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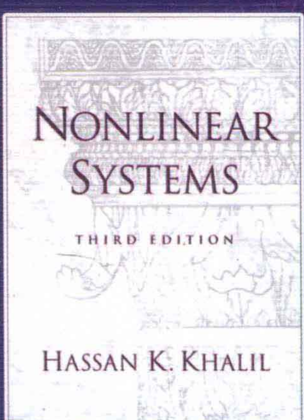
非线性系统（第三版）

Nonlinear Systems, Third Edition



英文版

［美］ Hassan K. Khalil 著

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内容简介

非线性系统的研究近年来受到越来越广泛的关注，国外许多工科院校已将“非线性系统”作为相关专业研究生的学位课程。本书是美国密歇根州立大学电气与计算机工程专业的研究生教材，全书内容按照数学知识的由浅入深分成了四个部分。基本分析部分介绍了非线性系统的基本概念和基本分析方法；反馈系统分析部分介绍了输入－输出稳定性、无源性和反馈系统的频域分析；现代分析部分介绍了现代稳定性分析的基本概念、扰动系统的稳定性、扰动理论和平均化以及奇异扰动理论；非线性反馈控制部分介绍了反馈线性化，并给出了几种非线性设计工具，如滑模控制、李雅普诺夫再设计、反步设计法、基于无源性的控制和高增益观测器等。此外本书附录还汇集了一些书中用到的数学知识，包括基本数学知识的复习、压缩映射和一些较为复杂的定理证明。本书已根据作者于2012年4月2日更新过的勘误表进行过更正。

本书既可以作为研究生第一学期非线性系统课程的教材，也可以作为工程技术人员、应用数学专业人员的自学教材或参考书。

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出 版 说 明

21世纪初的5至10年是我国国民经济和社会发展的重要时期，也是信息产业快速发展的关键时期。在我国加入WTO后的今天，培养一支适应国际化竞争的一流IT人才队伍是我国高等教育的重要任务之一。信息科学和技术方面人才的优劣与多寡，是我国面对国际竞争时成败的关键因素。

当前，正值我国高等教育特别是信息科学领域的教育调整、变革的重大时期，为使我国教育体制与国际化接轨，有条件的高等院校正在为某些信息学科和技术课程使用国外优秀教材和优秀原版教材，以使我国在计算机教学上尽快赶上国际先进水平。

电子工业出版社秉承多年来引进国外优秀图书的经验，翻译出版了“国外计算机科学教材系列”丛书，这套教材覆盖学科范围广、领域宽、层次多，既有本科专业课程教材，也有研究生课程教材，以适应不同院系、不同专业、不同层次的师生对教材的需求，广大师生可自由选择和自由组合使用。这些教材涉及的学科方向包括网络与通信、操作系统、计算机组织与结构、算法与数据结构、数据库与信息处理、编程语言、图形图像与多媒体、软件工程等。同时，我们也适当引进了一些优秀英文原版教材，本着翻译版本和英文原版并重的原则，对重点图书既提供英文原版又提供相应的翻译版本。

在图书选题上，我们大都选择国外著名出版公司出版的高校教材，如Pearson Education培生教育出版集团、麦格劳—希尔教育出版集团、麻省理工学院出版社、剑桥大学出版社等。撰写教材的许多作者都是蜚声世界的教授、学者，如道格拉斯·科默(Douglas E. Comer)、威廉·斯托林斯(William Stallings)、哈维·戴特尔(Harvey M. Deitel)、尤利斯·布莱克(Uyless Black)等。

为确保教材的选题质量和翻译质量，我们约请了清华大学、北京大学、北京航空航天大学、复旦大学、上海交通大学、南京大学、浙江大学、哈尔滨工业大学、华中科技大学、西安交通大学、国防科学技术大学、解放军理工大学等著名高校的教授和骨干教师参与了本系列教材的选题、翻译和审校工作。他们中既有讲授同类教材的骨干教师、博士，也有积累了几十年教学经验的老教授和博士生导师。

在该系列教材的选题、翻译和编辑加工过程中，为提高教材质量，我们做了大量细致的工作，包括对所选教材进行全面论证；选择编辑时力求达到专业对口；对排版、印制质量进行严格把关。对于英文教材中出现的错误，我们通过与作者联络和网上下载勘误表等方式，逐一进行了修订。

此外，我们还将与国外著名出版公司合作，提供一些教材的教学支持资料，希望能为授课老师提供帮助。今后，我们将继续加强与各高校教师的密切联系，为广大师生引进更多的国外优秀教材和参考书，为我国计算机科学教学体系与国际教学体系的接轨做出努力。

