国外电子信息精品著作（影印版）

Microwave Circuits for 24GHz Automotive Radar in Silicon-based Technologies

基于硅技术24GHz汽车雷达微波电路

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科学出版社

北京

《国外电子信息精品著作》序

20 世纪 90 年代以来，信息科学技术成为世界经济的中坚力量。随着经济全球化的进一步发展，以微电子、计算机、通信和网络技术为代表的信息技术，成为人类社会进步过程中发展最快、渗透性最强、应用面最广的关键技术。信息技术的发展带动了微电子、计算机、通信、网络、超导等产业的发展，促进了生命科学、新材料、能源、航空航天等高新技术产业的成长。信息产业的发展水平不仅是社会物质生产、文化进步的基本要素和必备条件，也是衡量一个国家的综合国力、国际竞争力和发展水平的重要标志。在中国，信息产业在国民经济发展中占有举足轻重的地位，成为国民经济重要支柱产业。然而，中国的信息科学支持技术发展的力度不够，信息技术还处于比较落后的水平，因此，快速发展信息科学技术成为我国迫在眉睫的大事。

要使我国的信息技术更好地发展起来，需要科学工作者和工程技术人员付出艰辛的努力。此外，我们要从客观上为科学工作者和工程技术人员创造更有利于发展的环境，加强对信息技术的支持与投资力度，其中也包括与信息技术相关的图书出版工作。

从出版的角度考虑，除了较好较快地出版具有自主知识产权的成果外，引进国外的优秀出版物是大有裨益的。洋为中用，将国外的优秀著作引进到国内，促进最新的科技成就迅速转化为我们自己的智力成果，无疑是值得高度重视的。科学出版社引进一批国外知名出版社的优秀著作，使我国从事信息技术的广大科学工作者和工程技术人员能以较低的价格购买，对于推动我国信息技术领域的科研与教学是十分有益的事。

此次科学出版社在广泛征求专家意见的基础上，经过反复论证、仔细遴选，共引进了接近 30 本外版书，大体上可以分为两类，第一类是基础理论著作，第二类是工程应用方面的著作。所有的著作都涉及信息领域的最新成果，大多数是 2005 年后出版的，力求" 层次高、内容新、参考性强"。在内容和形式上都体现

了科学出版社一贯奉行的严谨作风。

当然，这批书只能涵盖信息科学技术的一部分，所以这项工作还应该继续下去。对于一些读者面较广、观点新颖、国内缺乏的好书还应该翻译成中文出版，这有利于知识更好更快地传播。同时，我也希望广大读者提出好的建议，以改进和完善丛书的出版工作。

总之，我对科学出版社引进外版书这一举措表示热烈的支持，并盼望这一工作取得更大的成绩。

王越

中国科学院院士

中国工程院院士

2006 年 12 月

Preface

There are continuous efforts focussed on improving road traffic safety worldwide. Numerous vehicle safety features have been invented and standardized over the past decades. Particularly interesting are the driver assistance systems, since these can considerably reduce the number of accidents by supporting drivers' perception of their surroundings. Many driver assistance features rely on radar-based sensors. Nowadays the commercially available automotive front-end sensors are comprised of discrete components, thus making the radar modules highly-priced and suitable for integration only in premium class vehicles. Realization of low-cost radar front-end circuits would enable their implementation in inexpensive economy cars, considerably contributing to traffic safety.

Cost reduction requires high-level integration of the microwave front-end circuitry, specifically analog and digital circuit blocks co-located on a single chip. Recent developments of silicon-based technologies, e.g. CMOS and SiGe:C bipolar, make them suitable for realization of microwave sensors. Additionally, these technologies offer the necessary integration capability. However, the required output power and temperature stability, necessary for automotive radar sensor products, have not yet been achieved in standard digital CMOS technologies. On the other hand, SiGe bipolar technology offers excellent high-frequency characteristics and necessary output power for automotive applications, but has lower potential for realization of digital blocks than CMOS.

This work presents the design, implementation, and characterization of microwave receiver circuits in CMOS and SiGe bipolar technologies. The applicability of a standard digital 0.13 μm CMOS technology for realization of a 24 GHz narrow-band radar front-end sensor is investigated. The unlicensed industrial, scientific and medical (ISM) frequency band at 24 GHz is particularly interesting for radar applications, due to its worldwide availability and the possibility of inexpensive packaging in this frequency range.

