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网 络 科 学 与 工 程 从 书

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# Introduction to Complex Networks: Models, Structures and Dynamics

英 文 版

## 复杂网络引论 ——模型、结构与动力学

■ 陈关荣 汪小帆 李翔 编著



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# 序

随着以互联网为代表的网络信息技术的迅速发展，人类社会已经迈入了复杂网络时代。人类的生活与生产活动越来越多地依赖于各种复杂网络系统安全可靠和有效的运行。作为一个跨学科的新兴领域，“网络科学与工程”已经逐步形成并获得了迅猛发展。现在，许多发达国家的科学界和工程界都将这个新兴领域提上了国家科技发展规划的议事日程。在中国，复杂系统包括复杂网络作为基础研究也已列入《国家中长期科学和技术发展规划纲要（2006—2020年）》。

网络科学与工程重点研究自然科学技术和社会政治经济中各种复杂系统微观性态与宏观现象之间的密切联系，特别是其网络结构的形成机理与演化方式、结构模式与动态行为、运动规律与调控策略，以及多关联复杂系统在不同尺度下行为之间的相关性等。网络科学与工程融合了数学、统计物理、计算机科学及各类工程技术科学，探索采用复杂系统自组织演化发展的思想去建立全新的理论和方法，其中的网络拓扑学拓展了人们对复杂系统的认识，而网络动力学则更深入地刻画了复杂系统的本质。网络科学既是数学中经典图论和随机图论的自然延伸，也是系统科学和复杂性科学的创新发展。

为了适应这一高速发展的跨学科领域的迫切需求，中国工业与应用数学学会复杂系统与复杂网络专业委员会偕同高等教育出版社出版了这套“网络科学与工程丛书”。这套丛书将为中国广大的科研教学人员提供一个交流最新研

究成果、介绍重要学科进展和指导年轻学者的平台，以共同推动国内网络科学与工程研究的进一步发展。丛书在内容上将涵盖网络科学的各个方面，特别是网络数学与图论的基础理论，网络拓扑与建模，网络信息检索、搜索算法与数据挖掘，网络动力学（如人类行为、网络传播、同步、控制与博弈），实际网络应用（如社会网络、生物网络、战争与高科技网络、无线传感器网络、通信网络与互联网），以及时间序列网络分析（如脑科学、心电图、音乐和语言）等。

“网络科学与工程丛书”旨在出版一系列高水准的研究专著和教材，使其成为引领复杂网络基础与应用研究的信息和学术资源。我们热切希望通过这套丛书的出版，进一步活跃网络科学与工程的研究气氛，推动该学科领域的普及，并为其深入发展作出贡献。

金芳蓉（Fan Chung）院士  
美国加州大学圣地亚哥分校  
2011年元月

# Preface

The extensive study on complex networks is pervading sciences and engineering today, from physical, technological, biological, to social sciences. Their impacts on engineering and technology, in particular, are prominent and their influence is deemed to be far-reaching. Familiar complex networks include the Internet, the World Wide Web, wireless communication networks, biological neural networks, power grids, social relation and scientific cooperation networks, and so on. Research on fundamental properties and dynamical behaviors of various complex networks has become overwhelming recently.

The field of complex networks is indeed developing so fast and so wide that most new comers typically feel difficult to start their leaning and research on the subject. Although there are some well-written textbooks and research monographs that can be adopted for studies by the new comers, these references are generally too advanced or too broad for them to comprehend especially in a relatively short period of time; they are not easy to be used as a textbook for a short course on the subject either. Driven by such teaching and learning demands, this book is designed to serve as a concise textbook for new comers to the field. It is written as a one-semester introductory text for upper-division undergraduate or first-year graduate students in natural science, mathematics and engineering, or as an edited volume for self-study, or as a handy reference for research.

The book title keeps the adjective “complex” to reflect the historical perspective and to emphasize the nature of the subject, which is in line with the common phrases of complex systems and complex dynamics alike, therefore it should not be seen as redundant.

