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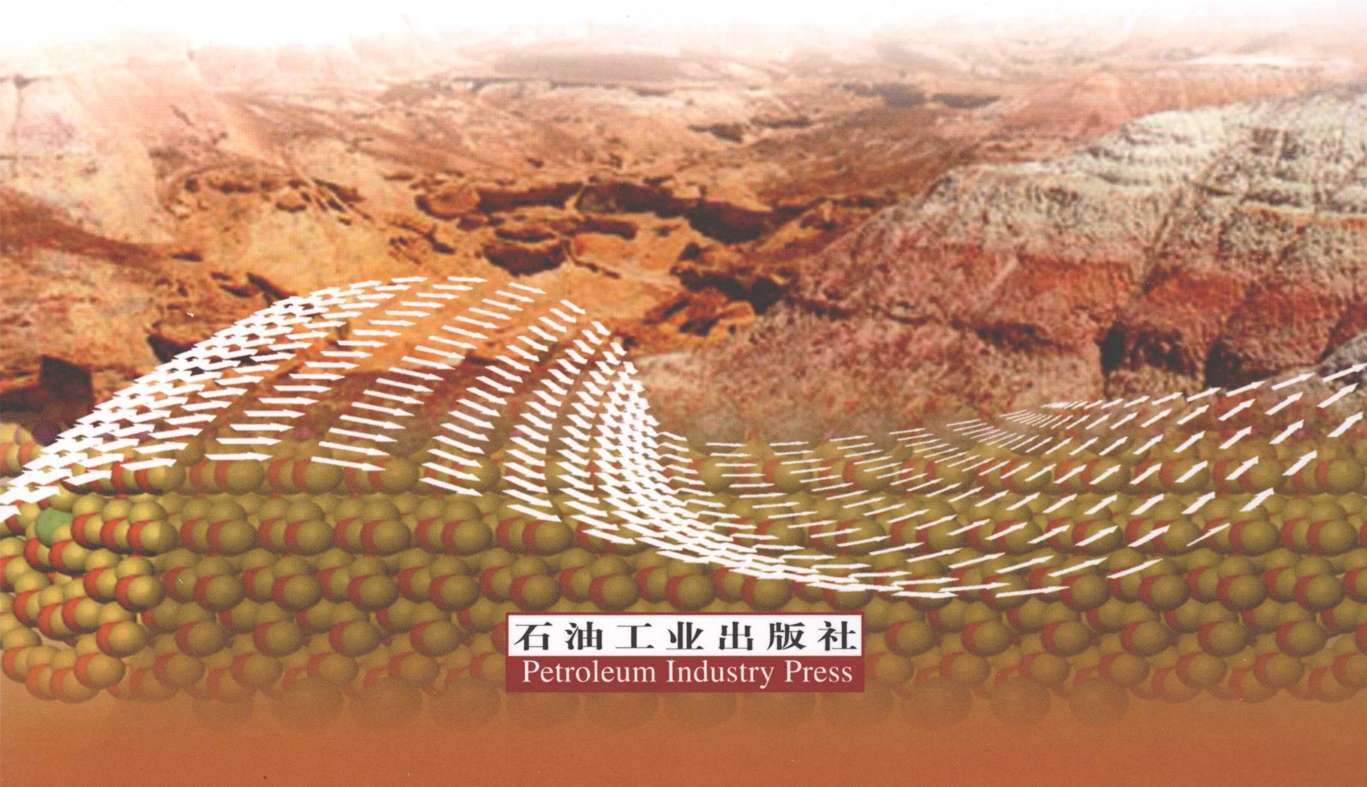
Mechanics Of Oil And Gas Flow In Porous Media

油气渗流力学

（英文版）

李 鋆 陈军斌 编著



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内 容 提 要

本书为《油气渗流力学》的英文版。全书从驱动力和驱动方式出发，在对达西定律分析的基础上，遵循由浅入深的认识规律，详细介绍了单相不可压缩液体的稳定渗流理论、刚性水压驱动下的油井干扰理论、微可压缩流体的不稳定渗流理论、天然气的渗流规律、水驱油理论、油气两相渗流理论、流体在双重介质中的渗流理论、非牛顿液体渗流理论等。

