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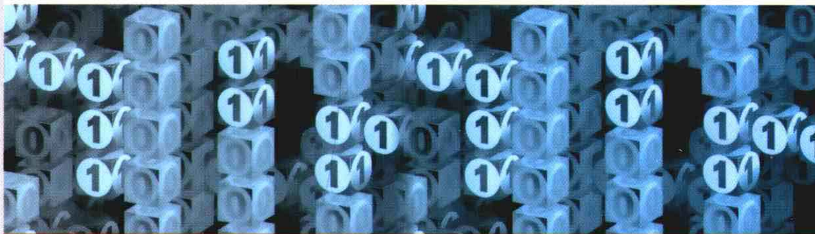
Discrete and Combinatorial Mathematics: An Applied Introduction

(Fifth Edition)



离散及组合数学

（第五版）

Ralph P. Grimaldi 著

（英文影印版）

科学出版社

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内 容 简 介

本书内容主要由四部分组成：(1)基本离散结构，包括集合论与逻辑、函数与关系、语言与有限状态自动机；(2)组合数学，包括排列组合、容斥原理、生成函数、递推关系、鸽巢原理；(3)图论及其应用，包括图论基本知识、树、最优化与匹配；(4)现代应用代数，包括环论与模算术、布尔代数与交换函数、群、编码理论、波利亚计数方法、有限域与组合设计。

本书可作为计算机、软件工程和电子类相关专业的本科生或研究生教材，也可供工程技术人员参考。

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Preface

It has been more than twenty years since September 2, 1982, when I signed the contract to develop what turned into the first edition of this present textbook. At that time the idea of further editions never crossed my mind. Consequently, I continue to find myself simultaneously very humbled and very pleased with the way this textbook has been received by so many instructors and especially students. The first four editions of this textbook have found their way into many colleges and universities here in the United States. They have also been used in other nations such as Australia, Canada, England, Ireland, Japan, Mexico, the Netherlands, Scotland, Singapore, South Africa, and Sweden. I can only hope that this fifth edition will continue to enlighten and challenge all those who wish to learn about some of the many facets of the fascinating area of mathematics called discrete mathematics.

The technological advances of the last four decades have resulted in many changes in the undergraduate curriculum. These changes have fostered the development of many single-semester and multiple-semester courses where some of the following are introduced:

1. Discrete methods that stress the finite nature inherent in many problems and structures;
2. Combinatorics — the algebra of enumeration, or counting, with its fascinating interrelations with so many finite structures;
3. Graph theory with its applications and interrelations with areas such as data structures and methods of optimization; and
4. Finite algebraic structures that arise in conjunction with disciplines such as coding theory, methods of enumeration, gating networks, and combinatorial designs.

A primary reason for studying the material in any or all of these four major topics is the abundance of applications one finds in the study of computer science — especially in the areas of data structures, the theory of computer languages, and the analysis of algorithms. In addition, there are also applications in engineering and the physical and life sciences, as well as in statistics and the social sciences. Consequently, the subject matter of discrete and combinatorial mathematics provides valuable material for students in many majors — not just for those majoring in mathematics or computer science.

The major purpose of this new edition is to continue to provide an introductory survey in both discrete and combinatorial mathematics. The coverage is intended for the beginning student, so there are a great number of examples with detailed explanations. (The examples are numbered separately and a thick line is used to denote the end of each example.) In addition, wherever proofs are given, they too are presented with sufficient detail (with the novice in mind).

The text strives to accomplish the following objectives:

1. To introduce the student at the sophomore-junior level, if not earlier, to the topics and techniques of discrete methods and combinatorial reasoning. Problems in counting, or enumeration, require a careful analysis of structure (for example, whether or not order and repetition are relevant) and logical possibilities. There may even be a question of existence for some situations. Following such a careful analysis, we often find that the solution of a problem requires simple techniques for counting the possible outcomes that evolve from the breakdown of the given problem into smaller subproblems.

2. To introduce a wide variety of applications. In this regard, whenever data structures (from computer science) or structures from abstract algebra are required, only the basic theory needed for the application is developed. Furthermore, the solutions of some applications lend themselves to iterative procedures that lead to specific algorithms. The algorithmic approach to the solution of problems is fundamental in discrete mathematics, and this approach reinforces the close ties between this discipline and the area of computer science.

