

高等学校经济类双语教学推荐教材

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经济学经典教材·金融系列

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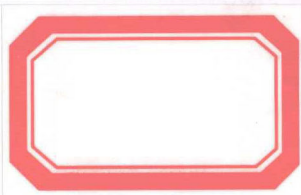
投资科学

(国际版)

戴维·G·卢恩伯格 著

 中国人民大学出版社

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Investment Science

戴维·G·卢恩伯格 (David G. Luenberger) 著
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前言

投资方面的理论目前受到那些极具聪明才智的人们的关注——一方面是由于在金融理论方面有非常大的发展，另一方面是因为信息和电脑技术爆炸式的增长，还有一方面是因为投资活动的全球性扩张。投资学理论最近的这些新发展正在进入大学课堂，进入金融服务机构，进入商业企业，并且正在被很多私人投资者所意识到。本书旨在成为传播过程中的一种工具。

本书致力于强调基本原则，并且说明如何掌握这些基本原则并转化为现实投资问题的完备和实际的解决方案。本书的组织结构反映了这种方法：从前到后的章节所覆盖到的内容是从简单的概念逐步深入到更加高深的内容。本书用最多的篇幅研究一些特殊的金融产品和投资问题，是为了让它们沿着本书的概念进度的线索进展下去，对它们的分析不仅用来说明概念，同时也用来描述投资环境的某些特征。

本书是为那些有与工程、数学或理科的本科教育大致相同的专业知识背景的个人，或者对基础数学很了解的个人而设计的。投资学的语言大部分是数学的，这个学科的某些方面只能以数学术语的方式来表述。然而，本书里用到的数学并不复杂——例如，只需要微积分里最基本的部分——但是读者必须要适应把数学作为推导和解决问题的方法。这样的读者才能够利用他们的专业知识背景来加快和深入学习。

实际上，本书能够从几个层次上去阅读，不同的层次要求不同的数学知识，包括不同的学习范围。对于这些不同层次的最简单的说明就在于本书的编码排版方式。有的部分或者小标题，其后面标有星号（*），比如，“2.6 应用和扩展”，星号就表明这部分是特殊的，其内容可能有点偏，或者比本书其他部分对数学要求更高，在第一次阅读的时候这些部分可以略过去。这种编排方法仅仅是一个大概，教材解释了每个部分开头的内容并且指导读者如何继续读下去。

每章后面的练习是教材的重点，读者应该尝试一下把每章的练习都做几个。这些练习也是编了号码的：做了◇记号的是比一般的练习在数学上难度更大的；作了⊕记号的要求用数值计算方法（通常要用一个表格软件程序）。

本书很大程度上受电脑表格软件包的影响，几乎投资学的所有基本思想——比如现值、资产组合免疫、现金匹配、项目最优化、因素模型、二项式网格风险中性定价和模拟——都能很容易地用表格软件包来说明。因此，就可以在那些从概念上的内容看达到最新技术发展水平的章节中给出各种例子。更进一步说，学生能够用现成的软件系统地说明和解决现实的和富有挑战性的投资问题。通过使学生全面了解问题的所有方面，这个过程加深了他们的理解。很多学了这门课的学生都说，完成课程的各个项目（特别是标了⊕记号的练习题里面难度很大的那部分）使他们学得更好。

在写作本书的过程中有很大的乐趣，在某种程度上是因为我从我的同事和学生那里得到了很多的鼓励和帮助。特别要由衷地感谢 Graydon Barz, Kian Esteghamat, Charles Feinstein, Marius Holtan, Blake Johnson, Robert Maxfield, Paul McEntire, James Smith, Lucie Tepla 和 Lauren Wang，他们都阅读了不断改进的文稿的相当大的部分，并提出了修改建议。几位同仁的真知灼见使本书的最后这一版本改进不少，他们是：波士顿大学的 Joseph Cherian, 田纳西大学的 Phillip Daves, 斯特拉克莱德大学的 Jaime Cuevas Dermody, 多伦多大学的 Myron Gordon, 英属哥伦比亚大学的 Robert Heinkel, 威斯康星大学的 James Hodder, 多伦多大学的 Raymond Kan, 亚利桑那大学的 Chris Lamoureux, 卡内基·梅隆大学的 Duane Seppi, 多伦多大学的 Suresh Sethi, 西北大学的 Costas Skiadas, 特瑞纳资金管理公司的 Jack Treynor。

