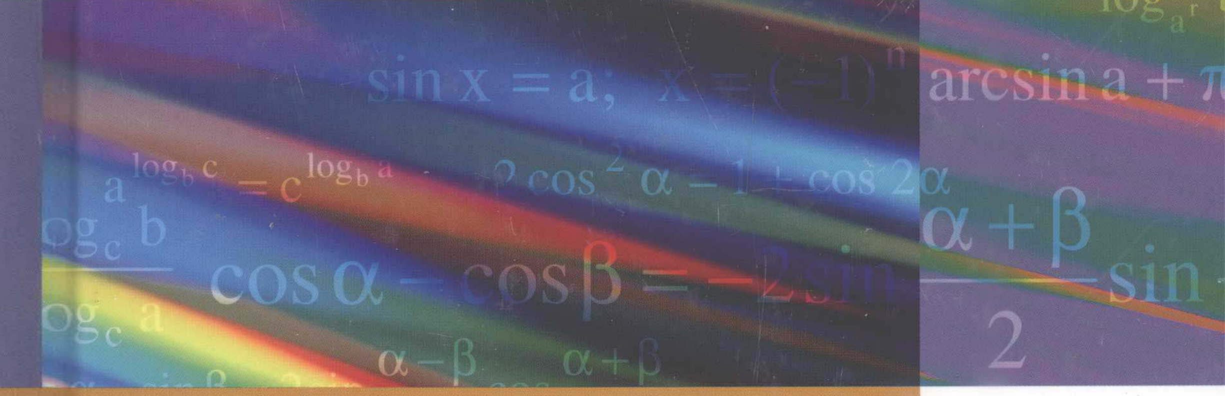
新一代信息科学与技术

Rational Function Systems and Electrical Networks with Multi-parameters

Kai-Sheng Lu

多元有理函数系统与电网络

（英文版）



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DUOYUAN YOULIHANSHU XITONG YU DIANWANGLUO

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Preface

The doctoral course "Rational Function Systems and Electrical Networks with Multi-parameters" has been delivered by the author, for many years based on his extensive research. With a lack of corresponding books in this field, there is a challenge for higher education to have a framework and text to work from. It is quite necessary to transform the course lecture notes and the knowledge acquired over a long career focused in this area into a systematic, comprehensive book to assist in teaching and research.

Due to the inconvenience in exploring structural properties using linear system theory and electrical network theory over the real field R, the author uses the matrices over the field $F(z)$ of rational functions in multi-parameters to describe coefficient matrices of systems and electrical networks and investigates their structural properties based on the description of systems and electrical networks over $F(z)$. The book *Rational Function Systems and Electrical Networks with Multi-parameters* is divided into five chapters. Chapter 1 introduces the background and meaning of systems and electrical networks over $F(z)$. In chapter 2, Matrix theory is extended to the field $F(z)$; the reducibility condition of a class of matrix and its polynomial over $F(z)$ is discussed in detail; the definition of type-1 matrix and two basic properties is given; the fact that type-1 matrix is of the two properties is proved and the variable replacement condition for independent parameters is introduced. Chapter 3 explores the structural controllability and observability of linear systems over $F(z)$; and introduces some new conclusions in time domain and frequency domain. Chapter 4 shows the structural properties of electrical networks over $F(z)$: the separability, reducibility, controllability, and observability of RLC networks over $F(z)$ and structural conditions of controllability and observability over $F(z)$; the separability, reducibility, controllability, and observability of RLCM networks over $F(z)$; the state equation existence condition of active networks over $F(z)$ and controllability and observability conditions over $F(z)$. Chapter 5 describes further thoughts on the field.

Since the object of research in this book is Rational Function Systems and Electrical Networks with Multi-parameters, the results obtained are usually clear and intuitive, and are convenient to analyze and design systems and electric networks. For instance, there is such a conclusion that for an RLC network over $F(z)$ (this means in this kind of RLC networks all the resistors, capacitors, and inductances are regarded as independent parameters) without sources, if it has no all-capacitor cut-sets or all-inductor loops and it is inseparable (these are all structural conditions), then the characteristic polynomial of this network is irreducible over $F(z)$ (it does not need to calculate the characteristic polynomial, and we just need to observe the network structure). The reducibility of the characteristic polynomial has a direct relationship with controllability, observability, and stability. So it is easy to analyze and design an RLC network whose characteristic polynomial is irreducible according to this conclusion. The description of systems and electrical networks over $F(z)$ is a useful tool to investigate the structural properties of systems and electrical networks. Furthermore, the real field R is the subfield of $F(z)$, so the conclusions over $F(z)$ are more general than those over R.