3. To develop the mathematical maturity of the student through the study of an area that is so different from the traditional coverage in calculus and differential equations. Here, for example, there is the opportunity to establish results by counting a certain collection of objects in more than one way. This provides what are called combinatorial identities; it also introduces a novel proof technique. In this edition the nature of proof, along with what constitutes a valid argument, is developed in Chapter 2, in conjunction with the laws of logic and rules of inference. The coverage is extensive, keeping the student (with minimal background) in mind. [For the reader with a logic course (or something comparable) in his or her background, this material can be skipped over with little or no difficulty.] Proofs by mathematical induction (along with recursive definitions) are introduced in Chapter 4 and then used throughout the subsequent chapters.

With regard to theorems and their proofs, in many instances an attempt has been made to motivate theorems from observations on specific examples. In addition, whenever a finite situation provides a result that is not true for the infinite case, this situation is singled out for attention. Proofs that are extremely long and/or rather special in nature are omitted. However, for the very small number of proofs that are omitted, references are supplied for the reader interested in seeing the validation of these results. (The amount of emphasis placed on proofs will depend on the goals of the individual instructor and on those of his or her student audience.)

4. To present an adequate survey of topics for the computer science student who will be taking more advanced courses in areas such as data structures, the theory of computer languages, and the analysis of algorithms. The coverage here on groups, rings, fields, and Boolean algebras will also provide an applied introduction for mathematics majors who wish to continue their study of abstract algebra.

The prerequisites for using this book are primarily a sound background in high school mathematics and an interest in attacking and solving a variety of problems. No particular programming ability is assumed. Program segments and procedures are given in pseudocode, and these are designed and explained in order to reinforce particular examples. With regard to calculus, we shall mention later in this preface its extent in Chapters 9 and 10.

My primary motivation for writing the first four editions of this book has been the encouragement I had received over the years from my students and colleagues, as well as from the students and instructors who used the first four editions of the textbook at many different colleges and universities. Those four editions reflected both my interests and concerns and those of my students, as well as the recommendations of the Committee on the Undergraduate Program in Mathematics and of the Association of Computing Machinery. This fifth edition continues along the same lines, reflecting the suggestions and recommendations made by the instructors and especially the students who have used or are using the fourth edition.

Features

Following are brief descriptions of some of the major features of this newest edition. These are designed to assist the reader (student or otherwise) in learning the fundamentals of discrete and combinatorial mathematics.

Emphasis on algorithms and applications. Algorithms and applications in many areas are presented throughout the text. For example:

1. Chapter 1 includes several instances where the introductory topics on enumeration are needed — one example, in particular, addresses the issue of over-counting.

2. Section 7 of Chapter 5 provides an introduction to computational complexity. This material is then used in Section 8 of this chapter in order to analyze the running times of some elementary pseudocode procedures.

3. The material in Chapter 6 covers languages and finite state machines. This introduces the reader to an important area in computer science — the theory of computer languages.

4. Chapters 7 and 12 include discussions on the applications and algorithms dealing with topological sorting and the searching techniques known as the depth-first search and the breadth-first search.

5. In Chapter 10 we find the topic of recurrence relations. The coverage here includes applications on (a) the bubble sort, (b) binary search, (c) the Fibonacci numbers, (d) the Koch snowflake, (e) Hasse diagrams, (f) the data structure called the stack, (g) binary trees, and (h) tilings.

6. Chapter 16 introduces the fundamental properties of the algebraic structure called the group. The coverage here shows how this structure is used in the study of algebraic coding theory and in counting problems that require Polya's method of enumeration.

Detailed explanations. Whether it is an example or the proof of a theorem, explanations are designed to be careful and thorough. The presentation is primarily focused on improving understanding on the part of the reader who is seeing this type of material for the first time.

Exercises. The role of the exercises in any mathematics text is a crucial one. The amount of time spent on the exercises greatly influences the pace of the course. Depending on the interest and mathematical background of the student audience, an instructor should find that the class time spent on discussing exercises will vary.

There are over 1900 exercises in the 17 chapters. Those that appear at the end of each section generally follow the order in which the section material is developed. These exercises are designed to (a) review the basic concepts in the section; (b) tie together ideas presented in earlier sections of the chapter; and (c) introduce additional concepts that are related to the material in the section. Some exercises call for the development of an algorithm, or the writing of a computer program, often to solve a certain instance of a general problem. These usually require only a minimal amount of programming experience.