本书可作为石油工程、石油地质、地下水工程、油田化学等专业本科生教材，也可作为相关专业研究生的参考书，还可供从事油气田勘探与开发的科研技术人员参考。

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前　　言

改革开放以来，石油出版界在出版我国的石油科技学术专著、引进和翻译出版西方石油科技文献方面成绩斐然，但是将我国学术著作和教材译成英文出版的并不多。作为一个经济和石油大国，将我国石油科研和教学成果以专著和教材形式介绍给国际社会是一项十分必要的工作。这是让世界了解中国所必需的，也是我们推进世界石油科研和教学发展所应当做的。

教高〔2007〕1号文件《教育部财政部关于实施高等学校本科教学质量与教学改革工程的意见》中提出：鼓励和支持校内及聘请国内外著名专家学者和高水平专业人才承担教学任务和开设讲座，推动双语教学课程建设，探索有效的教学方法和模式，切实提高大学生的专业英语水平和直接使用英语从事科研的能力。但是，目前我国高等院校实施双语教学的过程中所遇到的主要问题是教材选择无所适从，部分学生无法承受外文原版教材的价格。

为此，我们在《油气渗流力学》中文版的基础上编写本书。目的不外乎两方面：一是为了提高“渗流力学”双语课程的教学质量以及大学生的专业英语水平；二是希望本书的英文版能向世界的渗流力学界展示我们的教学研究成果，了解我们的渗流力学的教学水平，同时接受国际同行的检验，期待他们的批评指正。

由于本书是由《油气渗流力学》中文版翻译而来，所以它对外国在华的留学生学习渗流力学和大学本科生学习专业外语均有所裨益。

本书第一、五、六章由陈军斌翻译，第二、三、四章由双立娜翻译，第七、八章和附录由黄海翻译，全书由李璗审定。

我们虽然从事渗流力学教学研究多年，但理论水平及写作能力都还不够高，特别是英文水平，离准确而漂亮的行文还差得远，敬请海内外同行不吝指正。

最后，我们诚挚感谢西安石油大学和石油工业出版社的支持，使得本书能够与广大读者见面。

李　璗

2012年3月

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Chapter 1 Basic law of percolation

The flow of fluid (liquid, gas and their mixture) in porous medium is called percolation. Solid containing pores or solid collection is called porous medium. Large pores are called cavern and its diameter is from 2mm to tens of meters; the porous media with micron diameter called pores, as for the fractures that divide the solid into non-contact parts, its width has a wide range, while the throats connecting pores are rather narrow. No matter whether they are cavern, fractures or pores, there is no strict definition at home and abroad at present. Sandstone, limestone are commonly seen as porous media; shale, igneous and metamorphic rocks with cracks that are seen as porous media too. The process of oil and gas development is the procedure that the oil, gas and water flow from the reservoir to the well bottom and then from bottom to the surface. Only the percolation principles of oil, gas, water and their mixture are mastered can more and more oil and gas be produced from reservoir. Mechanics of oil and gas flow through porous media is the science that studies movement principles of oil, gas, water and their mixture in reservoir.

Most of pores in sandstone reservoir are intergranular and they are more evenly distributed compared with the fractures. Shale, igneous and metamorphic rocks are fractured reservoir and the distribution of fractures is of great random. Sandstone is known as the porous medium; while shale with cracks, igneous and metamorphic rocks are known as fractured medium. Limestone, dolomite and some sandstone with both pores and fractures are known as double medium. The rocks talked above are the main reservoir of oil and gas of industrial value in China.

According to the current teaching program of relevant speciality, fluid statics in porous media and character of fluid and porous medium are introduced in the course of oil and gas reservoir physics. In order to suit the needs of petroleum engineering and without loss of generality, the static distribution of oil, gas and water in reservoir is introduced first in this chapter, and several oil displacement forces are introduced focusing on teaching the concepts. On this basis, the basic basic law of percolation—Darcy's law is introduced and then is used to analyze a variety of practical problems.

1.1 Static state in reservoirs

1.1.1 Distribution of oil and gas in reservoirs

Most of oil and gas reservoirs remain a state of relative equilibrium before development, the distribution of oil, gas and water in reservoir is associated with the properties of rocks and fluid in it. If the reservoir contains oil, gas and water at the same time, as the lightest component, gas will occupy the top of the structure, so it is known as gas cap; oil will gather in the lower wings, and water with greater density will gather under oil, as shown in figure 1 - 1.