我也衷心感谢我的妻子南茜，感谢她的鼓励和我对长时间的文字处理的理解。最后，我要衷心地感谢很多热情的学生，他们课堂上的问题，以及练习和课外活动的辛勤劳动，为初稿的成形提供了宝贵的信息反馈。

戴维·G·卢恩伯格

1997年4月

目 录

第 1 章	引言.....	1
	1.1 现金流.....	2
	1.2 投资和市场.....	3
	1.3 典型的投资问题.....	6
	1.4 本书的结构.....	8

第一部分 确定性的现金流

第 2 章	利息基础理论.....	13
	2.1 本金和利息.....	13
	2.2 现值.....	18
	2.3 现金流的现值和终值.....	19
	2.4 内部收益率.....	22
	2.5 评估准则.....	24
	2.6 应用和扩展.....	28
	2.7 小结.....	34
	练习.....	35
	参考文献.....	38
第 3 章	固定收益证券.....	40
	3.1 未来现金流的市场.....	41
	3.2 价值公式.....	44
	3.3 债券的详细介绍.....	49
	3.4 收益率.....	52
	3.5 久期.....	57
	3.6 免疫.....	62
	3.7 凸度.....	65
	3.8 小结.....	66
	练习.....	68
	参考文献.....	71

第 4 章	利率期限结构	72
	4.1 收益率曲线	72
	4.2 期限结构	73
	4.3 远期利率	77
	4.4 对利率期限结构的几种解释	80
	4.5 期望动态	83
	4.6 连续现值	88
	4.7 浮动利率债券	90
	4.8 久期	91
	4.9 免疫	94
	4.10 小结	96
	练习	97
	参考文献	101
第 5 章	应用利率分析	102
	5.1 资本预算	103
	5.2 最优资产组合	108
	5.3 动态现金流过程	111
	5.4 最优化管理	114
	5.5 一致性定理	121
	5.6 公司的估值	124
	5.7 小结	128
	练习	130
	参考文献	134

第二部分 单期随机现金流

第 6 章	均值—方差资产组合理论	137
	6.1 资产收益	137
	6.2 随机变量	141
	6.3 随机收益	146
	6.4 投资组合均值和方差	150
	6.5 可行集合	155
	6.6 马克维茨模型	157
	6.7 两基金定理	162
	6.8 包含无风险资产的投资组合	165
	6.9 单基金定理	166
	6.10 小结	169
	练习	170
	参考文献	172

第 7 章	资本资产定价模型.....	173
	7.1 市场均衡.....	173
	7.2 资本市场线.....	175
	7.3 定价模型.....	177
	7.4 证券市场线.....	181
	7.5 投资含义.....	183
	7.6 绩效评估.....	184
	7.7 作为定价公式的资本资产定价模型.....	187
	7.8 项目选择 [*]	190
	7.9 小结.....	192
	练习.....	193
	参考文献.....	195
第 8 章	模型和数据.....	197
	8.1 引言.....	197
	8.2 因素模型.....	198
	8.3 作为因素模型的资本资产定价模型.....	205
	8.4 套利定价理论 [*]	207
	8.5 数据和统计.....	212
	8.6 其他参数的估计.....	217
	8.7 偏离均衡 [*]	218
	8.8 多期谬论.....	221
	8.9 小结.....	222
	练习.....	224
	参考文献.....	227
第 9 章	一般原理.....	228
	9.1 引言.....	228
	9.2 效用函数.....	228
	9.3 风险厌恶.....	231
	9.4 效用函数评述 [*]	234
	9.5 效用函数与均值—方差准则 [*]	237
	9.6 线性定价.....	240
	9.7 投资组合选择.....	242
	9.8 对数最优定价 [*]	245
	9.9 有限状态模型.....	247
	9.10 风险中性定价.....	251
	9.11 定价方法的选择 [*]	252
	9.12 小结.....	254
	练习.....	255
	参考文献.....	258