This book summarizes the author's research achievements over the past 20 years with four projects for National Nature Science Foundation of China (subjects include: *Research on Electrical Network Theory over* $F(z)$ *and Computer Assistant Analysis*, 1995 – 1997; *Researching Electrical Network Structural Properties Using Matrix over* $F(z)$, 2002 – 2004; *Research on Structural Controllability and Observability of Systems over* $F(z)$, 2006 – 2008; *Research on Separability, Reducibility, Controllability, Observability and Stability of Active Networks over* $F(z)$, 2010 – 2012) and two projects for the Nature Science Foundation of Hubei Province of China, which is at the leading edge of scientific research.

The description of systems, electrical networks, and matrices over $F(z)$ in this book is different from other relevant books, which introduce linear systems, electrical networks and matrices over R. In this book we explore systems, electrical networks, and matrices over $F(z)$. This book involves three subject areas: systems, networks and matrices over $F(z)$, which is an achievement of interdisciplinary research. So there is a close connection among the three subjects in this book. For example, the reducibility condition of a class of rational function matrix introduced in Chapter 2 is the important base of Chapter 3 and Chapter 4. Usually systems, electrical networks, and matrices are introduced by three distinct subject area books, which are independent to each other.

This book can be used by postgraduate students, Ph.D students, college teachers, researchers and engineers in the field of electronic and electrical engineering, automatic control, and applied mathematics matrix theory .

If any deficiency or mistake should appear in the book due to my oversight, I welcome your comments and feedback to improve future editions and expanding the journey of scientific research.

Kai-Sheng Lu

Acknowledgments

This book is dedicated to my mother Gong-Gao Liu. She entered Tsinghua University in 1938 and graduated from Southwest Union University, which consisted of Beijing University, Tsinghua University and Nankai University during the period of World War Two, and was an assistant of Mr. Yi-Duo Wen at the Institute of Arts Research in Tsinghua University. I am sad to say my mother lost her husband in a car accident at the age of 31. Although her schoolmate fell in love with her, she put her children's needs above all and stayed a widow. She sacrificed her happiness and responded to all the duties of bringing us up despite all kinds of hardships. Without her instruction, edification, and encouragement, I could not have completed this book in my poor health.

The research achievements in this book were supported by the National Nature Science Foundation (NNSF) mentioned above under grant: 69472008, 50977069, 60574012 and 50177024. I would like to offer my sincere gratitude to the NNSF committee.

I would also like to extend my thanks to my students: Xiao-Yu Feng, Guo-Zhang Gao, Ke Zhang, Zong-Tao Wang, Guan-Min Liu, Qiang Ma and Hu-Ping Xu who have taken part in the research works of the NNSF; Qiang Ma and Yu-Peng Yuan, especially Qiang Ma, who have been devoted to the tedious work of editing the e-document of the book.

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1 Introduction

A matrix $A = (a_{ij})_{n \times m}$ is called the matrix over the real field or real number matrix, if each entry a_{ij} of this matrix is a real number. A system $\dot{X} = AX + BU$, $Y = CX + DU$ is called a linear system (including an electrical network, or a network for short) over the real field or a real number system, if its coefficient matrices A, B, C, and D are real number matrices.

We know that linear systems and networks over the real field are explored very well and are successfully used for analysis and design of systems and networks[Chen, 1984 ; Zheng, 2002 ; Chen et al., 1992 ; Chen, 1976; Balabanian, et al., 1969; Chen, 1987]. However, people find that real number matrices are not convenient to analyze structural properties of physical systems (e.g., structural controllability and structural observability). Take the liquid level control system shown in Fig. 1.1 as an example.

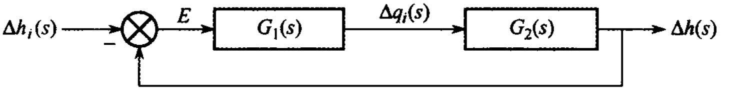
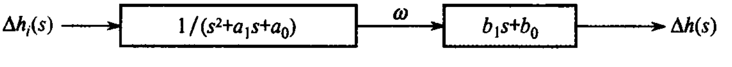


Fig. 1.1 Liquid level control system

In Fig. 1.1, $G_1(s) = \dfrac{\Delta q_i(s)}{E(s)} = K_p\left(1 + \dfrac{1}{T_i s}\right)$, $G_2(s) = \dfrac{\Delta h(s)}{\Delta q_i(s)} = \dfrac{K}{Ts+1}$.