In reservoir the oil-water contact surface is called oil-water interface, and it shall be the oil-bearing boundary when projected onto the horizontal plane. Strictly speaking, oil-bearing boundary should be classified as inner boundary and outer boundary. In fact, average of the two oil boundaries is generally taken as the oil-bearing boundary. The intersecting line of oil-gas interface and reservoir top is called gas boundary. If the outer-ring of reservoir connects to source that supply for reservoir, then the reservoir is called open-reservoir, the projection of overall profile is called supply boundary (Figure 1 - 1). If the outer-ring of reservoir is closed and its altitude is the same as oil-water interface, then the reservoir is called closed reservoir, and its overall profile is called closed boundary.

According to the distribution of oil, gas and water, water lies outside the oil-bearing boundary is called edge water; if the reservoir is of large thickness or relatively flat structure that makes the water under the oil, then the water is called bottom water.

In actual oil field, it is rare to see reservoir with only a single oil layer, most of reservoirs are of sandwich-type oil layers. The lithologic character, oil-water interface and oil-gas interface are not always the same from layer to layer. Therefore, the distribution and features of oil, gas and water should be understood first before oil fields development.

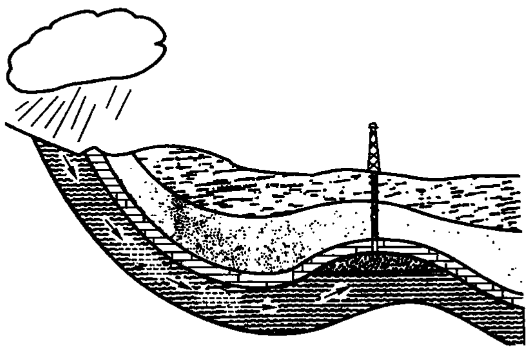

Figure 1 - 1 Reservoir with recharge area

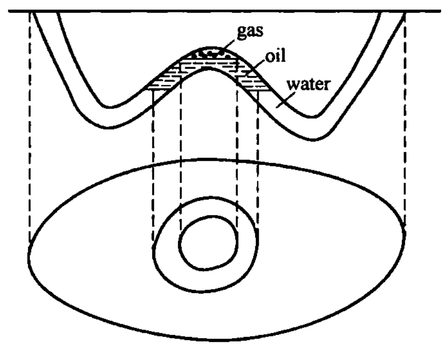

Figure 1 - 2 Oil and gas boundary

1.1.2 Concept of various pressures

Reservoir pressure is the character of formation energy, if the pressure field of reservoir is obtained; it is equivalent to know the flow state of fluid in oil layers. In the oil field development process, the concept of different pressures are often encountered, some will introduced as follows.

1. Initial formation pressure p_i

The reservoir pressure is the fluid pressure within the pores in the middle of oil formation. The reservoirs remain a state of relative equilibrium before development generally, and at this point the fluid pressure is called initial formation pressure. When the dip angle of formation is relatively big, the mid-depth of each well is usually not the same, wells lie on the top of structure have a shallower mid-depth while the wells lie in the wings have a deeper mid-depth. The total energy (total water head) $H = Z + p/(\rho g)$ of a arbitrary point M is a constant before oil field development, as shown in Figure 1 - 3. In this figure, OO' is base level, and initial oil-water interface is always chosen as the base level; Z is the height from the base level; ρ is the density of fluid; p is the pressure of point M, g is acceleration of gravity.

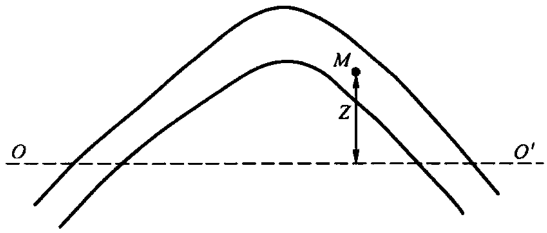

Figure 1 - 3 Converted pressure figure

The original formation pressure of each well is not of equality. Each well should be shut down and test its steady state formation pressure after the first exploration well discovers commercial oil and gas flow, and pressure should be tested in the beginning of each exploration well, the tested pressure of mid-depth of reservoir is the initial formation pressure of each well.