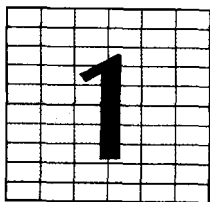
第三部分 衍生证券

第 10 章	远期、期货和互换	263
	10.1 引言.....	263
	10.2 远期合约.....	264
	10.3 远期价格.....	266
	10.4 远期合约的价值.....	273
	10.5 互换*.....	273
	10.6 期货合约基础.....	275
	10.7 期货价格.....	278
	10.8 与期望现货价格的关系*.....	281
	10.9 完美套期.....	282
	10.10 最小方差套期.....	283
	10.11 最优套期*.....	285
	10.12 非线性风险套期.....	287
	10.13 小结.....	291
	练习.....	291
	参考文献.....	295
第 11 章	资产动态模型	296
	11.1 二项式网格模型.....	297
	11.2 可加模型.....	299
	11.3 倍数模型.....	300
	11.4 典型参数值*.....	303
	11.5 对数正态随机变量.....	304
	11.6 随机游走和维纳过程.....	305
	11.7 股票价格过程.....	308
	11.8 伊藤引理*.....	312
	11.9 再论二项式网格模型.....	313
	11.10 小结.....	315
	练习.....	316
	参考文献.....	318
第 12 章	期权基础理论	319
	12.1 期权概念.....	320
	12.2 期权价值的实质.....	322
	12.3 期权组合和看涨——看跌平价.....	325
	12.4 提前执行.....	327

	12.5 单期二项式期权理论.....	327
	12.6 多期期权.....	330
	12.7 更一般的二项式问题.....	333
	12.8 评估实际投资机会.....	337
	12.9 一般的风险中性定价 [*]	344
	12.10 小结.....	345
	练习.....	346
	参考文献.....	350
第 13 章	其他期权问题.....	351
	13.1 引言.....	351
	13.2 布莱克 - 斯科尔斯 (Black-Scholes) 方程.....	351
	13.3 看涨期权公式.....	355
	13.4 风险中性定价 [*]	357
	13.5 得尔塔.....	358
	13.6 复制、综合期权和组合保险 [*]	360
	13.7 计算方法.....	362
	13.8 特异性期权.....	368
	13.9 储存成本和股息 [*]	371
	13.10 鞅定价 [*]	373
	13.11 小结.....	375
	附录: 布莱克 - 斯科尔斯方程的另一种推导.....	376
	练习.....	378
	参考文献.....	381
第 14 章	利率衍生证券.....	382
	14.1 利率衍生证券的例子.....	382
	14.2 理论需要.....	384
	14.3 二项式方法.....	385
	14.4 定价的应用.....	389
	14.5 校准和可调整利率贷款 [*]	391
	14.6 前推方程.....	395
	14.7 匹配期限结构.....	397
	14.8 免疫.....	400
	14.9 担保抵押债券 [*]	402
	14.10 利率动态模型 [*]	406
	14.11 连续时间解 [*]	408
	14.12 小结.....	410
	练习.....	411
	参考文献.....	413

第四部分 一般现金流

第 15 章	最优组合增长	417
	15.1 投资轮盘	417
	15.2 增长的对数效用方法	419
	15.3 对数—最优策略的性质 *	425
	15.4 替代方法 *	425
	15.5 连续时间增长	427
	15.6 可行区域	430
	15.7 对数最优定价公式 *	435
	15.8 对数最优定价和布莱克—斯科尔斯方程 *	438
	15.9 小结	440
	练习	441
	参考文献	443
第 16 章	一般投资评估	444
	16.1 多期证券	444
	16.2 风险中性定价	447
	16.3 最优定价	448
	16.4 双网格	452
	16.5 双网格定价	454
	16.6 具有个体不确定性的投资	458
	16.7 买入价格分析	463
	16.8 连续时间定价 *	469
	16.9 小结	471
	练习	472
	参考文献	474
附录 A	概率基础理论	475
	A.1 一般概念	475
	A.2 正态分布随机变量	476
	A.3 对数正态随机变量	477
附录 B	微积分和最优化	479
	B.1 函数	479
	B.2 微积分	480
	B.3 最优化	481
	练习答案	484
	中文练习答案	489



INTRODUCTION

Traditionally, investment is defined as the current commitment of resources in order to achieve later benefits. If resources and benefits take the form of money, investment is the present commitment of money for the purpose of receiving (hopefully more) money later. In some cases, such as the purchase of a bank certificate of deposit, the amount of money to be obtained later is known exactly. However, in most situations the amount of money to be obtained later is uncertain.