The closed loop transfer function of this system is $\dfrac{\Delta h(s)}{\Delta h_i(s)} = \dfrac{b_1 s + b_0}{s^2 + a_1 s + a_0}$.

The block diagram representation of the system can be denoted by



where $a_0 = \dfrac{KK_p}{T_i T}$, $a_1 = \dfrac{KK_p + 1}{T}$, $b_0 = \dfrac{KK_p}{T_i T}$, $b_1 = \dfrac{KK_p}{T}$.

Then the state equation of the feedback system can be denoted by

$$\dot{X} = AX + B\Delta h_i, \quad \Delta h = CX,$$

where $X = (\omega, \dot{\omega})^{\mathrm{T}}$, ω is the output signal of the left block, $\dot{\omega}$ is a derivative of ω, and $(\omega, \dot{\omega})^{\mathrm{T}}$ denotes the transpose of $(\omega, \dot{\omega})$,

$$A = \begin{pmatrix} 0 & 1 \\ -\dfrac{KK_p}{TT_i} & -\dfrac{1+KK_p}{T} \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad C = \begin{pmatrix} \dfrac{KK_p}{TT_i} & \dfrac{KK_p}{T} \end{pmatrix}. \tag{1.1}$$

For this system with given structure, only when all physical parameters K_p, T_i, K, and T take values, A, B, and C are real number matrices and the system is real number system. So analysis results of this real number system (such as the controllability of $(A,\ B)$, observability of (A, C^{T}), reducibility of characteristic polynomial det $(\lambda I - A)$, and so on) are determined by two factors: physical system structure and parameter values. But what is the individual effect of system structure is not distinguishable.

To explore the effect of system structures, various parametrizations were appeared:

Paper [Lin, 1974] proposed a structured matrix (SM), whose entry is either constant zero or independent nonzero, that is, the nonzero entries are independent parameters. For example,

$$A = \begin{pmatrix} z_1 & 0 \\ z_2 & z_3 \end{pmatrix},$$

where the three nonzero entries z_1, z_2, and z_3 are independent parameters. Papers [Shields et al., 1976; Glover et al., 1976; Davison, 1977; Hosoe et al., 1979; Mayeda, 1981; Li et al., 1996] used SM investigating structural controllability of MIMO systems.

Papers [Corpmat et al., 1976; Anderson et al., 1982; Willems, 1986] introduced one-degree polynomial matrix, whose entries are one-degree polynomials in independent parameters. Such as

$$A = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} + z_1 \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} + z_2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} z_1 + z_2 + 1 & z_1 + 1 \\ z_1 & z_2 \end{pmatrix}$$

is a one-degree polynomial matrix in the independent parameters z_1 and z_2.

A matrix is called a column-structured matrix (CSM) if the different entries in a column of the matrix contain the same parameter factor but the factors in different columns are independent of each other [Yamada et al., 1985]. For example,

$$A = \begin{pmatrix} 3z_1 & z_2 \\ 2z_1 & 4z_2 \end{pmatrix},$$

where z_1 and z_2 are parameter factors in the first column and second column, respectively, and they are independent.

A matrix of the form $M = Q + T$ is said to be a mixed matrix if the nonzero entries of T are algebraically independent over the field K to which the entries of Q belong [Murota, 1987, 1989a, 1989b, 1993, 1998]. Take the following matrix as example,

$$A = Q + T = \begin{pmatrix} 0 & \sqrt{2} \\ 3 & 0 \end{pmatrix} + \begin{pmatrix} z_1 & 0 \\ 0 & z_2 \end{pmatrix},$$

where Q is a matrix over the real field and the nonzero entries z_1 and z_2 in T are algebraic independent over the real field, that is, the real field does not contain the two members. A is called a mixed matrix, where K is the real field.

Clearly, the inverse matrix of the full rank square SM, CSM, one-degree polynomial matrix, or mixed matrix is generally not SM, CSM, one-degree polynomial matrix, or mixed matrix. To overcome this problem, papers [Lu et al., 1991, 1994] introduced rational function matrix in multi-parameters (RFM) to describe the coefficient matrices of systems and networks and based the description of systems and networks on RFM to research their structural properties.