The style of writing in this book is intended to be informal, emphasizing on basic ideas and methodologies with elementary and sometimes heuristic mathematical arguments, easily readable by anyone having minimal knowledge of calculus, linear algebra and ordinary differential equations. In this regard, and to be self-contained, a preliminary chapter on graph theory is included after the first overview chapter. The book is divided into two parts: Part I *Fundamental Theory* is a detailed text consisting of three chapters, presenting background information and basic materials needed to learn the subject, with a variety of exercises for illustrating fundamental concepts and familiarizing related modeling and analysis techniques. Part II *Applications: Selected Topics* contains several selected application-oriented topics, which are all independent of each other, in the sense that one can choose any chapters to

teach or to learn individually without referring to the contents of the other chapters in this part. Of course, the current arrangement of the few chapters would be a more logical ordering if one decides to read through this whole portion of the book. The last chapter of this part provides only outlines of several emerging topics which are believed important and promising, with sufficient numbers of key references provided for interested readers' future studies.

The basic materials of this book have been used in teaching a postgraduate course on *Complex Networks: Modeling, Dynamics and Control* at the City University of Hong Kong since 2007, received some valuable feedback from the students which made the present version better suit to a text style for both teaching and learning.

Owing to the introductory nature of the book, it does not cover most advanced developments in the field especially those in the last five years or so. It is the authors' hope that after learning this elementary text, the readers are ready to read the recent literature so as to pursue state-of-the-art research in the field of network science and engineering. For Chinese readers, there is a complementary volume written in Chinese by the same authors, entitled *Network Science: An Introduction* (Higher Education Press, Beijing, 2012), which has very little overlapping with the present book and hence should be fairly referential and informative.

## Acknowledgements

The authors would like to express their appreciations to their families for the long-term strong supports with great patience and understanding.

The authors also wish to thank those colleagues who had provided many helpful comments and suggestions to enhancing the contents or to improving the descriptions of the book, especially Zhengping Fan (Section 3.6.4), Jun-An Lu (Section 8.2), and Shi Zhou (Section 3.2.3), as well as the following individuals who had provided basic information and materials for Chapter 10: Lin Wang (Section 10.2), Yuting Liu and Zhiming Ma (Section 10.3), Tao Zhou (Section 10.4), Linyuan Lü (Section 10.5), and Luonan Chen (Section 10.6).

The authors appreciate the courtesy of the following publishers for taking some simulation and illustration figures from their journals, which have mostly been modified and all been cited and acknowledged: American Association for the Advancement of Science, American Physical Society, Association for Computing Machinery, Elsevier, European Physical Society, IEEE, National Academy of Sciences of the America, Nature Publishing, SIAM, Springer, Wiley, and World Scientific, among others.

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Guanrong Chen  
Xiaofan Wang  
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February 2012

City University of Hong Kong  
Shanghai Jiao Tong University  
Fudan University

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# **Part I**

# **Fundamental Theory**



# Chapter 1 Introduction

## 1.1 Background and Motivation

Between two randomly selected persons in the world, roughly how many friends are there connecting them together? When searching from one webpage to another through the World Wide Web (WWW), how many clicks are needed in average? How can computer viruses propagate so fast and so wide through the Internet? How are people infected by epidemics such as AIDS, SARS, and bird flu all over the world? How do rumors spread out in human societies? How does an economic crisis occur and then being recovered regionally or even globally? How does electric power blackout emerge from local system failures through the huge power grid? How can traffic jams in metropolitan cities be regulated effectively? How can the human brain work so efficiently? ... All these seemingly different issues have something to do with “networks” — Internet, WWW, social relationship networks, viruses and rumors propagation networks, economic trading and competition networks, power and traffic flow networks, wired and wireless communication networks, biological neural networks, biological metabolic networks, and so on. Most important above all, these apparently different networks have a lot in common.

Since the 1990s, the rapid growth of the Internet as an icon of the high-tech era has led our life to an age of networks. The influence of various complex and dynamical networks is currently pervading all kinds of sciences, ranging from physical to biological, even to social sciences. Its impact on modern engineering and technology is prominent and will be far-reaching. There is no doubt that we are living in a networked world today. On one hand, networks bring us with convenience and benefits, improve our efficiency of work and quality of life, and create tremendous opportunities which we never had before. On the other hand, however, networks also generate harms and damages to humans and societies, typically with epidemic spreading, computer virus propagation, and power blackout, to name just a few. Therefore, the increasing demand for networks and networking also requires a correct view and a serious investigation of the complex properties of various networks. For a long time in the history, studies of communication networks, power networks, biological networks, economic networks, social networks, etc., were carried out separately and independently. How-