Anders C. Nilsson and Bilong Liu

Vibro-Acoustics

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PREFACE

Noise pollution is an environmental problem. Structures excited by dynamic forces radiate noise. The art of noise reduction requires an understanding of vibro-acoustics. This topic describes how structures are excited, energy flows from an excitation point to a sound radiating surface, and finally how a structure radiates noise to a surrounding fluid. The aim of this text is to give a fundamental analysis and a mathematical presentation of these phenomena. The text is intended for graduate students, researchers and engineers working in the field of sound and vibration.

Part of the text has evolved from an advanced course on acoustics initially given at Chalmers University, Sweden, in the early nineteen seventies. Over the years, these lectures were transformed to MSc and PhD courses on vibro-acoustics. These courses were given at MWL, KTH, Sweden. During the years, much material has been added as inspired by research work, colleagues and PhD students at many of the universities and research institutes. I have been fortunate enough to be associated with.

The present text is published as two volumes. In the text, frequent references are made to the behaviour of simple vibratory systems and their response in the frequency domain. Therefore and for the sake of completeness, though well-known to the reader, the first chapter includes discussions of simple one degree of freedom systems. In this way, energy and power of simple vibratory systems are introduced. Various types of losses are discussed. In the second chapter the vibration of linear mechanical systems is studied in the frequency domain. In particular, the response of systems excited by harmonic and random forces is analysed. In Chapter 3, the basic differential equations governing longitudinal, transverse and bending waves are discussed. The equations are derived based on the concept of stresses and strains in solids. Energy stored and energy flow in structures caused by the various wave types are analysed. The general wave equation is introduced in Chapter 4. This equation is shown to govern all elastic motion of a solid. The generalised wave equation is utilised to describe the bending of thick plates, sandwich beams and I-beams. It is demonstrated that in-plane waves like longitudinal and transverse waves are strongly coupled. In Chapter 5, it is shown that in-plane and bending waves also are well coupled. As presented, waves are attenuated by internal losses and added damping. More importantly, any discontinuity or junction will influence the energy flow in a structure. A number of examples are given. Measurement techniques to determine losses across junctions are introduced. Chapter 6 deals with longitudinal waves in finite beams. Eigenfunctions and eigenvalues are discussed for various boundary conditions. The results are used to model free and forced vibrations of beams. Green's function is derived for some cases and used for the calculation of the forced response of beams. The mobility concept is used to determine the response of coupled systems. In addition, the transfer matrix system is introduced. Flexural vibrations of finite beams are discussed in Chapter 7. Again, eigenfunctions and eigenvalues are derived for a number of boundary conditions. Free and forced vibrations are considered. The free and forced vibration of isotropic rectangular and circular plates is investigated in Chapter 8. The responses of structures excited by random and harmonic forces are compared. The mobility concepts of finite and infinite plates are investigated, and simplified models for the calculation of the energy of plates are introduced.

Volume II of Vibro-Acoustics includes eight chapters together with solutions to the problems given at the end of each chapter. Chapter 9 in Volume II is on variational methods. The variational technique is used for the derivation of equations governing the vibration of sandwich and other composite elements and some simple shell elements. In the following Chapter 10, the coupling between mechanical systems is explored. This includes an introduction to the vibration of rubber mounts, resilient mountings and the design of foundations. The topics of Chapter 11 are waves in fluids including discussions on various types of monopole, dipole and multipole sources. Also included are out-door and room acoustics. Chapter 12 is on fluid structure interaction and radiation of sound. Sound radiation from infinite and finite plates as well as the fluid loading on structures is demonstrated. Chapter 13 is on sound transmission loss of panels. The sound transmission through infinite and finite panels is discussed. The influence of boundary conditions and geometry of transmission rooms on the measured and predicted sound transmission loss is illustrated. The title of Chapter 14 is Wave Guides. In this chapter the prediction and reduction of energy flow in structural wave guides typical for ship, aircraft and train constructions are investigated. Also included are discussions on energy flow and sound transmission through composite structures and shells. In particular, the prediction of sound transmission loss of aircraft structures is highlighted. The contents of Chapter 15 include random excitation of structures and flow induced vibrations. Both phenomena are of importance for interior noise in aircraft, ships and fast trains. Finally, Chapter 16 is on the prediction of velocity and noise levels in large structures like vehicles. The basic concept of the Statistical Energy Analysis is presented. Predictions based on the wave guide technique are also illustrated in the same chapter.

