钱学森

力学手稿

4

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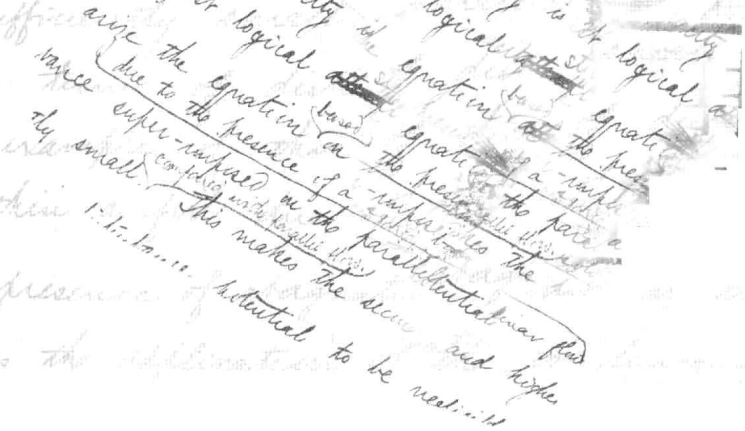

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出版前言

2011年12月11日是西安交通大学杰出校友钱学森先生的百年诞辰。为缅怀钱学森学长，学习他的科学思想和卓越风范，展示其丰功伟绩和人格魅力，西安交通大学举办了“纪念钱学森诞辰100周年”系列活动：作为制片方之一，参与西部电影集团摄制传记故事片《钱学森》；与中央电视台合作，出品纪录片《实验班的故事——沿着钱学森走过的路》；扩建钱学森生平业绩展馆，向校内外开放；举办钱学森科学与教育思想研讨会；出版发行《钱学森力学手稿》、《钱学森年谱（初编）》、《钱学森第六次产业革命思想探微丛书》等。

钱学森先生在美国深造和工作期间留下大量珍贵手稿，这些手稿真实展示了钱学森先生博大精深的学识、开拓求实的精神和严谨奋进的作风，是钱老勇攀科学高峰和严谨治学的集中体现。这里，我们将部分原稿整理汇集成册，出版《钱学森力学手稿》，作为钱老百年诞辰的献礼。

《钱学森力学手稿》共10卷，包含两部分内容。第一部分是草稿，包括扁壳、球壳和圆柱壳屈曲分析的公式推导和数值演算。在研究圆柱壳轴压屈曲问题时，为了求得圆柱壳体的临界压力，在有关的五百多页草稿中，对多达二十多种可能的屈曲模

态逐一进行公式推演和数值计算，最终才找到满意的并在论文中采用的屈曲模态。仔细观察草稿中的数据列表，每个数字有效位数都长达八位，在手摇机械式计算机作为主要计算工具的年代，这串串数字凝聚着多少现今难以想象的艰辛劳动。

第二部分是手稿，以航空航天工程为核心，涵盖空气动力学、固体力学、火箭技术、工程控制论和物理力学等领域的部分学术论文手稿、打印稿和讲义。

《钱学森力学手稿》是在西安交通大学校领导的大力支持下，由西安交通大学航天航空学院沈亚鹏教授整理完成。图书出版过程中得到了西安交通大学党委宣传部、校友关系发展部、图书馆、航天航空学院等的积极协助，在此深表感谢。

Preliminary Calculation of Circular Cylinder (Ⅲ)

New Calculation

for the circular region,

$$\frac{w_0}{R} = \frac{1}{2}\left(\frac{a}{R}\right)^2\left\{1-\left(\frac{r}{a}\right)^2\sin^2\theta\right\}$$

$$\frac{w}{R} = \frac{1}{2}\left(\frac{a}{R}\right)^2\left\{1-\left(\frac{r}{a}\right)^2\sin^2\theta - f_1\left[1-\left(\frac{r}{a}\right)^2\right]^2 - f_2\left[1-\left(\frac{r}{a}\right)^2\right]^6\right\}$$

$$\frac{1}{r}\frac{\partial w}{\partial r} = \frac{1}{R}\left\{-\sin^2\theta + 2f_1\left[1-\left(\frac{r}{a}\right)^2\right] + 6f_2\left[1-\left(\frac{r}{a}\right)^2\right]^5\right\}$$

$$\frac{1}{r}\frac{\partial w_0}{\partial r} = \frac{1}{R}\left\{-\sin^2\theta\right\}$$