There is also a broader viewpoint of investment—based on the idea of flows of expenditures and receipts spanning a period of time. From this viewpoint, the objective of investment is to tailor the pattern of these flows over time to be as desirable as possible. When expenditures and receipts are denominated in cash, the net receipts at any time period are termed **cash flow**, and the series of flows over several periods is termed a **cash flow stream**. The investment objective is that of tailoring this cash flow stream to be more desirable than it would be otherwise. For example, by taking out a loan, it may be possible to exchange a large negative cash flow next month for a series of smaller negative cash flows over several months, and this alternative cash flow stream may be preferred to the original one. Often future cash flows have a degree of uncertainty, and part of the design, or tailoring, of a cash flow stream may be concerned with controlling that uncertainty, perhaps reducing the level of risk. This broader definition of investment, as tailoring a pattern of cash flows, encompasses the wide assortment of financial activities more fully than the traditional view. It is this broader interpretation that guides the presentation of this book.

Investment science is the application of scientific tools to investments. The scientific tools used are primarily mathematical, but only a modest level of mathematics is required to understand the primary concepts discussed in this book. The purpose of this book is to convey both the principles of investment science and an understanding of how these principles can be used in practice to make calculations that lead to good investment decisions.

There is also an art to investment. Part of this art is knowing what to analyze and how to go about it. This part of the art can be enhanced by studying the material in this

book. However, there is also an intuitive art of being able to evaluate an investment from an assortment of qualitative information, such as the personality characteristics of the people involved (the principals), whether a proposed new product will sell well, and so forth. This part of the art is not treated explicitly in this book, although the reader will gain some appreciation for just what this art entails.

1.1 CASH FLOWS

According to the broad interpretation, an investment is defined in terms of its resulting cash flow sequence—the amounts of money that will flow to and from an investor over time. Usually these cash flows (either positive or negative) occur at known specific dates, such as at the end of each quarter of a year or at the end of each year. The stream can then be described by listing the flow at each of these times. This is simplest if the flows are known deterministically, as in bank interest receipts or mortgage payments. In such cases the stream can be described by a series of numbers. For example, if the basic time period is taken as one year, one possible stream over a year, from beginning to end, is $(-1, 1.2)$, corresponding to an initial payment (the investment) of \$1 at the beginning of the year and the receipt of \$1.20 a year later. An investment over four years might be $(-1, .10, .10, .10, 1.10)$, where an initial investment of \$1 leads to payment of \$.10 at the end of each year for three years and then a final payment of \$1.10. Note that for a span of one year, two cash flow numbers are specified—one at the beginning and one at the end. Likewise, the four-year example involves five cash flow numbers.

Cash flow streams can also be represented in diagram form, as illustrated in Figure 1.1. In such a figure a time axis is drawn and a cash flow at a particular time is indicated by a vertical line at that time, the length of the line being proportional to the magnitude of the flow.

If the magnitudes of some future cash flows are uncertain (as is frequently the case), a more complex representation of a cash flow stream must be employed. There are a few different techniques for doing this, and they are presented later in the book. But whether or not uncertainty is present, investments are described in terms of cash flow streams.

A diversity of investment issues can be stated in terms of cash flow streams, such as the following: Which of two cash flow streams is most preferable? How much would I be willing to pay to own a given stream? Are two streams together worth more to me than the sum of their individual values? If I can purchase a share of a stream, how much should I purchase? Given a collection of available cash flow streams, what is the most favorable combination of them?

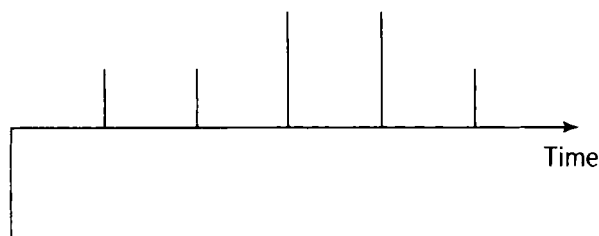


FIGURE 1.1 Cash flow stream. The cash flow stream of an investment can be represented by a diagram. In the example shown, the cash flows occur periodically. The first of these flows is negative, representing a cash outlay, and the subsequent flows are all positive.