The completion of this text would not have been possible without the determined support of Professor Jing Tian and the Key Laboratory of Noise and Vibration Research, Institute of Acoustics, Chinese Academy of Sciences in Beijing and the assistance and support of my coauthor Professor Bilong Liu. I would also like to express my gratitude to many former colleagues in many countries and in particular to Professor Leping Feng, MWL, KTH, for always being an inspiring discussion partner.

Anders C. Nilsson
Beijing, December 2010

NOTATIONS

b	width
c	damping factor, viscous losses
c_{b}	phase velocity, bending/flexural waves
c_{g}	group velocity
c_{l}	phase velocity, longitudinal waves
c_{r}	phase velocity, Rayleigh waves
c_{t}	phase velocity, transverse waves
f	frequency
f_n	natural frequency corresponding to mode n
f_{c}	critical frequency
f_{r}	dilatation frequency
g	acceleration due to gravity
$g_n(t)$	time dependent solution corresponding to eigenfunction $\varphi_n(x)$
$h(t)$	response function due to unit pulse, eq. (1-36)
i	$\sqrt{-1}$
k	$k_0(1 + i\delta)$, complex spring constant
k_0	real part of spring constant
k_{l}	wavenumber longitudinal waves
k_{t}	wavenumber transverse waves
k_{r}	wavenumber Rayleigh waves
m	mass
m'	mass per unit length
r	radial distance

s	spring constant per unit length or unit area
t	time
v	velocity
w	transverse displacement
B	torsional rigidity
$C(\tau)$	memory function
D'	bending stiffness beam
D	bending stiffness plate
D_x, D_y	bending stiffness, orthotropic plate
E	Young's modulus of elasticity
E_0	real part of Young's modulus of elasticity
E_x	σ_x/ε_x
F_d	damping force
F_n, F_{mn}	modal force
$F(t)$	force as function of time
F'	force per unit length
G	shear modulus
$G(x\|x_1)$	Green's function, 1-dimension
$G_{xx}(\omega)$	power spectral density, one sided
$G_{xy}(\omega)$	cross power spectral density, one sided
$H(\omega)$	frequency response function
$I(t)$	impulse
I	moment of inertia
$\boldsymbol{I}$	(I_x, I_y, I_z) intensity vector
K	bulk modulus
K_n, K_{mn}	modal stiffness
L, L_x, L_y	length
M_n, M_{mn}	modal mass
M'_x	bending moment per unit width around x-axis
M'_{xy}	bending moment per unit width due to torsional stress
$R_{xx}(\tau)$	auto correlation function

$R_{xy}(\tau)$	cross correlation function
$\mathbf{R}$	$10\log(1/\tau)$
S	cross section area
$S_{xx}(\omega)$	power spectral density, two sided
$S_{xy}(\omega)$	cross power spectral density, two sided
T	time period, harmonic oscillations
T_b	Timoshenko constant
T_x	shear force per unit width
$Y(\omega)$	point mobility
$Y(x, y\|x_0, y_0)$	transfer mobility, plates
Y_∞	point mobility, infinite structure
$\mathcal{E}$	total energy
$\mathcal{T}$	kinetic energy
$\mathcal{V}$	potential energy
$\mathcal{X}$	shear parameter
Υ	stiffness parameter
β	$c/(2m)$
γ	Euler constant
γ_{xy}	shear angle
$\gamma^2_{xy}(f)$	coherence function
δ	loss factor
$\delta(t)$	Dirac function
δ_{ij}	Kronecker delta
ε	strain
ε_x	strain in x-direction
ζ	displacement in z-direction
η	displacement in y-direction
η_0	loss factor
η_{tot}	total loss factor
κ	wavenumber, flexural waves
κ_0	real part of wavenumber, flexural waves