Strictly speaking, the initial equilibrium is destroyed and the initial formation pressure cannot be obtained any more as long as only once well produces. If the pressure manometer cannot be put in the mid-depth of reservoir, the initial formation pressure can be calculated from tested pressure gradient. Clearly, in order to improve the measurement accuracy, the pressure manometer should be put in the mid-depth of reservoir as far as possible.

2. Current formation pressure p

In the process of oil and gas reservoir development, if a well is off production and others are still producing stably, the bottom-hole pressure of the shut-down well will gradually increased. After a long time, the pressure will no longer increase and stabilize. The measured pressure of mid-depth of reservoir of this well is called current formation pressure, and it is also known as static pressure.

3. Supply pressure p_e

When there is a fluid supply zone, the pressure of supply boundary is called supply pressure, with p_e stands for it.

4. Bottom hole pressure p_w

The pressure measured in the mid-depth of well bottom in the process of production is called well bottom pressure, also known as flow pressure, with p_w stands for it.

5. Converted pressure p_r

As is known from fluid mechanics, except pressure energy, the fluid in the reservoir also has potential energy, if it is in kinematical state, it is also has kinetic energy. As a point M in reservoir shown in figure 1 - 3, the total energy of unit mass fluid is:

$$H = Z + \frac{p}{\rho g} + \frac{v^2}{2g}$$

Where: v —The movement velocity of point M.

Of course, as for the original state, $v = 0$.

Because flow rate is very small in reservoir and the order is 10^{-7} m/s, and its square is smaller than hydrostatic head which should be omitted. So the total energy can be written as:

$$H = Z + \frac{p}{\rho g}$$

The total energy can be also expressed in the form of pressure:

$$p_r = \rho g H = p + \rho g Z$$

In the formula above: p_r is the converted pressure at point M, it stands for the total energy of fluid point M, and p stands for the magnitude of pressure energy.

Before oil and gas reservoir development, the pressure that is converted to the initial oil-water interface of each well should be equal. Then the fluid flows from the place with high converted pressure to the place with lower converted pressure after oil and gas reservoir development. Because the reservoir uplift is always much lower than its lateral extension, therefore for the sake of convenience for analysis, three-dimensional percolation is usually simplified to the planar percolation, hence the concept of converted pressure must be used. In the days to come when studying the percolation of whole reservoir, all the pressures are converted pressures if there is no added illustrations.

Example 1 − 1 The mid-depth of formation at a given well is −940m above sea-level, its oil-water interface altitude is −1200m, the formation crude oil density is 800kg/m^3. The reservoir middle pressure actually measured is 12.5MPa, and then what is the converted pressure on the oil-water interface?

Solution: the elevation of middle reservoir of this well is:

$$Z = 1200 - 940 = 260(\text{m})$$

$$\begin{aligned} p_r &= p + \rho g Z \\ &= 12.5 + 800 \times 9.81 \times 260 \times 10^{-6} \\ &= 14.54(\text{MPa}) \end{aligned}$$

In the course of well performance analysis, the formation pressure of each well need to be compared, and at this moment only use converted pressure can we get right conclusions.

1.2 Driving forces and driving mode in reservoir

When the bottom hole pressure of a well in reservior decreases, the oil or gas will flow to the bottom of well. What is the energy driving the oil in reservoir?

Some kind of energy is summarized as follows.

1.2.1 Hydrostatic pressure

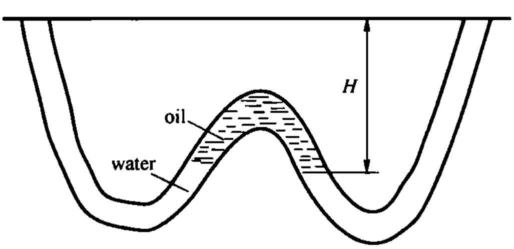

Figure 1 − 4 Pressure on oil-water interface

When there is the existence of water supply area which is connected with oil and gas reservoir, the hydrostatic pressure of water supply area is the driving force. The fluid will flow as long as the bottom hole pressure is lower than the pressure of supply area. When the water supply is ample, the pressure on the oil-water interface can be considered unchanged. If the depth of oil-water interface is H (Figure 1 − 4), then the hydrostatic pressure on the oil-water interface is:

$$p = 10^{-6}\rho_w g H$$

Where: ρ_w—The density of water, kg/m^3;

g—Acceleration of gravity, m/s^2;

H—The depth of oil-water interface, m;

p—The pressure on the oil-water interface, MPa.