The low-noise amplifier (LNA) and mixer receiver building blocks have been designed in CMOS and bipolar technologies. These building blocks have been integrated into receiver and transceiver front-ends. The performance stability of the circuits is compared over a very wide temperature range from -40 to 125 °C. Additionally, ESD protection techniques are considered. Further, advanced modeling and de-embedding techniques, required for accurate circuit characterization, are investigated. The presented circuits are suitable for automotive, industrial and consumer applications, as e.g. lane-change assistant, door openers or alarms.

This manuscript is based on the dissertation entitled "Microwave Circuits for 24 GHz Radar Front-End Applications in CMOS and Bipolar Technologies" submitted to the University of Paderborn. The research work was supported under the German BMBF funded project EMCpack/FASMZS 16SV3295 and was carried out in close collaboration with Infineon Technologies AG, Neubiberg, Germany.

I would like to express the deepest gratitude to my advisor Prof. Dr.-Ing. Andreas Thiede for his kind guidance, support, patience and insight throughout my research at the University of Paderborn. His valuable advice and inspiring ideas have advanced my work and encouraged me to research deeper. I highly appreciate his great efforts, amiable attention and understanding evinced in the guidance of my research work.

Furthermore, my debt of gratitude is owed to Prof. Dr.-Ing. Andreas Thiede and Prof. Dr.-Ing. Dr.-Ing. habil. Robert Weigel for reviewing this manuscript.

In addition, I would like to express my sincere appreciation to Dr. Werner Simbürger for enabling and supporting my activities at Infineon Technologies AG, Neubiberg, Germany. His sustained encouragement and valuable discussions have contributed a great deal to this work.

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Contents

Chapter 1
Introduction

Increasing road traffic safety is a major objective of governments across the world. In particular, the European Union (EU) has set a challenging objective of halving the number of road accident victims by 2010 [1]. Active on-board safety features offer an approach with a high potential for achieving this target. It has been observed over the past decades that the decrease in the number of victims is related to technological innovations of the automotive safety, such as seatbelts, anti-lock braking system (ABS), airbags or electronic stability programme (ESP), as shown in Fig. 1.1 (*data source*: ADAC). Future generations of active safety equipment will be based on the advanced driver assistance systems (ADAS) including e.g. adaptive-cruise control (ACC), lane-change assistant, collision avoidance systems and parking aids. Implementation of these systems can considerably reduce the number of road accidents and mitigate the consequences. However, the low integration level and high cost of the commercially available modules to date, hamper the mass volume integration and standardization of these systems. Thus, there are research efforts, supported by the EU [2], to develop low-cost driver assistance systems that could be suitable also for low-budget cars.

The cost reduction can be achieved by high level integration of the building blocks on a single chip or in a package, referred to as system on chip (SoC) and system in package (SiP), respectively. Silicon-based technologies as CMOS or SiGe offer high on-chip integration capability and competitive performance compared to the III-V semiconductors as e.g. gallium-arsenide [3], which have been dominating the discrete microwave components market.

The standard digital CMOS process is particularly attractive, as it enables the high-level integration of analog and digital blocks. Recent advances in CMOS technology have enabled it to become an inexpensive alternative for realization of high-frequency integrated circuits. However, the required output power and temperature robustness, particularly for automotive radar sensor products, have not yet been achieved in standard digital CMOS technologies. Furthermore, metal-oxide-semiconductor (MOS) transistors suffer from very high flicker noise corner frequencies compared to bipolar transistors, making it difficult to build a direct down-

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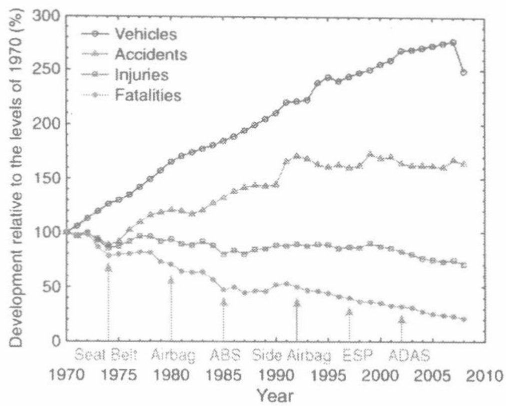

Fig. 1.1 Road traffic statistics in Germany.

conversion receiver in CMOS. This principle has the advantage of simplicity compared to the double-conversion or "sliding-IF" topologies.

The SiGe bipolar process offers transistors with excellent high-frequency characteristics, sufficient output power for automotive radar applications and the required robustness, but has the disadvantage of lower integration capability compared to CMOS. The use of a BiCMOS instead of a pure bipolar process resolves the integration drawback, but increases the costs and complexity.

