of several variables from vector-valued functions, so that these ideas are distinct in the minds of students. The book culminates when these two threads are joined in the last chapter, which is devoted to vector calculus.

## **Accuracy Assurance**

One of the challenges we face with a first edition is ensuring the book meets the high standards of accuracy that instructors expect. More than 200 mathematicians reviewed the manuscript for accuracy, level of difficulty, and effective pedagogy. Additionally, nearly 1000 students participated in class-testing this book before publication. A team of mathematicians carefully examined each example, exercise, and figure in multiple rounds of editing, proof-reading, and accuracy checking. From the beginning and throughout development, our goal has been to craft a textbook that is mathematically precise and pedagogically sound.

## Text Versions

#### Calculus

Complete (Chapters 1–15) ISBN 0-321-33611-9 | 978-0-321-33611-8 Single Variable Calculus (Chapters 1–11) ISBN 0-321-66407-8 | 978-0-321-66407-5 Multivariable Calculus (Chapters 9–15) ISBN 0-321-66415-9 | 978-0-321-66415-0

## Calculus: Early Transcendentals

Complete (Chapters 1–14) ISBN 0-321-57056-1 | 978-0-321-57056-7 Single Variable Calculus (Chapters 1–10) ISBN 0-321-66414-0 | 978-0-321-66414-3 Multivariable Calculus (Chapters 8–14) ISBN 0-321-66415-9 | 978-0-321-66415-0

## Media and Online Supplements

## **Technology Resource Manuals**

Maple Manual by James Stapleton, North Carolina State University
Mathematica Manual by Marie Vanisko, Carroll College
TI-Graphing Calculator Manual by Elaine McDonald-Newman, Sonoma State
University, and Luz De Alba, Drake University

These manuals cover Maple<sup>TM</sup> 13, Mathematica<sup>®</sup> 7, and the TI-83 Plus/TI-84 Plus and TI-89, respectively. Each manual provides detailed guidance for integrating a specific software package or graphing calculator throughout the course, including syntax and commands. These manuals are available to instructors and students through the Pearson Math and Stats Resources page, www.pearsonhighered.com/mathstatsresources, and MyMathLab<sup>®</sup>.

## MyMathLab® Online Course (access code required)

MyMathLab is a text-specific, easily customizable online course that integrates interactive multimedia instruction with textbook content. MyMathLab gives you the tools you need to deliver all or a portion of your course online, whether your students are in a lab setting or working from home.

- eBook featuring Interactive Figures that can be manipulated to illuminate difficult-to-convey concepts. Instructors can use these animations in the classroom to illustrate the important ideas of calculus, and students can try out the interactive animations while they are using MyMathLab. In each case, these animations help build geometric intuition of calculus.
- Interactive homework exercises, correlated to your textbook at the objective level, are algorithmically generated for unlimited practice and mastery. Most exercises are free response and provide guided solutions, sample problems, and tutorial learning aids for extra help.
- "Getting Ready" chapter includes hundreds of exercises that address prerequisite skills in algebra and trigonometry. Each student can receive remediation for those skills with which he or she needs help.
- Personalized Study Plan, generated when students complete a test or quiz, indicates
  which topics have been mastered and links to tutorial exercises for topics students have
  not mastered. You can customize the Study Plan so that the topics available match your
  course content, or so that students' homework results also determine mastery.
- Multimedia learning aids, such as video lectures, Java applets, animations, and a complete interactive eBook, help students independently improve their understanding and performance. You can assign these multimedia learning aids as homework to help your students grasp the concepts.
- Homework and Test Manager lets you assign homework, quizzes, and tests that are automatically graded. Select just the right mix of questions from the MyMathLab exercise bank, instructor-created custom exercises, and/or TestGen® test items.
- **Gradebook**, designed specifically for mathematics and statistics, automatically tracks students' results, lets instructors stay on top of student performance, and gives them control over how to calculate final grades. You can also add offline (paper-and-pencil) grades to the gradebook.
- MathXL® Exercise Builder allows you to create static and algorithmic exercises for your online assignments. You can use the library of sample exercises as an easy starting point, or you can edit any course-related exercise.

• Pearson Tutor Center (www.pearsontutorservices.com) access is automatically included with MyMathLab. The Tutor Center is staffed by qualified math instructors who provide textbook-specific tutoring for students via toll-free phone, fax, email, and interactive Web sessions.