This kind of driving force that relies on hydrostatic pressure of supply area to displace oil is called water drive. For most reservoirs, water supply is always less than the oil production. Apart from a few small oil and gas reservoirs, in the whole development stage, it is rare to see the reservoirs with natural water drive, and water injection is the main development scheme. The majority of oil and gas reservoirs in China are developed by water flooding.

1.2.2 Elastic energy of formation and the fluid in it

The crude oil in formation is in compression state under the original reservoir pressure long term. After the well is put into production the reservoir fluid will expand with the drop of reservoir pressure.

The physical quantity that characterizes the magnitude of elastic energy of fluid is compacting factor of fluid, it is the reciprocal of bulk modulus of elasticity, and it shows the relative change of fluid volume when unit pressure changes. It is expressed as the following formula:

$$C_L = -\frac{1}{V}\frac{\partial V}{\partial p} \tag{1-1}$$

In the formula above: V is volume of fluid, because fluid volume V is a decreasing function of pressure p, so its derivative is negative. In order to make C_L positive, a negative sign need to be added in front of the formula. Also, because fluid volume is function of temperature T, marked as $V(p, T)$, so there appears a partial derivative in the formula, and C_L is isothermal compressibility.

Normally, during development there is little change in temperature of reservoir, so the impact of temperature on the liquid volume is not considered generally. The coefficient of compressibility of water is considered as a constant within the reservoir pressure variation range. Its magnitude is about $(3.7 \sim 5) \times 10^{-3}\,MPa^{-1}$, the coefficient of compressibility is relevant to the natural gas content dissolved in it, and it has a big range, its magnitude usually is $(7 \sim 140) \times 10^{-3}\,MPa^{-1}$. While the coefficient of compressibility of gas is rather bigger than that of oil and water, and it cannot be considered as constant with pressure. When the temperature of reservoir is a constant, formula (1 − 1) can be simplified as:

$$C_L = -\frac{1}{V}\frac{dV}{dp} \tag{1-2}$$

When the coefficient of compressibility of fluid changes little, and it can be approximately considered as a constant, the formula above can be also expressed as:

$$C_L = -\frac{1}{V}\frac{\Delta V}{\Delta p} \tag{1-3}$$

If oil volume factor B_o is lead in, and another formula of compressibility of oil C_o can be educed from formula (1 -2) as:

$$C_o = -\frac{1}{B_o}\frac{dB_o}{dp} \quad (p > p_b) \qquad (1-4a)$$

or

$$C_o = \frac{1}{B_o}\frac{dB_o}{dp} \quad (p < p_b) \qquad (1-4b)$$

This is because when the formation pressure is lower than the saturation pressure, volume factor is the increasing function of pressure, so there is no negative sign in front of formula. The negative sign is needed when the formation pressure is higher than the saturation pressure.

Similar rock particles stand compression of formation pressure, when the formation pressure drops, the rock particles will expand. But coefficient of compressibility of solid is very small comparing with that of liquid and gas, so it can be omitted. The formation lies deep underground; it stands overburden force F, and this force is balanced by the sum of the product of rock frame stress σ_m multiplied by the contact area of cap formation and reservoir bed A_m plus of the product of formation pressure (hydrostatic pressure of fluid in reservoir pores) p multiplied by the cross-section area of pores A_p, the formula is shown below:

$$F = \sigma_m A_m + pA_p$$

In the process of oil recovery, the formation pressure will drop; especially arround the bottom hole, the pressure drop even more. Because the overburden acting force F is a constant, the stress that the rock frame stands will increase which causes its deformation and the decreasing pore volume, the reduction of porosity and permeability. Figure 1 -5 is the deformation figure of rock frame.