The aim of this work is the realization of integrated receiver front-ends for narrow-band radar sensors at 24 GHz in Infineon's CMOS and SiGe technologies. Both technologies are automotive-certified and offer moderate mask costs at the present market volumes. These sensors can be useful for multiple car safety features such as lane-change assistant, side-crash detection, rear-collision warning or Stop and Go assistant, as presented in Fig. 1.2. Presently, some of the features are implemented using various approaches, such as CCD or CMOS cameras, ultrasonic sensors or lidar. Highly-integrated low-cost radar sensors may offer cheaper alternative for these features. Furthermore, realization of cost-effective sensors can enable their implementation in consumer and industrial applications as e.g. door openers, motion sensors and alarms.

Currently the market is dominated by ultra-wideband (UWB) 24 GHz short-range radar (SRR) sensors [4], [5]. However, according to Electronic Communications Committee's (ECC) decision, these sensors are allowed on the market in the EU only until July 2013 [6]. The allocated frequency range 21.625 – 26.625 GHz is only a temporary solution, whilst 79 GHz is intended for future SRR applications. However, the unlicensed industrial, scientific and medical (ISM) frequency range 24 – 24.25 GHz is an attractive alternative for mid-range radar sensors due to a higher allowable transmit power. Furthermore, it is still possible to use standard inexpensive packaging solutions [7], classical mounting techniques and moderately-priced measurement equipment at this frequency range. A frequency-modulated

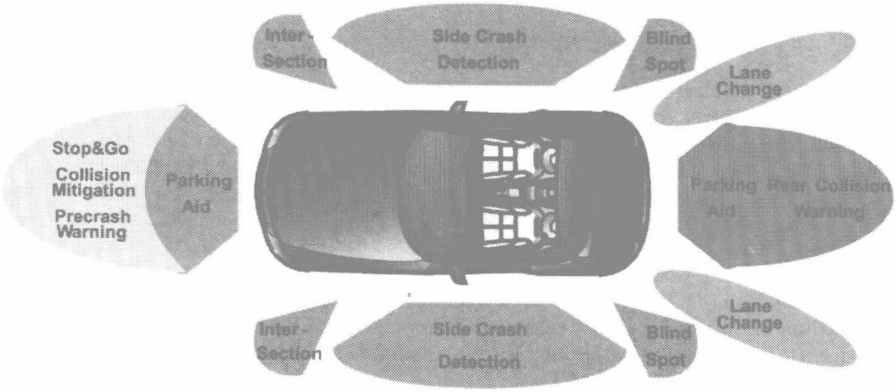

Fig. 1.2 Radar-based automotive safety features.

continuous-wave (FMCW) radar in the 24 GHz ISM band offering a 70 m outreach has been reported in [8].

Numerous publications report 24 GHz receivers in CMOS [9], [10] or SiGe [11], [12] technology. However, there are only a few publications that present fully ESD-protected receiver front-ends [13]. Sufficient ESD robustness and performance stability over a wide range of temperatures are required for hostile environment such as in automotive applications. Furthermore, there are few publications in the literature that offer direct comparison of receiver building blocks realized in different technologies [14].

This work presents the design, implementation, and characterization of building blocks and integrated receivers for 24 GHz narrow-band radar applications realized in CMOS and SiGe technologies. The performance stability of the circuits is compared over the extended automotive temperature range from -40 to 125 °C. The challenges posed to circuit design due to high ESD robustness requirements and corresponding circuit techniques are addressed. Furthermore, innovative circuit topologies for LNA, mixer and transceiver integration are proposed. Additionally, novel modeling and measurement techniques are presented.

This manuscript is organized as follows: chapter 2 provides an overview of radar principles, system architectures and the design challenges. The circuits in this work are realized in Infineon's CMOS and SiGe:C technologies, which are described in chapter 3. Modeling and simulation techniques are presented in chapter 4. Chapter 5 presents measurement techniques and discusses challenges of on-board measurement of differential devices. Circuit design and the experimental results of the building blocks and of the integrated receivers are described in chapter 6. Transceiver considerations and implementations are presented in chapter 7. Finally, chapter 8 summarizes the results and concludes this work.