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Preface

This text is intended for a first-year graduate-level course on nonlinear systems or control. It may also be used for self study or reference by engineers and applied mathematicians. It is an outgrowth of my experience teaching the nonlinear systems course at Michigan State University, East Lansing. Students taking this course have had background in electrical engineering, mechanical engineering, or applied mathematics. The prerequisite for the course is a graduate-level course in linear systems, taught at the level of the texts by Antsaklis and Michel [9], Chen [35], Kailath [94], or Rugh [158]. The linear systems prerequisite allowed me not to worry about introducing the concept of "state" and to refer freely to "transfer functions," "state transition matrices," and other linear system concepts. The mathematical background is the usual level of calculus, differential equations, and matrix theory that any graduate student in engineering or mathematics would have. In the Appendix, I have collected a few mathematical facts that are used throughout the book.

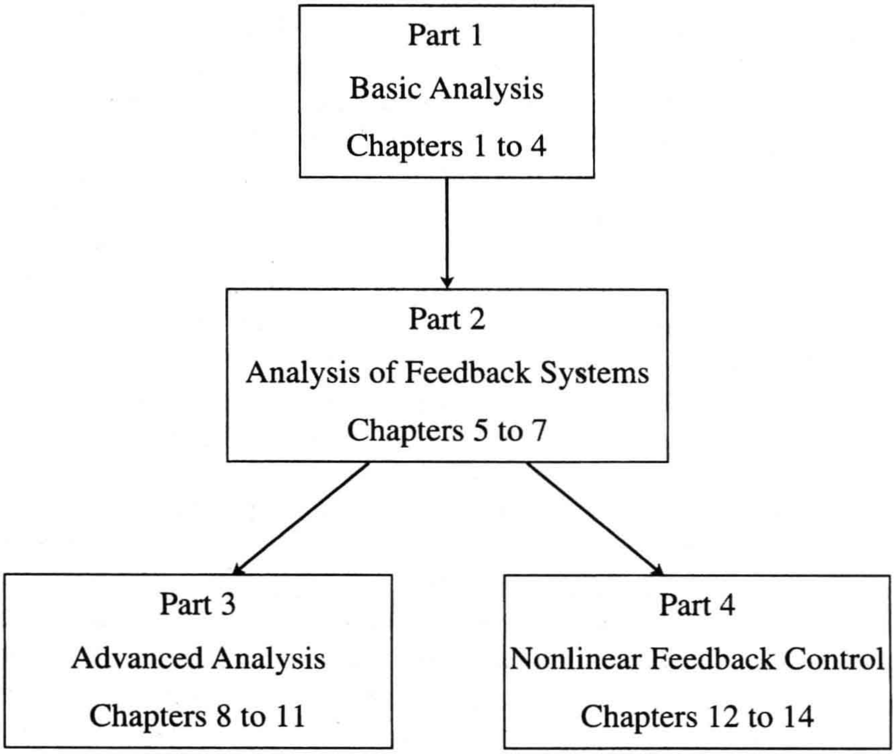
I have written the text in such a way that the level of mathematical sophistication increases as we advance from chapter to chapter. This is why the second chapter is written in an elementary context. Actually, this chapter could be taught at senior, or even junior, level courses without difficulty. This is also the reason I have split the treatment of Lyapunov stability into two parts. In Sections 4.1 through 4.3, I introduce the essence of Lyapunov stability for autonomous systems where I do not have to worry about technicalities such as uniformity, class $\mathcal{K}$ functions, etc. In Sections 4.4 through 4.6, I present Lyapunov stability in a more general setup that accommodates nonautonomous systems and allows for a deeper look into advanced aspects of the stability theory. The level of mathematical sophistication at the end of Chapter 4 is the level to which I like to bring the students, so that they can comfortably read the rest of the text.

There is yet a higher level of mathematical sophistication that is assumed in writing the proofs in the Appendix. These proofs are not intended for classroom use. They are included to make the text on one hand, self contained, and, on the other, to respond to the need or desire of some students to read such proofs, such as students continuing on to conduct Ph.D. research into nonlinear systems or control theory. Those students can continue to read the Appendix in a self-study manner.

This third edition has been written with the following goals in mind:

1. To make the book (especially the early chapters) more accessible to first-year graduate students. As an example of the changes made toward that end, note the change in Chapter 3: All the material on mathematical background, the contraction mapping theorem, and the proof of the existence and uniqueness theorem have been moved to the Appendix. Several parts of the book have been rewritten to improve readability.

2. To reorganize the book in such a way that makes it easier to structure nonlinear systems or control courses around it. In the new organization, the book has four parts, as shown in the flow chart. A course on nonlinear systems analysis will cover material from Parts 1, 2, and 3, while a course on nonlinear control will cover material from Parts 1, 2, and 4.