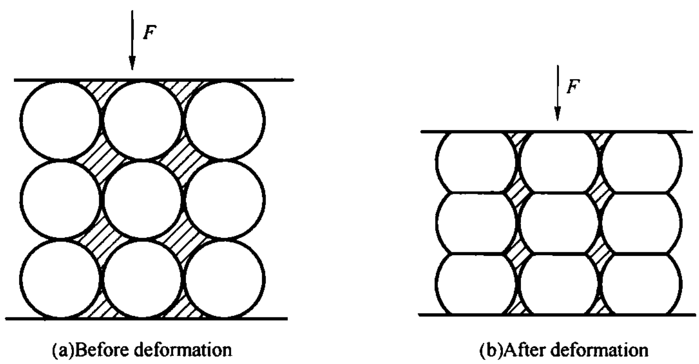

Figure 1 -5 The deformation figure of rock frame

If the water injection pressure is increased, the stress that the rock frame stands will decrease and rock frame will recover which makes the pore volume increase. In fact, this process is irreversible, the increase of rock frame stress will cause plastic deformation, such as the destruction of cementation texture, and it will not recover after the build up of pressure. To simplify, if the reservoir is not deep, the process can be considered as reversible, i. e. elastic, and the error is not great. If the depth of reservoir is more than 4000m or it is abnormal pressure reservoir, the change of porosity and permeability with pressure can not be ignored. The experience of low permeability oil and gas field development has shown that the change of permeability with pressure of low permeability oil and gas field is noticeable, and this problem can not be ignored.

From the analysis above we know: when the formation pressure decreases, the pore volume decreases; while the formation pressure increases, the pore volume increases. So the compression coefficient of formation pores (formation compressibility for short) C_f is defined as:

$$C_f = -\frac{1}{V_p}\frac{\partial V_p}{\partial p} \tag{1-5}$$

Assuming that the process of percolation is isothermal, and then the partial derivative form can be written as the total derivative form. In Russian literatures, the formation coefficient of compressibility β_n is defined by rock bulk volume V as denominator as shown:

$$\beta_n = \frac{1}{V}\frac{dV_p}{dp}$$

If the porosity of formation is ϕ, and the equation $V_p = \phi V$ can be obtained, then:

$$\beta_n = \phi C_f$$

From Russian and West literatures, this difference should be noticed. The formula (1-5) is used as the definition of coefficient of compressibility of formation pores.

In actual oil fields, the situation that oil-water two-phase exists or oil-gas-water three-phase co-exists always happens, then a total coefficient of compressibility of rock and fluid C_t can be introduced now. It shows the total fluid driven from unit pore volume depends on the expansion of oil, gas and water and the reduction of pore volume when the formation pressure drops one unit. When the saturation of oil, gas and water is respectively S_o, S_g and S_w, then:

$$C_t = C_f + C_o S_o + C_w S_w + C_g S_g \tag{1-6}$$

In the formula above: C_o, C_g and C_w is respectively the coefficient of compressibility of oil, gas and water.

The oil and gas flow from reservoir to hole bottom depending on elasticity of formation only is called elastic driving. Generally speaking, as a driving force, elastic drive accounts for less than 10% of whole drive but it exists in the entire developing process, especially when the work system is changed.

1.2.3 Elastic energy of dissolved gas

There is a large number of natural gas dissolved in crude oil, and when formation pressure drops below the oil saturation pressure, the previously dissolved gas will escape from oil and change to be free gas, the gas elastic expansion will drive the oil from the formation to the hole bottom. This kind of driving mode relying on only dissolved gas without any other energy is called dissolved gas drive. The driving energy of dissolved gas drive and elastic drive is uniformly distributed in the oil reservoirs, and it is different from the energy of water drive from outer boundary.

1.2.4 Elastic energy of gas cap

As for the saturated oil and gas reservoir with gas cap, when wells are put into production, the decrease of formation pressure will inevitably cause the pressure decline of gas cap which leads the natural gas in gas cap to expend and drive the oil to the bottom hole. Apparently, the gas cap drive must be along with dissolved gas drive.

1.2.5 Action of gravity

Oil will naturally flow from the structural higher place to the lower place, from the top of oil layer to the bottom; the gravity of oil is also a driving force when there is no other driving energy.

The driving forces listed above are the driving energy which are met often. As for a particular reservoir, several driving energy can exist simultaneously, but in different stages of production, one of them must play a dominant role and others are in secondary status. To depend on which energy to drive oil in the production process is called the driving mode of reservoir. Therefore, according to the different driving energy above, the driving mode can be divided into water drive, elastic drive, dissolved gas drive, gas cap drive and gravity drive.