$$\frac{\partial^2 w}{\partial r^2} = \frac{1}{R}\left\{-\sin^2\theta + 2f_1\left[1-3\left(\frac{r}{a}\right)^2\right] + 6f_2\left[1-\left(\frac{r}{a}\right)^2\right]^4\left[1-11\left(\frac{r}{a}\right)^2\right]\right\}$$

$$\frac{\partial^2 w_0}{\partial r^2} = \frac{1}{R}\left\{-\sin^2\theta\right\}$$

$$\frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2} = \frac{1}{r^2}\frac{\partial^2 w_0}{\partial \theta^2} = \frac{1}{R}\left\{-\cos 2\theta\right\}$$

$$-\left\{\frac{1}{r}\frac{\partial w}{\partial r}\frac{\partial^2 w}{\partial r^2}-\frac{1}{r}\frac{\partial w_0}{\partial r}\frac{\partial^2 w_0}{\partial r^2}\right\}$$

$$=\frac{1}{R^2}\left[(1-\cos 2\theta)\left\{2f_1\left[1-2\left(\frac{r}{a}\right)^2\right]+6f_2\left[1-\left(\frac{r}{a}\right)^2\right]^4\left[1-6\left(\frac{r}{a}\right)^2\right]\right\}\right.$$

$$\left.-\left\{4f_1^2\left[1-\left(\frac{r}{a}\right)^2\right]\left[1-3\left(\frac{r}{a}\right)^2\right]+24f_1f_2\left[1-\left(\frac{r}{a}\right)^2\right]^5\left[1-7\left(\frac{r}{a}\right)^2\right]+36f_2^2\left[1-\left(\frac{r}{a}\right)^2\right]^9\left[1-11\left(\frac{r}{a}\right)^2\right]\right\}\right.$$

$$-\left\{\frac{1}{r^2}\frac{\partial^2 w}{\partial r^2}\frac{\partial^2 w}{\partial\theta^2}-\frac{1}{r^2}\frac{\partial^2 w_0}{\partial r^2}\frac{\partial^2 w_0}{\partial\theta^2}\right\}=\frac{1}{R^2}\cos 2\theta\left\{2f_1\left[1-3\left(\frac{r}{a}\right)^2\right]+6f_2\left[1-\left(\frac{r}{a}\right)^2\right]^4\left[1-11\left(\frac{r}{a}\right)^2\right]\right\}$$

The terms for the particular integral is then, <u>multiplied by R^2</u>

$$2f_1\left[1-2\left(\frac{r}{a}\right)^2\right]+6f_2\left[1-\left(\frac{r}{a}\right)^2\right]^4\left[1-6\left(\frac{r}{a}\right)^2\right]-4f_1^2\left[1-\left(\frac{r}{a}\right)^2\right]\left[1-3\left(\frac{r}{a}\right)^2\right]$$

$$-24f_1f_2\left[1-\left(\frac{r}{a}\right)^2\right]^5\left[1-7\left(\frac{r}{a}\right)^2\right]-36f_2^2\left[1-\left(\frac{r}{a}\right)^2\right]^9\left[1-11\left(\frac{r}{a}\right)^2\right]$$

$$+\cos 2\theta\left\{-2f_1\left(\frac{r}{a}\right)^2-30f_2\left[1-\left(\frac{r}{a}\right)^2\right]^4\left(\frac{r}{a}\right)^2\right\}$$

1	$(\frac{1}{a})^2$	$(\frac{1}{a})^4$	$(\frac{1}{a})^6$	$(\frac{1}{a})^8$	$(\frac{1}{a})^{10}$	$(\frac{1}{a})^{12}$	$(\frac{1}{a})^{14}$	$(\frac{1}{a})^{16}$	$(\frac{1}{a})^{18}$	$(\frac{1}{a})^{20}$
$+2f_1$	$-4f_1$									
$+6f_2$	$-24f_2$	$+36f_2$	$-24f_2$	$+6f_2$						
	$-36f_2$	$+144f_2$	$-216f_2$	$+144f_2$	$-36f_2$					
$-4f_1^2$	$+16f_1^2$	$-12f_1^2$								
$-48f_1f_2$	$+240f_1f_2$	$-480f_1f_2$	$+480f_1f_2$	$-240f_1f_2$	$+48f_1f_2$					
	$+336f_1f_2$	$-1680f_1f_2$	$+3360f_1f_2$	$-3360f_1f_2$	$+1680f_1f_2$	$-336f_1f_2$	X	$\frac{1}{2}$		
$-36f_2^2$	$+324f_2^2$	$-1296f_2^2$	$+3024f_2^2$	$-4536f_2^2$	$+4536f_2^2$	$-3024f_2^2$	$+1296f_2^2$	$-324f_2^2$	$+36f_2^2$	
	$+396f_2^2$	$-3564f_2^2$	$+14256f_2^2$	$-33264f_2^2$	$+49896f_2^2$	$-49896f_2^2$	$+33264f_2^2$	$-14256f_2^2$	$+3564f_2^2$	$-396f_2^2$
$\frac{1}{16\cdot4}$	$\frac{1}{36\cdot16}$	$\frac{1}{64\cdot36}$	$\frac{1}{100\cdot64}$	$\frac{1}{144\cdot100}$	$\frac{1}{196\cdot144}$	$\frac{1}{256\cdot196}$	$\frac{1}{324\cdot256}$	$\frac{1}{400\cdot324}$	$\frac{1}{484\cdot400}$	$\frac{1}{576\cdot484}$
	$-2f_1$									
	$-30f_2$	$+120f_2$	$-180f_2$	$+120f_2$	$-30f_2$					
	$\frac{1}{32\cdot12}$	$\frac{1}{60\cdot32}$	$\frac{1}{96\cdot60}$	$\frac{1}{140\cdot96}$	$\frac{1}{192\cdot140}$					