Let $z_1, \ldots, z_q$ denote q independent parameters (can also be variables or indeterminates), not constants or numerical values. Let $z = (z_1, \ldots, z_q) \in R^q$, R^q is the domain of definition for z, and it can also be called parameter space. Let $F(z)$ denote the field of all rational functions with real coefficients in q parameters $z_1, \ldots, z_q$, and $F(z)[\lambda]$ denote the ring of all $F(z)$-coefficient polynomials in λ . Such as

$$\frac{\sqrt{5}\, z_1 + 2z_2 + 3}{4z_1 + z_3} \in F(z),$$

where $\sqrt{5}$, 2, 3, 4, and 1 are real coefficients; because polynomial ring in z is a subset of $F(z)$, $\sqrt{5}\, z_1 + 2z_2 + 3 \in F(z)$; because the real field is a sub-field of $F(z)$, any real number is a member of $F(z)$. It is clear that

$$(5z_1 + z_2^3)\lambda^2 + \sqrt{6}\, z_6\lambda + \frac{8z_3^2 + 1}{3z_4z_5 + z_2z_6 + \sqrt{2}} \in F(z)[\lambda].$$

Definition 1.1 If any entry of matrix M is a member of $F(z)$ (i.e., rational function in $z_1, \ldots, z_q$), then matrix M is called a rational function matrix (RFM) in z or a matrix over $F(z)$; if the coefficient matrices of a system (including network) are considered to be RFMs, the system is called the rational function system in multi-parameters, simply called rational function system (RFS), or system over $F(z)$.

Obviously, the inverse matrix of a full rank square RFM is also an RFM.

Papers [Lin, 1974; Shields et al., 1976; Glover et al., 1976; Davison, 1977; Hosoe et al., 1979; Mayeda, 1981; Li et al., 1996; Corpmat et al., 1976; Anderson et al., 1982; Willems, 1986; Yamada et al., 1985; Murota, 1987, 1989a, 1989b, 1993, 1998] are of mathematical significance. However usually these matrices defined in the papers can not describe the physical systems completely, so they are not directly used to explore the structural properties of physical systems. Take Fig. 1.1 as illustration. The independent parameters of this system should be the 4 physical parameters K_p, T_i, K, and T. From Eq. (1.1) and the above definitions we know that matrix over the real field, SM, one-degree polynomial matrix in independent parameters, CSM and mixed matrix can not describe the matrices A, B, and C completely, but all these three matrices are RFMs and the system (A, B, C) is RFS, where $z = (K_p, T_i, K, T)$. Why RFM can describe the structure of physical systems is that the conception of RFM is more general, and matrix over the real field, SM, one-degree polynomial matrix, CSM and mixed matrix can be treated as special RFMs. So the research work of RFS has both mathematical and physical significance.

It should be emphasized that the conclusions over $F(z)$ is determined by structures of systems and networks only, not the values of z, because parameters are not evaluated with regard to conclusions over $F(z)$, which eliminates the value effect and only leaves the structure effect. Let us see the following conclusions over $F(z)$:

1) If an RLC network over $F(z)$ without sources has no all-capacitor cut-set, or all-inductor loop and is unhinged[①] (this is structural condition), then the characteristic polynomial $\det(\lambda I - A)$ of the network is an irreducible polynomial over $F(z)[\lambda]$ (without calculating characteristic polynomial, and only observing the structure of this network) [Lu et al., 1998].

2) If an unhinged RLC network with sources over $F(z)$ has no all-voltage source and capacitor loop, or all-current source and inductor cut-set, and if its network without sources (let voltage sources be short circuited and current sources be open circuited) is unhinged and has no all-capacitor cut-set, or all-inductor loop, then this network with sources is controllable over $F(z)$ [Lu, 2003].

3) If an RLC network without sources over $F(z)$ is unhinged and neither all-capacitor network nor all-inductor network, then the network is observable over $F(z)$ when any network variables (node voltages and/or branch currents) are outputs [Lu et al., 2005b]; where all resistors, capacitors and inductors of the network are treated as independent parameters z.

---

① An RLC network is said to be separable (or hinged) if the network has at least one sub-network which has at most one node in common with its complement sub-network, otherwise it is an unhinged network.