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Chapter 2
Radar Systems

Automotive safety systems require information about the objects in the vicinity of the vehicle. These data are usually obtained by sensing the surroundings. A typical sensor system usually transmits a signal and estimates the attributes of the available targets, such as velocity or distance from the sensor, based on the measurement of the scattered signal. The signal used for this purpose in radar (radio detection and ranging) systems is an electromagnetic (EM) wave at microwave frequencies. The main advantage of radar systems compared to other alternatives such as sonar or lidar is the immunity to weather conditions and potential for lower cost realization.

Section 2.1 describes the principle of radar. There are two main operation principles, continuous wave (CW) and pulsed. The latter is not treated within this scope, since the frequency regulations in the ISM band result in a limitation on the absolute transmitter power. Thus, pulsed radar would result in a lower SNR due to a lower duty cycle compared to the CW operation. Radar operation is discussed in sections 2.2 - 2.4. Frequency regulations around 24 GHz are described in section 2.5. Typical radar architectures and circuit related challenges are presented in section 2.6. Section 2.7 provides an overview of the automotive radar systems and their application for car safety. Finally, section 2.8 concludes this chapter with considerations on technology features needed for radar realization.

2.1 Radar Principle

Radar systems are composed of a transmitter that radiates electromagnetic waves of a particular waveform and a receiver that detects the echo returned from the target. Only a small portion of the transmitted energy is re-radiated back to the radar, which is then amplified, down-converted and processed. The range to the target is evaluated from the travelling time of the wave. The direction of the target is determined by the arrival angle of the echoed wave. The relative velocity of the target is determined from the doppler shift of the returned signal.

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For automotive radar applications the separation between the transmitter and receiver is negligible compared to the distance to a target. Thus, these systems are monostatic in a classical sense. However, the automotive radar systems are usually referred to as *bistatic* when two separate antennas are used for transmit and receive and *monostatic* when the same antenna is used for these functions, as depicted in Fig. 2.1. The latter configuration requires a duplexer component to provide isolation between transmitter and receiver. This is usually realized using expensive external bulky transmit/receive (T/R) switch or circulator components. The solution of using hybrid ring coupler [1] offers a cost advantage at the expense of lower performance due to higher losses and increased noise figure.

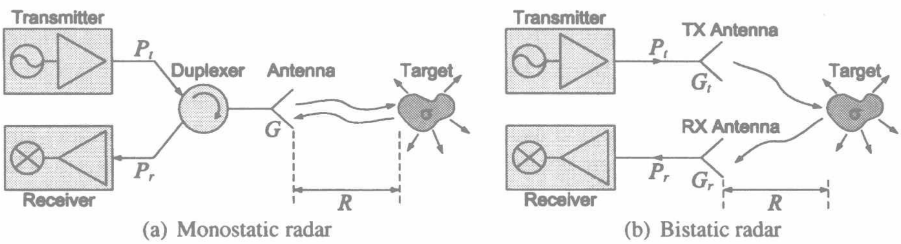

Fig. 2.1 Radar configurations.

2.2 Radar Equation and System Considerations

The radar equation provides the received power level as function of the characteristics of the system, the target and the environment. The well-known bistatic radar equation [2] for the system in Fig. 2.1(b) is given by

$$P_r = \frac{P_t A_{\text{er}} A_{\text{et}} \sigma}{4\pi R^4 \lambda^2 L_{\text{sys}}}, \tag{2.1}$$

where P_r is the received power, P_t is the transmitted power, A_{er} and A_{et} are the effective area of the receive and transmit antennas, respectively, R is the distance to the target, σ is the radar cross-section (RCS), defined as the ratio of the scattered power in a given direction to the incident power density and L_{sys} is the system loss due to misalignment, antenna pattern loss, polarization mismatch, atmospheric loss [3], but also due to analog to digital conversion and fast Fourier transform (FFT) windowing. Taking into consideration that the effective area of the receive and transmit antenna is related to the wavelength λ and to the antenna gain G_r and G_t, as $A_{\text{er}} = G_r\lambda^2/4\pi$ and $A_{\text{et}} = G_t\lambda^2/4\pi$, respectively, the radar equation can be rewritten as

$$P_r = \frac{P_t G_r G_t \lambda^2 \sigma}{(4\pi)^3 R^4 L_{\text{sys}}}. \tag{2.2}$$

Based on the system characteristics and the noise floor of the receiver a certain minimal signal power level $P_{\text{r,min}}$ is required in order to detect the target. Thus, from (2.2) the maximum achievable radar range can be calculated as follows

$$R_{\max} = \left(\frac{P_t G_r G_t \lambda^2 \sigma}{(4\pi)^3 P_{\text{r,min}} L_{\text{sys}}} \right)^{1/4}. \tag{2.3}$$