434

$$\frac{\Phi}{R^2} = E\left(\frac{a}{R}\right)^4\Big[\frac{1}{4}f_0\left(\frac{\lambda}{a}\right)^2 + \frac{1}{32}\left(f_1+3f_2-2f_1^2-12f_1f_2-18f_2^2\right)\left(\frac{\lambda}{a}\right)^4 + \frac{1}{144}\left(-f_1-15f_2+4f_1^2+72f_1f_2+180f_2^2\right)\left(\frac{\lambda}{a}\right)^6$$

$$+\frac{1}{192}\left(15f_2-f_1^2-90f_1f_2-405f_2^2\right)\left(\frac{\lambda}{a}\right)^8 + \frac{1}{400}\left(-15f_2+120f_1f_2+1080f_2^2\right)\left(\frac{\lambda}{a}\right)^{10}$$

$$+\frac{1}{96}\left(f_2-12f_1f_2-1252f_2^2\right)\left(\frac{\lambda}{a}\right)^{12} + \frac{1}{384}\left(-f_2+24f_1f_2+1512f_2^2\right)\left(\frac{\lambda}{a}\right)^{14}$$

$$+\frac{1}{896}\left(-3f_1f_2-945f_2^2\right)\left(\frac{\lambda}{a}\right)^{16} + \frac{5}{12}f_2^2\left(\frac{\lambda}{a}\right)^{18} - \frac{9}{80}f_2^2\left(\frac{\lambda}{a}\right)^{20} + \frac{9}{484}f_2^2\left(\frac{\lambda}{a}\right)^{22} - \frac{1}{104}f_2^2\left(\frac{\lambda}{a}\right)^{24}$$

$$+\cos 2\theta\Big\{-\frac{1}{192}\left(f_1+15f_2\right)\left(\frac{\lambda}{a}\right)^6 + \frac{1}{16}f_2\left(\frac{\lambda}{a}\right)^8 - \frac{1}{32}f_2\left(\frac{\lambda}{a}\right)^{10} + \frac{1}{112}f_2\left(\frac{\lambda}{a}\right)^{12} - \frac{1}{896}f_2\left(\frac{\lambda}{a}\right)^{14} + h_2\left(\frac{\lambda}{a}\right)^2 + h_2\left(\frac{\lambda}{a}\right)^4\Big\}\Big]$$

$$\frac{1}{\lambda}\frac{\partial\Phi}{\partial\lambda} = E\left(\frac{a}{R}\right)^2\Big[\frac{1}{2}f_0 + \frac{1}{8}A\left(\frac{\lambda}{a}\right)^2 + \frac{1}{24}B\left(\frac{\lambda}{a}\right)^4 + \frac{1}{24}C\left(\frac{\lambda}{a}\right)^6 + \frac{1}{40}D\left(\frac{\lambda}{a}\right)^8 + \frac{1}{8}E\left(\frac{\lambda}{a}\right)^{10} + \frac{1}{56}F\left(\frac{\lambda}{a}\right)^{12}$$

$$+\frac{1}{56}G\left(\frac{\lambda}{a}\right)^{14} + \frac{15}{2}H\left(\frac{\lambda}{a}\right)^{16} - \frac{9}{4}I\left(\frac{\lambda}{a}\right)^{18} + \frac{9}{22}J\left(\frac{\lambda}{a}\right)^{20} - \frac{3}{88}K\left(\frac{\lambda}{a}\right)^{22}$$