However, the driving mode is not unchangeable; it can be converted from one driving mode to another under some condition. For example: if the developing method used is unreasonable, and there is big recharge area around the reservoir, but, because the production of reservoir is too much making the energy supplement of edge water cannot catch up with the consumption of reservoir, then the pressure will decrease and below saturation pressure in part area of reservoir, and now in this parts of area the water drive with high efficiency change to the dissolved gas drive with low efficiency. Conversely, if there is no supply of natural recharge area, and we adopt the method of artificial water flooding, then the dissolved gas drive and elastic drive can be changed to water drive with high efficiency.

If the driving mode is different, then the percolation mechanism is different and also the percolation process, even the percolation fluid. In order to develop reservoir rapidly and efficiently, the percolation law under different driving modes must be studied to direct reservoir development practice.

There is no doubt that oil must overcome the resistance when flowing in formation. The flow resistance is mainly viscosity resistance because of the complex pore structure, uneven distributed and narrow porous channel and extremely rough channel surface. However, because of the existence of local loss caused by inertia, the Jamin action will cause extra resistance when there is multiphase flow.

1.3 Darcy's law

1.3.1 Continuous medium

All objects are composed of molecule. Even though its appearance is static, the molecule is constantly in motion. Strictly speaking, all the objects are not continuous, although molecular movement laws can be predicted when we know the initial state theoretically. In fact, it is full of trouble when studying the movement of more than three molecules, so it is impossible to study the moving law of fluid at molecular level.

What we care about are the assembly of many molecules and the study of the movement of this assembly. All the properties that characterize the movement of fluid (velocity for example) are the average value of each molecule in the assembly. If the assembly is taken as the basic unit, then fluid can be considered to be continuous, and the basic unit is called mass point, while it is not a mathematical point. The mass point must be as small as possible to fully reflect the flow property. Also, the mass point must include sufficient molecules to guarantee the steady state of the average value. The so called density of one point ρ express with formula is:

$$\rho = \lim_{\Delta V \to 0} \frac{\Delta m}{\Delta V}$$

Where: ΔV—The volume around given point, m^3;

Δm—Quality of fluid in ΔV, kg.

According to the continuum theory, $\Delta V \to 0$ is to approach one material point, while there are plenty of molecules in the mass point and it is not the infinitesimal in mathematics. To study the fluid movement is not starting from the molecular level but from the mass point of continuous medium, it is called microscopic level.

Percolation is the movement of fluid in porous media. Porous media include not only solid but also pores. So one mass point of porous media must include enough solid and pores. It can be imaged that the mass point of porous media is bigger than that of fluid or solid mass point. The so called property of one point in porous media (pressure and velocity for example) is the average value in this point. Because of the extreme complexity of the porous media constructure, in most cases the macroscopic field is studied in percolation mechanics in porous medium, which is the average value of mass point parameter and the microscopic mechanism is less related.

1.3.2 Darcy's law

Assuming that the porous medium is composed of isodiametric ball type particles, whose radius is the average value derived from grain size analysis. According to the result of ball type particles packing study, the porosity of porous media has nothing to do with the size of particles but the mode of arrangement of particles. The porosity is the smallest when the four centre of sphere arrange as rhombus and the value is 0.259, and the stable filling mode with the biggest porosity is not found until today, while the biggest porosity found already with stable arrangement is 0.875.

The actual shape of rock particles is not globular, there is cementing matter except particles. It's hard to explain the real issues with globular particle model. The smallest reservoir porosity around the world is less than 0.259. So far the effective way to simplify the porous media has not been found yet.

All the complex mechanical phenomena and movement in macro-world comply with the Newton's law without any exception. Although flowing form and fluid of percolation are vastly different, they are have general characters. And these general characters are the basic laws of percolation. Studying percolation mechanics in porous medium is to study the properties of each type of percolation—the special percolation law adapts to a given specific case.