λ	wavelength but also Lamé constant
μ	mass per unit area
ν	Poisson's ratio
ξ	displacement, x-direction
ρ	density
ρ_{a}	apparent density
σ	stress
σ_x	stress in x-direction
τ	transmission coefficient
$\tau(\alpha)$	transmission coefficient as function of angle of incidence, α
τ_{ij}	transmission coefficient between the structures i and j
τ_{xy}	shear stress component
φ	phase angle
$\varphi_n(x)$	one dimensional eigenfunction
$\varphi_{mn}(x, y)$	two dimensional eigenfunction
ω	angular frequency
ω_0	$\sqrt{k_0/m}$
ω_{r}	$\sqrt{\omega_0^2 - \beta^2}$
ω_{n_0}	real part of angular frequency corresponding to f_n
Θ	(torsional) angle
Π	power
Π_{d}	dissipated power
Π_x	$I_x \cdot S$, energy flow in x-direction
Π_{ij}	energy flow from structure i to j
ϕ	scalar potential
ψ	vector potential
ΔL_{v}	velocity level difference
$\dot{x}$	$\mathrm{d}x/\mathrm{d}t$
$\ddot{x}$	$\mathrm{d}^2x/\mathrm{d}t^2$
x^*	complex conjugate of x
$\langle x^2 \rangle$	space average of x^2

$\langle f\|g\rangle$	$\int f(x)g(x)\mathrm{d}x$
(m, n)	mode m, n of vibrating plate
$\mathrm{Re}z$	real part of z
$\mathrm{Im}z$	imaginary part of z
$E[x]$	expected value of x
$\mathrm{FT}(x)$	Fourier transform of x

Operators

Cartesian coordinates

grad u

$$\frac{\partial u}{\partial x}\cdot \boldsymbol{e}_x + \frac{\partial u}{\partial y}\cdot \boldsymbol{e}_y + \frac{\partial u}{\partial z}\cdot \boldsymbol{e}_z = \left(\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial z}\right)$$

div $\boldsymbol{u} = \boldsymbol{\nabla}\cdot\boldsymbol{u}$

$$\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z}$$

curl $\boldsymbol{u} = \boldsymbol{\nabla}\times\boldsymbol{u}$

$$\left(\frac{\partial u_z}{\partial y} - \frac{\partial u_y}{\partial z}\right)\cdot \boldsymbol{e}_x + \left(\frac{\partial u_x}{\partial z} - \frac{\partial u_z}{\partial x}\right)\cdot \boldsymbol{e}_y + \left(\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y}\right)\cdot \boldsymbol{e}_z$$

$\boldsymbol{\nabla}^2\, u$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$$

Cylindrical coordinates

$x = r\cdot\cos\varphi, y = r\cdot\sin\varphi$

grad u

$$\frac{\partial u}{\partial r}\cdot \boldsymbol{e}_r + \frac{1}{r}\frac{\partial u}{\partial \varphi}\cdot \boldsymbol{e}_\varphi + \frac{\partial u}{\partial z}\cdot \boldsymbol{e}_z$$

div $\boldsymbol{u}$

$$\frac{1}{r}\frac{\partial}{\partial r}(ru_r) + \frac{1}{r}\frac{\partial u_\varphi}{\partial \varphi} + \frac{\partial u_z}{\partial z}$$

curl $\boldsymbol{u}$

$$\left(\frac{1}{r}\frac{\partial u_z}{\partial \varphi} - \frac{\partial u_\varphi}{\partial z}\right)\cdot \boldsymbol{e}_r + \left(\frac{\partial u_r}{\partial z} - \frac{\partial u_z}{\partial r}\right)\cdot \boldsymbol{e}_\varphi + \left(\frac{1}{r}\frac{\partial (ru_\varphi)}{\partial r} - \frac{1}{r}\frac{\partial u_r}{\partial \varphi}\right)\cdot \boldsymbol{e}_z$$