For example, consider the network, as shown in Fig. 1.2, where $z = (C, L, R_1, \ldots, R_4)$. Clearly, its network without sources (let the voltage source be short circuited) is unhinged and has no all-capacitor cut-set, or all-inductor loop. So from conclusion 1) we know that the characteristic polynomial $\det(\lambda I - A)$ of this network is an irreducible polynomial in ring $F(z)[\lambda]$. From Fig. 1.2, we know that there is a source in this unhinged network and no all-voltage source and capacitor loop, or all-current source and inductor cut-set; its network without sources is unhinged and has no all-capacitor cut-set, or all-inductor loop. So from conclusion 2), this network is controllable over $F(z)$. If take the node voltage u_{R1} of resistor R_1 as output in its network without sources, then for the network without sources is unhinged and is not all-capacitor or all-inductor network, the network is observable over $F(z)$ from conclusion 3). Application of these conclusions only needs to observe network structure. So conclusions over $F(z)$ are only determined by structure, and the properties of systems over $F(z)$ or simply called properties over $F(z)$ (such as controllability and observability over $F(z)$) are equivalent to structural properties (such as structural controllability and observability). The description of systems over $F(z)$ is a useful tool to explore structural properties.

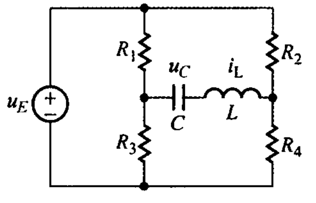


Fig. 1.2 An RLC network

We make some primary investigation to structural properties of RFSs using this tool: Research on the reducibility for a class of square RFM and its characteristic polynomial; controllability and observability over $F(z)$; separability, reducibility, controllability, and observability of networks over $F(z)$. Some obtained conclusions will be introduced in Chapters 2 – 5.

Now we will show the differences to the existing well known theories. Linear system theory can be divided into four parallel embranchments: state-space method, frequency domain method, geometric theory, and algebra theory according to mathematic tool and system description [Zheng, 2002].

The basic description of state-space (time domain) method is

$$\dot{x} = Ax + Bu, \quad y = Cx + Du,$$

where $x \in R^n, u \in R^m, y \in R^p$, A, B, C, and D are $n \times n, n \times m, p \times n$ and $p \times m$ real number matrices respectively. This is a linear time-invariable system, which is a real number system above (the term "real number system" in this book is to distinguish to RFS). In this book, A, B, C, and D are considered to be matrices over $F(z)$, and systems are treated to be RFSs, and structural properties depending on system structures not on parameter values are researched.

Frequency domain method is based on the state-space method. The transfer function matrix $C(sI - A)^{-1}B + D$ of the real number system is a matrix over the field $F(s)$ of rational functions in only one complex variable s, where $F(s)$ denotes the field of all rational functions with real coefficients in s, that is, the coefficients of s are real numbers. However although the entry of the transfer function matrix $C(sI - A)^{-1}B + D$ of RFS is rational function in s, the coefficients of s are members over $F(z)$, which are rational functions in $z_1, \ldots, z_q$ and not just real numbers — real numbers are a special situation. Or we can say that this transfer function matrix is a matrix over $F(z;\ s)$, where $F(z;\ s)$ denotes the field of all rational functions with real coefficients in $q+1$ independent parameters or indeterminates $z_1, \ldots, z_q, s$.

The algebra theory of linear systems (see Chapter 10 of [Kalman et al., 1969]) explores the linear systems over arbitrary and certain field K of numbers — generally it is the real field R, where the state vector $x \in K^n$, input vector $u \in K^m$, output vector $y \in K^p$, and the coefficient matrices F, G, and H (now customarily denoted by A, B, and C) are $n \times n, n \times m$ and $p \times n$ matrices respectively over K. Since K is the field of numbers, the description of linear systems over K is not easy to express and explore the structural properties of physical systems.

The geometric method of linear system [Wonham, 1979] explores linear space over the real field and the field of complex numbers, other than $F(z)$, so it is not easy to describe the structure of physical systems yet.

In this book, we utilize the generally used and effective method of the state-space and frequency domain, and try to extend the theory of linear time-invariable systems (real number systems) to RFSs. The significance of this is as follows:

First, description over $F(z)$ is a useful tool to explore the structural properties of systems and networks. Research of systems and networks over $F(z)$ is a work of mathematic and physical significance.

Furthermore, researching and analyzing properties of systems and networks over $F(z)$ (structural properties) has more practicable meaning than that over the real field. For instance, the controllability and observability of system are as shown in Fig. 1.1. When $K_p = 2, T_i = 1, K = 3, T = 1$, or denoted by $z = (K_p, T_i, K, T) = \overline{z} = (2, 1, 3, 1)$, substituting into Eq. (1.1) yields