The nature of percolation is the actual fluid flow through the porous media. Deducing from the property that the diameter of pores is very small and the contact surface is very big when fluid flow through porous medium, the conclusion can be obtained that the viscous loss is the main loss. Therefore, if a kind of liquid with viscosity μ passes through a formation with the cross-section area A and length L with flow rate Q and causes the differential pressure of formation is Δp, it is easy to know:

$$Q \propto \frac{A\Delta p}{\mu L}$$

If the formula above is written in the form of equation:

$$Q = \frac{KA\Delta p}{\mu L} \quad (1-7)$$

This is the Darcy's law. In 1856 a French engineer Darcy proved this law by laboratory experiments. Darcy made a straight metal cylinder about 1m tall with loose sand in it, and sealed the top and bottom of tube with screen, then fixed piezometer tube on both top and bottom, the water flows from top to bottom (Figure 1 - 6). He found that no matter how the rate of flow changed, the percolation velocity v is directly proportional to hydraulic slope.

$$v = K_s \frac{h_1 - h_2}{L} = K_s \frac{\Delta h}{L}$$

Where: v —The percolation velocity, it is the quotient of flow rate dividing by cross section area of sand column, m/s;

Δh—The height difference of water column in the piezometer, m;

L—The length of sand column, m;

K_s—The percolation coefficient, m.

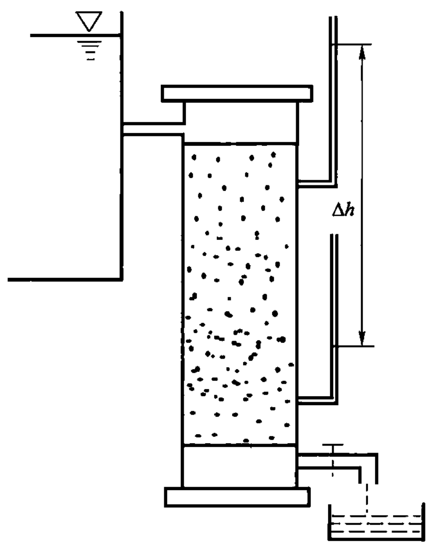

Figure 1 - 6 The flow diagram of Darcy's experiment

If only the percolation coefficient K_s is changed and the sand type remains unchanged, the relationship between hydraulic slope and velocity is unchangeable. Darcy only cared about water at that time, so he used hydraulic slope instead of pressure gradient. While as for each kind of sand the percolation coefficient of water is a constant, Darcy's law of this form is very convenient on studying the percolation of water; it is still used in groundwater percolation mechanics until today. It is not convenient to use hydraulic slope and percolation coefficient to study the movement of oil, gas, water and their mixture in formation, and using pressure gradient and the percolation coefficient which get rid of the impact of fluid property is better. It is apparently to see that:

$$K_s = K\frac{\rho g}{\mu}$$

In the formula above: scale coefficient is K called permeability, it completely depends on the properties of porous media and has nothing to do with the properties of fluid. Then the Darcy's law changes to be:

$$v = \frac{K\Delta p}{\mu L} \tag{1-8}$$

As the result that imagined. Many people repeated Darcy's experiment after him. No matter the cylinder is put straight or inclined, the differential pressure should be the converted pressure that consider the position head.

1.3.3 Percolation velocity and actual velocity

According to the Darcy's law, the percolation velocity v is:

$$v = \frac{Q}{A} = \frac{K\Delta p}{\mu L}$$

It shows the rate of flow through the unit cross-sectional area of rock. However, the percolation velocity is not the actual velocity of the fluid mass point in pores. Because in the given cross section of rock, there are the section of pores that the fluid can pass through and the solid particles that the fluid cannot pass through. Taking any cross section of core and assume its pore area is A_p, then:

$$n = \frac{A_p}{A}$$

In the formula above: n is called the transparence of the cross section, also called surface porosity. Apparently, the actual velocity on any cross section is:

$$v_t = \frac{Q}{A_p} = \frac{Q}{nA}$$

Because the heterogeneity of rock, even though the area A of each section remains unchanged, the pore area of each section is different and the transparence is not the same, therefore, the actual velocity is also different. In order to study easily, the average value $\bar{n}$ of transparence along the flow route is taken to determine the actual velocity of fluid as shown below:

$$v_t = \frac{Q}{\bar{n}A}$$

$$\bar{n} = \frac{1}{L}\int_0^L n\mathrm{d}x$$

And the pore volume of rock can be written as:

$$V_p = \int_0^L An\mathrm{d}x = AL\left(\frac{1}{L}\int_0^L n\mathrm{d}x\right) = A\bar{n}L$$