$\boldsymbol{\nabla}^2\, u$

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial u}{\partial r}\right] + \frac{1}{r^2}\frac{\partial^2 u}{\partial \varphi^2} + \frac{\partial^2 u}{\partial z^2}$$

Spherical coordinates

$x = r \cdot \cos\varphi \sin\theta, y = r \cdot \sin\varphi \cdot \sin\theta, z = r \cdot \cos\theta$

grad u $\quad \dfrac{\partial u}{\partial r} \cdot \boldsymbol{e}_r + \dfrac{1}{r}\dfrac{\partial u}{\partial \theta} \cdot \boldsymbol{e}_\theta + \dfrac{1}{r\sin\theta}\dfrac{\partial u}{\partial \varphi} \cdot \boldsymbol{e}_\varphi$

div $\boldsymbol{u}$ $\quad \dfrac{1}{r^2}\dfrac{\partial}{\partial r}(r^2 u_r) + \dfrac{1}{r \cdot \sin\theta}\dfrac{\partial}{\partial \theta}(u_\theta \sin\theta) + \dfrac{1}{r \cdot \sin\theta}\dfrac{\partial u_\varphi}{\partial \varphi}$

curl $\boldsymbol{u}$

$$\left(\frac{1}{r\sin\theta}\frac{\partial(u_\varphi \sin\theta)}{\partial\theta} - \frac{1}{r\sin\theta}\frac{\partial u_\theta}{\partial\varphi}\right) \cdot \boldsymbol{e}_r + \left(\frac{1}{r\sin\theta}\frac{\partial u_r}{\partial\varphi} - \frac{1}{r}\frac{\partial(r u_\varphi)}{\partial r}\right) \cdot \boldsymbol{e}_\theta + \left(\frac{1}{r}\frac{\partial(r u_\theta)}{\cdot\,\partial r} - \frac{1}{r}\frac{\partial u_r}{\partial\theta}\right) \cdot \boldsymbol{e}_\varphi$$

$\boldsymbol{\nabla}^2 u$ $\quad \dfrac{1}{r^2}\dfrac{\partial}{\partial r}\left(r^2\dfrac{\partial u}{\partial r}\right) + \dfrac{1}{r^2\sin\theta}\dfrac{\partial}{\partial\theta}\left(\sin\theta\dfrac{\partial u}{\partial\theta}\right) + \dfrac{1}{(r\sin\theta)^2}\dfrac{\partial^2 u}{\partial\varphi^2}$

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Chapter 1

MECHANICAL SYSTEMS WITH ONE DEGREE OF FREEDOM

In noise reducing engineering the consequences of changes made to a system must be understood. Questions posed could be on the effects of changes to the mass, stiffness or losses of the system and how these changes can influence the vibration of or noise radiation from some structures. Real constructions certainly have many or in fact infinite modes of vibration. However, to a certain extent, each mode can often be modelled as a simple vibratory system. The most simple vibratory system can be described by means of a rigid mass, mounted on a vertical mass less spring, which in turn is fastened to an infinitely stiff foundation. If the mass can only move in the vertical direction along the axis of the spring, the system has one degree of freedom (1-DOF). This is a vibratory system never actually encountered in practice. However, certain characteristics of systems with many degrees of freedom, or rather, continuous systems with an infinite degree of freedom, can be demonstrated by means of the very simple 1-DOF model. For this reason, the basic mass spring system is used in this chapter to illustrate some of the basic concepts concerning free vibrations, transient, harmonic and other types of forced excitation. Kinetic and potential energies are discussed as well their dependence on the in put power to the system and its losses.

1.1 A simple mass-spring system

A simple mass-spring system is shown in Fig. 1-1. The mass is m and