3. To update the material of the book to include topics or results that have proven to be useful in nonlinear control design in recent years. New to the third addition are the: expanded treatment of passivity and passivity-based control, integral control, sliding mode control, and high-gain observers. Moreover, bifurcation is introduced in the context of second-order systems. On the technical side, the reader will find Kurzweil's converse Lyapunov theorem, nonlocal results in Chapters 10 and 11, and new results on integral control and gain scheduling.

4. To update the exercises. More than 170 new exercises have been included.

I am indebted to many colleagues, students, and readers, who helped me in writing the book, through discussions, suggestions, corrections, constructive comments, and feedback on the first two editions. There are, probably, more than 100 names that I would like to acknowledge, but my fear of inadvertently omitting some names, leads to me settle for a big *thank you* to each one of you.

I am grateful to Michigan State University for providing an environment that allowed me to write this book, and to the National Science Foundation for supporting my research on nonlinear feedback control.

The book was typeset using LaTeX. All computations, including numerical solution of differential equations, were done using MATLAB and SIMULINK. The figures were generated using MATLAB or the graphics tool of LaTeX.

As much as I wish the book to be free of errors, I know this will not be the case. Therefore, reports of errors, sent electronically to

khalil@msu.edu

will be greatly appreciated. An up-to-date errata list will be available at the homepage of the book:

www.egr.msu.edu/~khalil/NonlinearSystems

The homepage also will contain a list of changes from the second edition, additional exercises, and other useful material.

HASSAN KHALIL
East Lansing, Michigan

Contents

Chapter 1

Introduction

When engineers analyze and design nonlinear dynamical systems in electrical circuits, mechanical systems, control systems, and other engineering disciplines, they need to absorb and digest a wide range of nonlinear analysis tools. In this book, we introduce some of the these tools. In particular, we present tools for the stability analysis of nonlinear systems, with emphasis on Lyapunov's method. We give special attention to the stability of feedback systems from input–output and passivity perspectives. We present tools for the detection and analysis of "free" oscillations, including the describing function method. We introduce the asymptotic tools of perturbation theory, including averaging and singular perturbations. Finally, we introduce nonlinear feedback control tools, including linearization, gain scheduling, integral control, feedback linearization, sliding mode control, Lyapunov redesign, backstepping, passivity-based control, and high-gain observers.

1.1 Nonlinear Models and Nonlinear Phenomena

We will deal with dynamical systems that are modeled by a finite number of coupled first-order ordinary differential equations

$$
\begin{aligned}
\dot{x}_1 &= f_1(t, x_1, \ldots, x_n, u_1, \ldots, u_p) \\
\dot{x}_2 &= f_2(t, x_1, \ldots, x_n, u_1, \ldots, u_p) \\
\vdots \quad & \quad \vdots \\
\dot{x}_n &= f_n(t, x_1, \ldots, x_n, u_1, \ldots, u_p)
\end{aligned}
$$

where $\dot{x}_i$ denotes the derivative of x_i with respect to the time variable t and u_1, u_2, …, u_p are specified input variables. We call the variables x_1, x_2, …, x_n the state variables. They represent the memory that the dynamical system has of its past.

We usually use vector notation to write these equations in a compact form. Define

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ \vdots \\ x_n \end{bmatrix}, \quad u = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_p \end{bmatrix}, \quad f(t,x,u) = \begin{bmatrix} f_1(t,x,u) \\ f_2(t,x,u) \\ \vdots \\ \vdots \\ f_n(t,x,u) \end{bmatrix}$$

and rewrite the n first-order differential equations as one n-dimensional first-order vector differential equation

$$\dot{x} = f(t,x,u) \tag{1.1}$$

We call (1.1) the state equation and refer to x as the *state* and u as the *input*. Sometimes, another equation

$$y = h(t,x,u) \tag{1.2}$$

is associated with (1.1), thereby defining a q-dimensional *output* vector y that comprises variables of particular interest in the analysis of the dynamical system, (e.g., variables that can be physically measured or variables that are required to behave in a specified manner). We call (1.2) the output equation and refer to equations (1.1) and (1.2) together as the state-space model, or simply the state model. Mathematical models of finite-dimensional physical systems do not always come in the form of a state model. However, more often than not, we can model physical systems in this form by carefully choosing the state variables. Examples and exercises that will appear later in the chapter will demonstrate the versatility of the state model.

A good part of our analysis in this book will deal with the state equation, many times without explicit presence of an input u, that is, the so-called unforced state equation

$$\dot{x} = f(t,x) \tag{1.3}$$

Working with an unforced state equation does not necessarily mean that the input to the system is zero. It could be that the input has been specified as a given function of time, $u = \gamma(t)$, a given feedback function of the state, $u = \gamma(x)$, or both, $u = \gamma(t,x)$. Substituting $u = \gamma$ in (1.1) eliminates u and yields an unforced state equation.