Students do their assignments in the Flash®-based MathXL Player, which is compatible with almost any browser (Firefox®, Safari™, or Internet Explorer®) on almost any platform (Macintosh® or Windows®). MyMathLab is powered by CourseCompass™, Pearson Education's online teaching and learning environment, and by MathXL, our online homework, tutorial, and assessment system. MyMathLab is available to qualified adopters. For more information, visit www.mymathlab.com or contact your Pearson representative.

## MathXL® Online Course (access code required)

MathXL is an online homework, tutorial, and assessment system that accompanies Pearson's textbooks in mathematics or statistics.

- Interactive homework exercises, correlated to your textbook at the objective level, are algorithmically generated for unlimited practice and mastery. Most exercises are free response and provide guided solutions, sample problems, and learning aids for extra help.
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- Personalized Study Plan, generated when students complete a test, quiz, or homework, indicates which topics have been mastered and links to tutorial exercises for topics students have not mastered. Instructors can customize the available topics in the study plan to match their course concepts.
- Multimedia learning aids, such as video lectures, Java applets, and animations, help students independently improve their understanding and performance. These are assignable as homework, to further encourage their use.
- Gradebook, designed specifically for mathematics and statistics, automatically tracks students' results, lets you stay on top of student performance, and gives you control over how to calculate final grades.
- MathXL Exercise Builder allows you to create static and algorithmic exercises for your online assignments. You can use the library of sample exercises as an easy starting point or use the Exercise Builder to edit any of the course-related exercises.
- Homework and Test Manager lets you create online homework, quizzes, and tests that
  are automatically graded. Select just the right mix of questions from the MathXL exercise bank, instructor-created custom exercises, and/or TestGen test items.

The new, Flash®-based MathXL Player is compatible with almost any browser (Firefox®, Safari<sup>TM</sup>, or Internet Explorer®) on almost any platform (Macintosh® or Windows®). MathXL is available to qualified adopters. For more information, visit our website at **www.mathxl.com**, or contact your Pearson representative.

#### TestGen®

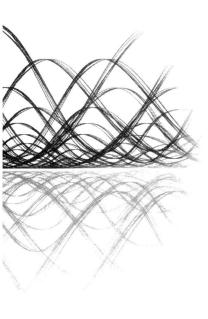
TestGen (www.pearsoned.com/testgen) enables instructors to build, edit, print, and administer tests using a computerized bank of questions developed to cover all the objectives of the text. TestGen is algorithmically based, allowing instructors to create multiple but equivalent versions of the same question or test with the click of a button. Instructors can also modify test bank questions or add new questions. The software and test bank are available for download from Pearson Education's online catalog.

## **Video Lectures With Optional Captioning**

The Video Lectures With Optional Captioning feature an engaging team of mathematics instructors who present comprehensive coverage of topics in the text. The lecturers' presentations include illustrative examples and exercises and support an approach that emphasizes visualization and problem solving. Available only through MyMathLab and MathXL.

## PowerPoint® Lecture Slides

These PowerPoint slides contain key concepts, definitions, figures, and tables from the textbook. These files are available to qualified instructors through the Pearson Instructor Resource Center, **www.pearsonhighered/irc**, and MyMathLab.



## Note to Students

We offer several practical suggestions about how to gain the most from this book and from your calculus course.

- 1. Our experience in teaching calculus over many years tells us that the greatest obstacle to learning calculus is not the new ideas of calculus, which are often easily understood. Rather, students find a greater struggle with prerequisite skills—most notably algebra and trigonometry. Your progress with calculus will be far less difficult if you have a solid understanding of algebra and trigonometry before you begin Chapter 2. Take advantage of the material in Chapter 1 and Appendix A, as well as the review that your instructor may provide, so that your prerequisite skills are strong *before* you embark on the study of calculus.
- 2. An old saying is worth repeating: *Mathematics is not a spectator sport*. No one can expect to learn calculus merely by reading the book and listening to lectures. Your participation and engagement are essential. Read the book actively with a pencil and paper nearby. Use the margins for your notes, answer the Quick Check questions as you go, and work as many exercises as possible. Working exercises will accelerate your learning of calculus more than anything else you do.
- 3. The use of graphing calculators and computer software is a major issue in teaching and learning calculus. Instructors differ in their emphasis on technology, so it is important to understand your instructor's approach to the use of technology and to become proficient with the required technology as quickly as possible. You should strive for a balance between the use of technology and the use of what are called *analytical*, or pencil-and-paper, methods. Technology should be used to extend and check your analytical skills but never to replace them.