Each chapter concludes with a set of supplementary exercises. These provide further review of the ideas presented in the chapter, and also use material developed in earlier chapters.

Solutions are provided at the back of the text for almost all parts of all the odd-numbered exercises.

Chapter summaries. The last numbered section in each chapter provides a summary and historical review of the major ideas covered in that chapter. This is intended to give the reader an overview of the contents of the chapter and provide information for further study and applications. Such further study can be readily assisted by the list of references that is supplied.

In particular, the summaries at the ends of Chapters 1, 5, and 9 include tables on the enumeration formulas developed within each of these chapters. Sometimes these tables include results from earlier chapters in order to make comparisons and to show how the new results extend the prior ones.

Organization

The areas of discrete and combinatorial mathematics are somewhat new to the undergraduate curriculum, so there are several options as to which topics should be covered in these courses. Each instructor and each student may have different interests. Consequently, the coverage here is fairly broad, as a survey course mandates. Yet there will always be further topics that some readers may feel should be included. Furthermore, there will also be some differences of opinion with regard to the order in which some topics are presented in this text.

The nature and importance of the algorithmic approach to problem solving is stressed throughout the text. Ideas and approaches on problem solving are further strengthened by the interrelations between enumeration and structure, two other major topics that provide unifying threads for the material developed in the book.

The material is subdivided into four major areas. The first seven chapters form the underlying core of the book and present the fundamentals of discrete mathematics. The coverage here provides enough material for a one-quarter or one-semester course in discrete mathematics. The material in Chapter 2 can be reviewed by those with a background in logic. For those interested in developing and writing proofs, this material should be examined very carefully. A second course — one that emphasizes combinatorics — should include Chapters 8, 9, and 10 (and, time permitting, sections 1, 2, 3, 10, 11, and 12 of Chapter 16). In Chapter 9 some results from calculus are used; namely, fundamentals on differentiation and partial fraction decompositions. However, for those who wish to skip this chapter, sections 1, 2, 3, 6, and 7 of Chapter 10 can still be covered. A course that emphasizes the theory and applications of finite graphs can be developed from Chapters 11, 12, and 13. These chapters form the third major subdivision of the text. For a course in applied algebra, Chapters 14, 15, 16, and 17 (the fourth, and final, subdivision) deal with the algebraic structures — group, ring, Boolean algebra, and field — and include applications on cryptology, switching functions, algebraic coding theory, and combinatorial designs. Finally, a course on the role of discrete structures in computer science can be developed from the material in Chapters 11, 12, 13, 15, and sections 1–9 of Chapter 16. For here we find applications on switching functions, the RSA cryptosystem, and algebraic coding theory, as well as an introduction to graph theory and trees, and their role in optimization.

Other possible courses can be developed by considering the following chapter dependencies.

Chapter	Dependence on Prior Chapters
1	No dependence
2	No dependence (Hence an instructor can start a course in discrete mathematics with either the study of logic or an introduction to enumeration.)
3	1, 2
4	1, 2, 3
5	1, 2, 3, 4
6	1, 2, 3, 5 (Minor dependence in Section 6.1 on Sections 4.1, 4.2)
7	1, 2, 3, 5, 6 (Minor dependence in Section 7.2 on Sections 4.1, 4.2)
8	1, 3 (Minor dependence in Example 8.6 on Section 5.3)
9	1, 3
10	1, 3, 4, 5, 9 (Minor dependence in Example 10.33 on Section 7.3)
11	1, 2, 3, 4, 5 (Although some graph-theoretic ideas are mentioned in Chapters 5, 6, 7, 8, and 10, the material in this chapter is developed with no dependence on the graph-theoretic material given in these earlier results.)
12	1, 2, 3, 4, 5, 11
13	3, 5, 11, 12
14	2, 3, 4, 5, 7 (The Euler phi function (ϕ) is used in Section 14.3. This function is derived in Example 8.8 of Section 8.1 but the result can be used here in Chapter 14 without covering Chapter 8.)
15	2, 3, 5, 7
16	1, 2, 3, 4, 5, 7
17	2, 3, 4, 5, 7, 14

In addition, the index has been very carefully developed in order to make the text even more flexible. Terms are presented with primary listings and several secondary listings. Also there is a great deal of cross referencing. This is designed to help the instructor who may want to change the order of presentation and deviate from the straight and narrow.