Furthermore, in most practical designs a minimal signal to noise ratio (SNR) at the output of the receiver $\text{SNR}_{\text{o,min}}$ is considered in order to ensure high probability of detection and low false-alarm rate. Typically, SNR values of higher than 12 dB are required. The noise factor of a receiver is defined as

$$F = \frac{S_i/N_i}{S_o/N_o}, \tag{2.4}$$

where S_i and S_o are the input and output signal levels, respectively, N_o is the noise level at the receiver output and N_i is the input noise level, given by

$$N_i = k_{\text{B}} T B, \tag{2.5}$$

where B is the system bandwidth, k_{B} is the Boltzmann constant and T is the temperature in Kelvin. Taking into consideration that there is an additional processing gain due to the integration over several pulses, approximately given by $G_{\text{int}} = T_{\text{CPI}} \cdot B$, where T_{CPI} is the coherent processing interval (CPI), the maximum radar range in (2.3) can be rewritten as a function of $\text{SNR}_{\text{o,min}}$ as follows

$$R_{\max} = \left(\frac{P_t G_r G_t \lambda^2 \sigma T_{\text{CPI}}}{(4\pi)^3 \cdot kTF \cdot \text{SNR}_{\text{o,min}} \cdot L_{\text{sys}}} \right)^{1/4}. \tag{2.6}$$

The attenuation for the propagation of the electromagnetic waves at 24 GHz is about 0.15 dB/km [4]. Taking into consideration that the typical range for automotive radar sensors is up to 200 m, the contribution of the atmospheric attenuation to L_{sys} is negligible. Even under heavy rain or fog conditions the attenuation over these distances is in the range of few decibels.

The RCS of typical targets in automotive applications ranges from 0.1 to 200 m^2. The antenna gain is usually in the range of 15 - 25 dBi. Antennas are typically realized as patch antenna arrays for beam shaping. Their large size at 24 GHz limits the dimensions of radar modules.

Equation (2.6) can be rearranged for the noise factor F. Plugging in the smallest RCS and the largest required distance of operation results in the required receiver noise figure. For example, for an object with a σ of 0.1 that has to be detected at a maximal distance of 100 m with transmit and receive antenna gains of 20 dB, transmitter power of 0 dBm, the system losses of 3 dB, the CPI time of 2 ms and minimum required SNR after the FFT of 12 dB, the required receiver front-end noise figure is 10.75 dB. For a typical narrow-band 24 GHz system a single side-band (SSB) noise figure (NF) of less than 10 dB is needed. The NF is related to the noise factor in (2.4) as $\mathrm{NF} = 10 \cdot \log(F)$. The gain of a receiver front-end is less crucial, since it can be compensated in the baseband stage. However, it still has to be above 10 dB for a low receiver NF, due to noise figure cascading.

Another limiting case, referred to as the *blocker* case, is the scenario of a large target with maximum RCS being present very close to a radar at a minimal distance of operation. This sets the requirement on the front-end linearity in terms of input-referred 1dB compression point (IP1dB), which should be typically above −15 dBm. Combination of both mentioned limiting cases results in a requirement on the receiver's dynamic range (DR), which usually should be above 70 dB.

2.3 CW and Frequency-Modulated Radar

2.3.1 Doppler Radar

A classical continuous wave (CW) or Doppler radar implementation uses a fixed transmit frequency to detect a moving target and its velocity. It is based on the Doppler frequency shift. If there is a non-zero relative velocity v_r between a radar transmitter sending a signal at frequency f_0, and a moving target, the returned signal has frequency $f_0 + f_d$, where f_d is the Doppler frequency shift given by

$$f_d = \frac{2v_r}{c} f_0, \tag{2.7}$$

where c is the speed of light. The relative velocity v_r of a target is determined by the velocity component along the line-of-sight of the radar and is given by

$$v_r = v_a \cos\theta, \tag{2.8}$$

where v_a is the actual velocity of a target and θ is the angle between the target trajectory and the line-of-sight, as depicted in Fig. 2.2.

It can be observed from (2.8) that for an acute angle $\theta < 90°$, corresponding to an approaching target, the Doppler shift is positive $f_d > 0$ and for an obtuse angle $\theta > 90°$, corresponding to a receding target, the Doppler shift is negative $f_d < 0$. Furthermore, for $\theta = 90°$ the Doppler shift is zero. Thus, the velocity component perpendicular to the line-of-sight cannot be determined.