With these thoughts in mind, it is time to begin the calculus journey. We hope it is as exciting for you as it is for us every time we teach calculus.

William Briggs

Lyle Cochran

Bernard Gillett

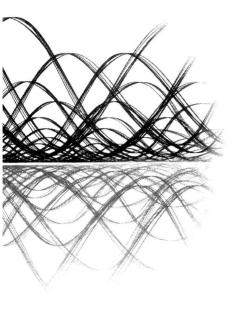
# 目录摘要

第1章	函数	·· 1
第2章	极限	• 32
第3章	导数	. 99
第4章	导数的应用 ·····	177
第5章	积分	250
	积分的应用 ······	
第7章	对数函数和指数函数 ·····	375
第8章	积分方法 ······	439

# 目 录

第1章 函数         1.1 函数的回顾         1.2 函数的表示法         1.3 三角函数         意复习题         第2章 极限         2.1 极限的概念         2.2 极限的定义         2.3 极限的计算方法         2.4 无穷极限         2.5 无穷远处的极限         2.6 连续性         2.7 极限的严格定义         总复习题         第3章 导数         3.1 导数的概念         3.2 导数的运算法则         3.3 积法则与商法则         3.4 三角函数的导数         3.5 作为变化率的导数	
1.1 函数的回顾 1.2 函数的表示法 1.3 三角函数 总复习题  第2章 极限 2.1 极限的概念 2.2 极限的定义 2.3 极限的计算方法 2.4 无穷极限 2.5 无穷远处的极限 2.6 连续性 2.7 极限的严格定义 总复习题  第3章 导数 3.1 导数的概念 3.2 导数的运算法则 3.3 积法则与商法则 3.4 三角函数的导数	1 10 22 29 32 32 38 47
1.1 函数的回顾 1.2 函数的表示法 1.3 三角函数 总复习题  第2章 极限 2.1 极限的概念 2.2 极限的定义 2.3 极限的计算方法 2.4 无穷极限 2.5 无穷远处的极限 2.6 连续性 2.7 极限的严格定义 总复习题  第3章 导数 3.1 导数的概念 3.2 导数的运算法则 3.3 积法则与商法则 3.4 三角函数的导数	1 10 22 29 32 32 38 47
1.2 函数的表示法 1.3 三角函数	10 22 29 32 32 38
1.3 三角函数	22 29 32 32 38
第2章 极限         2.1 极限的概念         2.2 极限的定义         2.3 极限的计算方法         2.4 无穷极限         2.5 无穷远处的极限         2.6 连续性         2.7 极限的严格定义         总复习题         第3章 导数         3.1 导数的概念         3.2 导数的运算法则         3.3 积法则与商法则         3.4 三角函数的导数	29 32 32 38 47
第2章 极限         2.1 极限的概念         2.2 极限的定义         2.3 极限的计算方法         2.4 无穷极限         2.5 无穷远处的极限         2.6 连续性         2.7 极限的严格定义         总复习题         第3章 导数         3.1 导数的概念         3.2 导数的运算法则         3.3 积法则与商法则         3.4 三角函数的导数	32 32 38 47
2.1 极限的概念 2.2 极限的定义 2.3 极限的计算方法 2.4 无穷极限 2.5 无穷远处的极限 2.6 连续性 2.7 极限的严格定义 总复习题 第3章 导数 3.1 导数的概念 3.2 导数的运算法则 3.3 积法则与商法则 3.4 三角函数的导数	32 38 47
2.2 极限的定义 2.3 极限的计算方法 2.4 无穷极限 2.5 无穷远处的极限 2.6 连续性 2.7 极限的严格定义 总复习题  第3章 导数 3.1 导数的概念 3.2 导数的运算法则 3.3 积法则与商法则 3.4 三角函数的导数	38 47
2.3 极限的计算方法         2.4 无穷极限         2.5 无穷远处的极限         2.6 连续性         2.7 极限的严格定义         总复习题         第3章 导数         3.1 导数的概念         3.2 导数的运算法则         3.3 积法则与商法则         3.4 三角函数的导数	47
2.4 无穷极限	
2.5 无穷远处的极限         2.6 连续性         2.7 极限的严格定义         总复习题         第3章 导数         3.1 导数的概念         3.2 导数的运算法则         3.3 积法则与商法则         3.4 三角函数的导数	56
2.6 连续性····································	
2.6 连续性····································	65
2.7 极限的严格定义         总复习题         第3章 导数         3.1 导数的概念         3.2 导数的运算法则         3.3 积法则与商法则         3.4 三角函数的导数	
总复习题	
第3章 导数         3.1 导数的概念         3.2 导数的运算法则         3.3 积法则与商法则         3.4 三角函数的导数	
3.1 导数的概念	
3.2       导数的运算法则         3.3       积法则与商法则         3.4       三角函数的导数	
3.3 积法则与商法则	
3.4 三角函数的导数	
3.5 作为变化率的异数	
3.6 链法则	
3.7 隐函数求导法	
3.8 相关变化率	
总复习题	
第 <b>4</b> 章 导数的应用 ····································	
4.1 最大值与最小值	···· 177
4.2 导数提供的信息	177
4.3 函数作图	177
4.4 最优化问题	···· 177 ···· 185
4.5 线性逼近与微分	177 185 199
1.0 XIEELYWA	177 185 199 208