Changes in the Fifth Edition

The changes here in the fifth edition of *Discrete and Combinatorial Mathematics* reflect the observations and recommendations of students and instructors who have used earlier editions of the text. As with the first four editions, the tone and purpose of the text remain intact. The author's goal is still the same: to provide within these pages a sound, readable, and understandable introduction to the foundations of discrete and combinatorial mathematics — for the beginning student or reader. Among the changes one will find in this fifth edition we mention the following:

- The examples in Section 4 of Chapter 1 now include material on *runs*, a concept that arises in the study of statistics — in particular, in the area of quality control.
- Exercise 13 for Section 3 of Chapter 2 develops the rule of inference known as *resolution*, a rule that serves as the basis for many computer programs designed to automate a reasoning system.
- The earlier editions of this text included a section that introduced the notion of probability. This section has now been expanded and three additional optional sections have been added for those who wish to further examine some of the introductory ideas associated with discrete probability — in particular, the axioms of probability, conditional probability, independence, Bayes' Theorem, and discrete random variables.

• The coverage on partial orders and total orders in Section 3 of Chapter 7 now includes an optional example where the Catalan numbers arise in this context.

• The introductory material in Section 1 of Chapter 8 has been rewritten to provide a more readable transition between the coverage on counting and Venn diagrams in Section 3 of Chapter 3 and the more general technique known as the Principle of Inclusion and Exclusion.

• One of the fascinating features of discrete and combinatorial mathematics is the variety of ways a given problem can be solved. In the fourth edition (in Chapters 1 and 3) the reader learned, in two different contexts, that a positive integer n had 2^{n-1} compositions — that is, there are 2^{n-1} ways to write n as an ordered sum of positive-integer summands. This result is now established in three other ways: (i) by the Principle of Mathematical Induction in Chapter 4; (ii) using generating functions in Chapter 9; and (iii) by solving a recurrence relation in Chapter 10.

• For those who want even more on discrete probability, Section 2 of Chapter 9 includes an example that deals with the geometric random variable.

• Section 2 of Chapter 10 now includes a discussion of the work by Gabriel Lamé in estimating the number of divisions used in the Euclidean algorithm to find the greatest common divisor of two positive integers.

• The Master theorem (of importance in the analysis of algorithms) is introduced and developed in an exercise for Section 6 of Chapter 10.

• The material on transport networks (in Section 3 of Chapter 13) has been updated and now incorporates the Edmonds-Karp algorithm in the procedure originally developed by Lester Ford and Delbert Fulkerson.

• The coverage on modular arithmetic in Section 3 of Chapter 14 now includes applications dealing with the linear congruential pseudorandom number generator, private-key cryptosystems, and modular exponentiation. Further, in Section 4 of Chapter 14, the material dealing with the Chinese Remainder Theorem, which was only stated in previous editions, now includes a proof of this result as well as an example dealing with how it is applied.

• Section 4 of Chapter 16 is new and optional. The material here provides an introduction to the RSA public-key cryptosystem and shows how one can apply some of the theoretical results developed in prior sections of the text.

• As with the second, third, and fourth editions, a great deal of effort has been applied in updating the summary and historical review at the end of each chapter. Consequently, new references and/or new editions are provided where appropriate.

• For this fifth edition, the following pictures and photographs have been added to the summary and historical review of certain chapters: a picture of Thomas Bayes and a photograph of Andrei Nikolayevich Kolmogorov in Chapter 3; a picture of Al-Khowârizmî in Chapter 4; a photograph of David A. Huffman in Chapter 12; and a photograph of Joseph B. Kruskal in Chapter 13.

Ancillaries

• There is an *Instructor's Solutions Manual* that is available, from the publisher, for those instructors who adopt the textbook for their classes. It contains the solutions and/or answers for all of the exercises within the 17 chapters and the three appendices of this textbook.

• There is also a *Student's Solutions Manual* that is available separately. It contains the solutions and/or answers for all of the odd-numbered exercises in the textbook. In some cases more than one solution is presented.

• The following Web site provides additional resources for learning more about discrete and combinatorial mathematics. In addition it also provides a way for readers to contact the author with comments, suggestions, or possible errors they have found.

www.aw.com/grimaldi

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Alas, the remaining errors, ambiguities, and misleading comments are once again the sole responsibility of the author.

R.P.G.
Terre Haute, Indiana

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1

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