Other more complex questions also arise. For example, sometimes the timing of all cash flows is not fixed, but can be influenced by the investor. If I purchase stock in a company, I have a negative cash flow initially, corresponding to my purchase payment; while I hold the stock, I perhaps receive dividends (relatively small positive cash flows) on a regular basis; finally, when I sell the stock, I obtain a major cash flow. However, the time of the last cash flow is not predetermined; I am free to choose it. Indeed, investments sometimes can be actively managed to influence both the amounts and the timing of all cash flows. For example, if I purchase a gold mine as an investment, I can decide how to mine it and thereby influence the cash flow every year. Determination of suitable management strategies is also part of investment science.

The view of investment science as the tailoring of cash flow streams gives the subject wide application. For individuals it applies to personal investment decisions, such as deciding on a home mortgage or planning for retirement. It also applies to business decisions, such as whether to invest in product development, whether to build a new manufacturing plant, and how to manage cash resources. Finally, it applies to government decisions, such as whether to build a dam or change the tax rate. Investment science guides us in the process of combining stocks, bonds, and other investment products into an overall package that has desirable properties. This process enhances total productivity by converting projects that in isolation may be too risky into members of attractive combinations.

1.2 INVESTMENTS AND MARKETS

At its root, investment analysis is a process of examining alternatives and deciding which alternative is most preferable. In this respect investment analysis is similar to the analysis of other decisions—operating a production facility, designing a building, planning a trip, or formulating an advertising campaign. Indeed, much of investment science relies on the same general tools used for analysis of these other decisions.

Investment problems differ from other decision problems in an important respect, however: most investments are carried out within the framework of a financial market, and these markets provide alternatives not found in other decision situations. This structure is what makes investment analysis unique and unusually powerful.

The Comparison Principle

Financial markets simplify decision making through a concept that we term the **comparison principle**. To introduce this principle, consider the following hypothetical situation.

Your uncle offers you a special investment. If you give him \$100 now, he will repay you \$110 in one year. His repayment is fully guaranteed by a trust fund of U.S. Treasury securities, and hence there is virtually no risk to the investment. Also, there

is no moral or personal obligation to make this investment. You can either accept the offer or not. What should you do?

To analyze this situation, you would certainly note that the investment offers 10% interest, and you could compare this rate with the prevailing rate of interest that can be obtained elsewhere, say, at your local bank or from the U.S. Government through, for example, a Treasury bill. If the prevailing interest rate were only 7%, you would probably invest in this special offer by your uncle (assuming you have the cash to invest). If on the other hand the prevailing interest rate were 12%, you would surely decline the offer. From a pure investment viewpoint you can evaluate this opportunity very easily without engaging in deep reflection or mathematical analysis. If the investment offers a rate above normal, you accept; if it offers a rate below normal, you decline.

This analysis is an example of the comparison principle. You evaluate the investment by comparing it with other investments available in the financial market. The financial market provides a basis for comparison.

If, on the other hand, your uncle offers to sell you a family portrait whose value is largely sentimental, an outside comparison is not available. You must decide whether, to you, the portrait is worth his asking price.

Arbitrage

When two similar investment alternatives are both available in the market, conclusions stronger than the comparison principle hold. For example, consider (idealized) banks that offer to loan money or accept deposits at the same rate of interest. Suppose that the rate used at one bank for loans and deposits is 10% and at another bank the rate is 12%. You could go to the first bank and borrow, say, \$10,000 at 10% and then deposit that \$10,000 in the second bank at 12%. In one year you would earn 2% of \$10,000, which is \$200, without investing any cash of your own. This is a form of **arbitrage**—earning money without investing anything. Presumably, you could even make more money by running your scheme at a higher level. It should be clear that this kind of thing does not occur—at least not very often. The interest rates in the two banks would soon equalize.

The example of the two banks assumed that the interest rate for loans and the interest rate paid for deposits were equal within any one bank. Generally, of course, there is a difference in these rates. However, in markets of high volume, such as the markets for U.S. Treasury securities, the difference between the buying price and the selling price is small. Therefore two different securities with identical properties must have approximately the same price—otherwise there would be an arbitrage opportunity.

Often it is assumed, for purposes of analysis, that no arbitrage opportunity exists. This is the **no-arbitrage** assumption.

Ruling out the possibility of arbitrage is a simple idea, but it has profound consequences. We shall find that the principle of no arbitrage implies that pricing relations are linear, that stock prices must satisfy certain relations, and that the prices of derivative securities, such as options and futures, can be determined analytically.