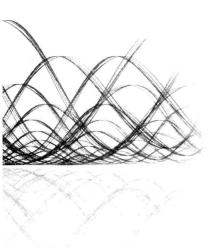
4.7	洛必达法则	
4.8		
总复	[习题	247
章	积分	250
5. 1	估计曲线下的面积	250
5.2	定积分	264
5.3	微积分基本定理	278
5.4	应用积分	293
5.5	换元法	301
总复	[习题	310
章	积分的应用	313
6.1	速度与净变化	313
6.2	曲线之间的区域	326
6.3	用切片法求体积	335
6.4	用柱壳法求体积	345
6.5	曲线的弧长	355
c c	物理应用	361
6.6	WELLIN	
	[习题	
		372
总复	見习题	372 375
总复 <b>章</b>	<ul><li>・ 対数函数和指数函数・・・・・・・・・・・・・・・・・・・・・・・・・・・・・・・・</li></ul>	<ul><li>372</li><li>375</li><li>375</li></ul>
总复 <b>章</b> 7.1	对数函数和指数函数       反函数         自然对数与指数函数	<ul><li>372</li><li>375</li><li>375</li><li>385</li></ul>
总复 <b>章</b> 7.1 7.2	<ul><li></li></ul>	372 375 375 385 397
总 <b>章</b> 7.1 7.2 7.3	可数函数和指数函数	372 375 375 385 397 407 416
总 <b>章</b> 7.1 7.2 7.3 7.4 7.5 7.6	夏习题····································	372 375 375 385 397 407 416 430
总 <b>章</b> 7.1 7.2 7.3 7.4 7.5 7.6	可数函数和指数函数	372 375 375 385 397 407 416 430
总 <b>章</b> 7.1 7.2 7.3 7.4 7.5 7.6	夏习题····································	372 375 375 385 397 407 416 430 436
总 <b>章</b> 7.1 7.2 7.4 7.6 8	可数       对数函数和指数函数         反函数       自然对数与指数函数         其他底的对数和指数函数       其数模型         反三角函数       冷必达法则与函数增长率         更习题       积分方法         和分方法       分部积分法	372 375 375 385 397 407 416 430 436 439
总 <b>章</b> 7.1 7.2 7.3 7.4 7.5 7.6 <b>章</b>	J > 题         对数函数和指数函数         反函数         自然对数与指数函数         其他底的对数和指数函数         指数模型         反三角函数         冷必达法则与函数增长率         是习题         积分方法         分部积分法	372 375 375 385 397 407 416 430 436 439
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	4.8 <b>章</b> 5.1 5.2 5.3 5.4 <b>章</b> 6.1 6.2 6.3 6.4	4.8 原函数         总复习题         5.1 估计曲线下的面积         5.2 定积分         5.3 微积分基本定理         5.4 应用积分         5.5 换元法         总复习题         章 积分的应用         6.1 速度与净变化         6.2 曲线之间的区域         6.3 用切片法求体积         6.4 用柱壳法求体积         6.5 曲线的弧长