A special case of (1.3) arises when the function f does not depend explicitly on t; that is,

$$\dot{x} = f(x) \tag{1.4}$$

in which case the system is said to be *autonomous* or *time invariant*. The behavior of an autonomous system is invariant to shifts in the time origin, since changing the time variable from t to $\tau = t - a$ does not change the right-hand side of the state equation. If the system is not autonomous, then it is called *nonautonomous* or *time varying*.

An important concept in dealing with the state equation is the concept of an equilibrium point. A point $x = x^*$ in the state space is said to be an equilibrium point of (1.3) if it has the property that whenever the state of the system starts at x^*, it will remain at x^* for all future time. For the autonomous system (1.4), the equilibrium points are the real roots of the equation

$$f(x) = 0$$

An equilibrium point could be isolated; that is, there are no other equilibrium points in its vicinity, or there could be a continuum of equilibrium points.

For linear systems, the state model (1.1)–(1.2) takes the special form

$$\begin{aligned} \dot{x} &= A(t)x + B(t)u \\ y &= C(t)x + D(t)u \end{aligned}$$

We assume that the reader is familiar with the powerful analysis tools for linear systems, founded on the basis of the *superposition principle*. As we move from linear to nonlinear systems, we are faced with a more difficult situation. The superposition principle does not hold any longer, and analysis tools involve more advanced mathematics. Because of the powerful tools we know for linear systems, the first step in analyzing a nonlinear system is usually to linearize it, if possible, about some nominal operating point and analyze the resulting linear model. This is a common practice in engineering, and it is a useful one. There is no question that, whenever possible, we should make use of linearization to learn as much as we can about the behavior of a nonlinear system. However, linearization alone will not be sufficient; we must develop tools for the analysis of nonlinear systems. There are two basic limitations of linearization. First, since linearization is an approximation in the neighborhood of an operating point, it can only predict the "local" behavior of the nonlinear system in the vicinity of that point. It cannot predict the "nonlocal" behavior far from the operating point and certainly not the "global" behavior throughout the state space. Second, the dynamics of a nonlinear system are much richer than the dynamics of a linear system. There are "essentially nonlinear phenomena" that can take place only in the presence of nonlinearity; hence, they cannot be described or predicted by linear models. The following are examples of essentially nonlinear phenomena:

- *Finite escape time.* The state of an unstable linear system goes to infinity as time approaches infinity; a nonlinear system's state, however, can go to infinity in finite time.

- *Multiple isolated equilibria.* A linear system can have only one isolated equilibrium point; thus, it can have only one steady-state operating point that attracts the state of the system irrespective of the initial state. A nonlinear system can have more than one isolated equilibrium point. The state may converge to one of several steady-state operating points, depending on the initial state of the system.

- *Limit cycles.* For a linear time-invariant system to oscillate, it must have a pair of eigenvalues on the imaginary axis, which is a nonrobust condition that is almost impossible to maintain in the presence of perturbations. Even if we do, the amplitude of oscillation will be dependent on the initial state. In real life, stable oscillation must be produced by nonlinear systems. There are nonlinear systems that can go into an oscillation of fixed amplitude and frequency, irrespective of the initial state. This type of oscillation is known as a limit cycle.

- *Subharmonic, harmonic, or almost-periodic oscillations.* A stable linear system under a periodic input produces an output of the same frequency. A nonlinear system under periodic excitation can oscillate with frequencies that are submultiples or multiples of the input frequency. It may even generate an almost-periodic oscillation, an example is the sum of periodic oscillations with frequencies that are not multiples of each other.

- *Chaos.* A nonlinear system can have a more complicated steady-state behavior that is not equilibrium, periodic oscillation, or almost-periodic oscillation. Such behavior is usually referred to as chaos. Some of these chaotic motions exhibit randomness, despite the deterministic nature of the system.

- *Multiple modes of behavior.* It is not unusual for two or more modes of behavior to be exhibited by the same nonlinear system. An unforced system may have more than one limit cycle. A forced system with periodic excitation may exhibit harmonic, subharmonic, or more complicated steady-state behavior, depending upon the amplitude and frequency of the input. It may even exhibit a discontinuous jump in the mode of behavior as the amplitude or frequency of the excitation is smoothly changed.

In this book, we will encounter only the first three of these phenomena.[1] Multiple equilibria and limit cycles will be introduced in the next chapter, as we examine second-order autonomous systems, while the phenomenon of finite escape time will be introduced in Chapter 3.

[1]To read about forced oscillation, chaos, bifurcation, and other important topics, the reader may consult [70], [74], [187], and [207].