This one principle, based on the existence of well-developed markets, permeates a good portion of modern investment science.

Dynamics

Another important feature of financial markets is that they are dynamic, in the sense that the same or similar financial instruments are traded on a continuing basis. This means that the future price of an asset is not regarded as a single number, but rather as a process moving in time and subject to uncertainty. An important part of the analysis of an investment situation is the characterization of this process.

There are a few standard frameworks that are used to represent price processes. These include binomial lattice models, difference equation models, and differential equation models, all of which are discussed in this text. Typically, a record of the past prices and other information are used to specify the parameters of such a model.

Because markets are dynamic, investment is itself dynamic—the value of an investment changes with time, and the composition of good portfolios may change. Once this dynamic character is understood and formalized, it is possible to structure investments to take advantage of their dynamic nature so that the overall portfolio value increases rapidly.

Risk Aversion

Another principle of investment science is **risk aversion**. Suppose two possible investments have the same cost, and both are expected to return the same amount (somewhat greater than the initial cost), where the term *expected* is defined in a probabilistic sense (explained in Chapter 6). However, the return is certain for one of these investments and uncertain for the other. Individuals seeking investment rather than outright speculation will elect the first (certain) alternative over the second (risky) alternative. This is the risk aversion principle.

Another way to state this principle is in terms of market rates of return. Suppose one investment will pay a fixed return with certainty—say 10%—as obtained perhaps from a government-guaranteed bank certificate of deposit. A second investment, say the stock in a corporation, has an uncertain return. Then the expected rate of return on that stock must be greater than 10%; otherwise investors will not purchase the stock. In general, we accept more risk only if we expect to get greater expected (or average) return.

This risk aversion principle can be formalized (and made analytical) in a few different ways, which are discussed in later chapters. Once a formalism is established, the risk aversion principle can be used to help analyze many investment alternatives.

One way that the risk aversion principle is formalized is through **mean–variance analysis**. In this approach, the uncertainty of the return on an asset is characterized by just two quantities: the mean value of the return and the variance of the return. The risk aversion principle then says that if several investment opportunities have the same mean but different variances, a rational (risk-averse) investor will select the one that has the smallest variance.

This mean–variance method of formalizing risk is the basis for the most well-known method of quantitative portfolio analysis, which was pioneered by Harry Markowitz (who won the Nobel prize in economics for his work). This approach leads to a comprehensive theory of investment and is widely considered to be the foundation for modern portfolio theory. We discuss this important theory in Chapter 6.

A more general way to formalize the risk aversion principle is through the introduction of individual **utility functions**. This approach is presented in Chapter 9.

Later, in Chapter 15, we find that risk aversion takes on a new character when investments are made repeatedly over time. In fact, short-term variance will be found to be *good*, not bad. This is one of the surprising conclusions of the comprehensive view of investment represented by investment science.

1.3 TYPICAL INVESTMENT PROBLEMS

Every investment problem has unique features, but many fit into a few broad categories or types. We briefly outline some of the most important problem types here. Fuller descriptions of these general types and more specific examples appear in the relevant chapters.

Pricing

Let us go back to our very first example of an investment situation, the first offer from your uncle, but now let us turn it around. Imagine that there is an investment opportunity that will pay exactly \$110 at the end of one year. We ask: How much is this investment worth today? In other words, what is the appropriate price of this investment, given the overall financial environment?

If the current interest rate for one-year investments is 10%, then this investment should have a price of exactly \$100. In that case, the \$110 paid at the end of the year would correspond to a rate of return of 10%. If the current interest rate for one-year investments is less than 10%, then the price of this investment would be somewhat greater than \$100. In general, if the interest rate is r (expressed as a decimal, such as $r = .10$), then the price of an investment that pays X after one year should be $X/(1 + r)$.

We determined the price by a simple application of the comparison principle. This investment can be directly compared with one of investing money in a one-year certificate of deposit (or one-year Treasury bill), and hence it must bear the same effective interest rate.

This interest rate example is a simple example of the general pricing problem: Given an investment with known payoff characteristics (which may be random), what is the reasonable price; or, equivalently, what price is consistent with the other securities that are available? We shall encounter this problem in many contexts. For example, early in our study we shall determine the appropriate price of a bond. Later we shall compute the appropriate price of a share of stock with random return characteristics. Still later we shall compute suitable prices of more complicated securities,