# Contents

	Preface	i
	Note to Students	vii
1	Functions	1
	1.1 Review of Functions 1 1.2 Representing Functions 10 1.3 Trigonometric Functions 22 Review Exercises 29	
2	Limits	32
	<ul> <li>2.1 The Idea of Limits 32</li> <li>2.2 Definitions of Limits 38</li> <li>2.3 Techniques for Computing Limits 47</li> <li>2.4 Infinite Limits 56</li> <li>2.5 Limits at Infinity 65</li> <li>2.6 Continuity 73</li> <li>2.7 Precise Definitions of Limits 85 <ul> <li>Review Exercises 96</li> </ul> </li> </ul>	
3	Derivatives	99
	3.1 Introducing the Derivative 99 3.2 Rules of Differentiation 113 3.3 The Product and Quotient Rules 121 3.4 Derivatives of Trigonometric Functions 129 3.5 Derivatives as Rates of Change 137 3.6 The Chain Rule 149 3.7 Implicit Differentiation 157 3.8 Related Rates 165 Review Exercises 173	*

4	Applications of the Derivative	177
	<ul> <li>4.1 Maxima and Minima 177</li> <li>4.2 What Derivatives Tell Us 185</li> <li>4.3 Graphing Functions 199</li> <li>4.4 Optimization Problems 208</li> <li>4.5 Linear Approximation and Differentials 218</li> <li>4.6 Mean Value Theorem 225</li> <li>4.7 L'Hôpital's Rule 231</li> <li>4.8 Antiderivatives 239  Review Exercises 247</li> </ul>	
5	Integration	250
	<ul> <li>5.1 Approximating Areas under Curves 250</li> <li>5.2 Definite Integrals 264</li> <li>5.3 Fundamental Theorem of Calculus 278</li> <li>5.4 Working with Integrals 293</li> <li>5.5 Substitution Rule 301  Review Exercises 310</li> </ul>	
6	Applications of Integration	313
	<ul> <li>6.1 Velocity and Net Change 313</li> <li>6.2 Regions Between Curves 326</li> <li>6.3 Volume by Slicing 335</li> <li>6.4 Volume by Shells 345</li> <li>6.5 Length of Curves 355</li> <li>6.6 Physical Applications 361  Review Exercises 372</li> </ul>	
7	Logarithmic and Exponential Functions	375
	<ul> <li>7.1 Inverse Functions 375</li> <li>7.2 The Natural Logarithmic and Exponential Functions 385</li> <li>7.3 Logarithmic and Exponential Functions with Other Bases 397</li> <li>7.4 Exponential Models 407</li> <li>7.5 Inverse Trigonometric Functions 416</li> <li>7.6 L'Hôpital's Rule and Growth Rates of Functions 430 Review Exercises 436</li> </ul>	
8	Integration Techniques	439
	<ul> <li>8.1 Integration by Parts 439</li> <li>8.2 Trigonometric Integrals 446</li> <li>8.3 Trigonometric Substitutions 454</li> <li>8.4 Partial Fractions 462</li> <li>8.5 Other Integration Strategies 472</li> <li>8.6 Numerical Integration 477</li> <li>8.7 Improper Integrals 488</li> <li>8.8 Introduction to Differential Equations 499</li> <li>Review Exercises 509</li> </ul>	

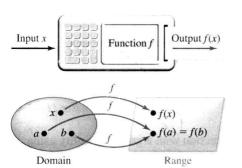


# **Functions**

- 1.1 Review of Functions
- 1.2 Representing Functions
- 1.3 Trigonometric Functions

Chapter Preview The goal of this chapter is to ensure that you begin your calculus journey fully equipped with the tools you will need. In this chapter, you will see many of the functions used in calculus: polynomials, rational functions, algebraic functions, and the trigonometric functions. (Logarithmic and exponential functions are introduced in Chapter 7.) It is imperative that you work hard to master the ideas in this chapter and refer to it when questions arise.

## **1.1** Review of Functions



If the domain is not specified, we take it to be the set of all values of x for which f is defined. We will see shortly that the domain and range of a function may be restricted by the context of the problem.

FIGURE 1.1

Mathematics is a language with an alphabet, a vocabulary, and many rules. If you are unfamiliar with set notation, intervals on the real number line, absolute value, the Cartesian coordinate system, or equations of lines and circles, please refer to Appendix A. Our starting point in this book is the fundamental concept of a function.

Everywhere around us we see relationships among quantities, or **variables**. For example, the consumer price index changes in time and the temperature of the ocean varies with latitude. These relationships can often be expressed by mathematical objects called **functions**. Calculus is the study of functions, and because we use functions to describe the world around us, calculus is a universal language for human inquiry.

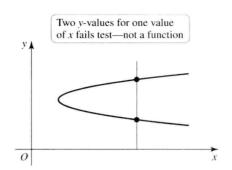
#### DEFINITION Function

A **function** f is a rule that assigns to each value x in a set D a *unique* value denoted f(x). The set D is the **domain** of the function. The **range** is the set of all values of f(x) produced as x varies over the domain (Figure 1.1).

The **independent variable** is the variable associated with the domain; the **dependent variable** belongs to the range. The **graph** of a function f is the set of all points (x, y) in the xy-plane that satisfy the equation y = f(x). The **argument** of a function is the expression on which the function works. For example, x is the argument when we write f(x). Similarly, 2 is the argument in f(2) and  $x^2 + 4$  is the argument in  $f(x^2 + 4)$ .

**QUICK CHECK 1** If 
$$f(x) = x^2 - 2x$$
, find  $f(-1)$ ,  $f(x^2)$ ,  $f(t)$ , and  $f(p-1)$ .

The requirement that a function must assign a *unique* value of the dependent variable to each value in the domain is expressed in the vertical line test (Figure 1.2).



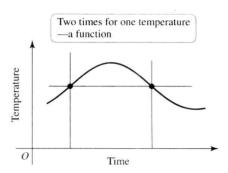


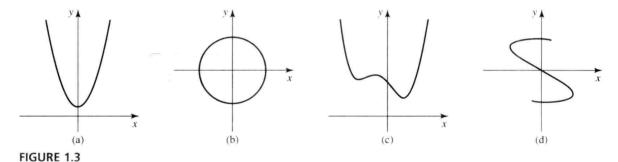
FIGURE 1.2

## **Vertical Line Test**

A graph represents a function if and only if it passes the vertical line test: Every vertical line intersects the graph at most once. A graph that fails this test does not represent a function.

A set of points or a graph that does not correspond to a function represents a relation between the variables. All functions are relations, but not all relations are functions.

**EXAMPLE 1** Identifying functions State whether each graph in Figure 1.3 corresponds to a function.



 $\triangleright$  A graphing window of  $[a,b] \times [c,d]$ means  $a \le x \le b$  and  $c \le y \le d$ .

**SOLUTION** The vertical line test indicates that only graphs (a) and (c) represent functions. In graphs (b) and (d), it is possible to draw vertical lines that intersect the graph more than once. Equivalently, it is possible to find values of x that correspond to more than one value of y. Therefore, graphs (b) and (d) do not pass the vertical line test and do not represent functions. Related Exercises 11-12 ◀

**EXAMPLE 2** Domain and range Graph each function with a graphing utility using

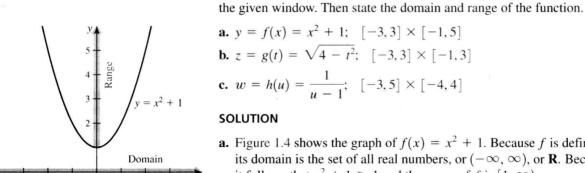


FIGURE 1.4

- **a.** Figure 1.4 shows the graph of  $f(x) = x^2 + 1$ . Because f is defined for all values of x, its domain is the set of all real numbers, or  $(-\infty, \infty)$ , or **R**. Because  $x^2 \ge 0$  for all x, it follows that  $x^2 + 1 \ge 1$  and the range of f is  $[1, \infty)$ .
- **b.** When n is even, functions involving nth roots are defined provided the quantity under the root is nonnegative. In this case, the function g is defined provided  $4 - t^2 \ge 0$ , which means  $t^2 \le 4$ , or  $-2 \le t \le 2$ . Therefore, the domain of g is [-2, 2]. By the