$$+\cos 2\theta\Big\{-\frac{6}{192}\left(f_1+15f_2\right)\left(\frac{\lambda}{a}\right)^4 + \frac{2}{16}f_2\left(\frac{\lambda}{a}\right)^6 - \frac{10}{32}f_2\left(\frac{\lambda}{a}\right)^8 + \frac{12}{112}f_2\left(\frac{\lambda}{a}\right)^{10} - \frac{14}{896}f_2\left(\frac{\lambda}{a}\right)^{12}$$

$$+2f_2+4h_2\left(\frac{\lambda}{a}\right)^2\Big\}\Big]$$

$$\frac{1}{r^2}\frac{\partial^2\Phi}{\partial\theta^2} = E\left(\frac{a}{R}\right)^2 \cos 2\theta \left\{ \frac{4}{192}(f_1+15f_2)\left(\frac{r}{a}\right)^4 - \frac{4}{16}f_2\left(\frac{r}{a}\right)^6 + \frac{4}{32}f_2\left(\frac{r}{a}\right)^8 - \frac{4}{112}f_2\left(\frac{r}{a}\right)^{10} + \frac{4}{896}f_2\left(\frac{r}{a}\right)^{12} - 4p_2 - 4h_2\left(\frac{r}{a}\right)^2 \right\}$$

$$\widehat{rr} = E\left(\frac{a}{R}\right)^2 \left[\frac{1}{2}\gamma_0 + \frac{1}{8}A\left(\frac{r}{a}\right)^2 + \frac{1}{24}B\left(\frac{r}{a}\right)^4 + \frac{1}{24}C\left(\frac{r}{a}\right)^6 + \frac{1}{40}D\left(\frac{r}{a}\right)^8 + \frac{1}{8}E\left(\frac{r}{a}\right)^{10} + \frac{1}{56}F\left(\frac{r}{a}\right)^{12} + \frac{1}{56}G\left(\frac{r}{a}\right)^{14} + \frac{15}{2}H\left(\frac{r}{a}\right)^{16} - \frac{9}{4}I\left(\frac{r}{a}\right)^{18} + \frac{9}{22}J\left(\frac{r}{a}\right)^{20} - \frac{3}{88}K\left(\frac{r}{a}\right)^{22} + \cos 2\theta \left\{ -\frac{2}{192}(f_1+15f_2)\left(\frac{r}{a}\right)^4 + \frac{4}{16}f_2\left(\frac{r}{a}\right)^6 - \frac{6}{32}f_2\left(\frac{r}{a}\right)^8 + \frac{8}{112}f_2\left(\frac{r}{a}\right)^{10} - \frac{10}{896}f_2\left(\frac{r}{a}\right)^{12} - 2p_2 \right\} \right]$$

$$\widehat{\theta\theta} = E\left(\frac{a}{R}\right)^2 \left[\frac{1}{2}\gamma_0 + \frac{3}{8}A\left(\frac{r}{a}\right)^2 + \frac{5}{24}B\left(\frac{r}{a}\right)^4 + \frac{7}{24}C\left(\frac{r}{a}\right)^6 + \frac{9}{40}D\left(\frac{r}{a}\right)^8 + \frac{11}{8}E\left(\frac{r}{a}\right)^{10} + \frac{13}{56}F\left(\frac{r}{a}\right)^{12} + \frac{15}{56}G\left(\frac{r}{a}\right)^{14} + \frac{15\times 17}{2}H\left(\frac{r}{a}\right)^{16} - \frac{9\times 19}{4}I\left(\frac{r}{a}\right)^{18} + \frac{9\times 21}{22}J\left(\frac{r}{a}\right)^{20} - \frac{3\times 23}{88}K\left(\frac{r}{a}\right)^{22} + \cos 2\theta \left\{ -\frac{30}{192}(f_1+15f_2)\left(\frac{r}{a}\right)^4 + \frac{56}{16}f_2\left(\frac{r}{a}\right)^6 - \frac{90}{32}f_2\left(\frac{r}{a}\right)^8 + \frac{132}{112}f_2\left(\frac{r}{a}\right)^{10} - \frac{168}{896}f_2\left(\frac{r}{a}\right)^{12} + 2p_2 + 12h_2\left(\frac{r}{a}\right)^2 \right\} \right]$$

$$\widehat{r\theta} = E\left(\frac{a}{R}\right)^2 \sin 2\theta \left\{ -\frac{10}{192}(f_1+15f_2)\left(\frac{r}{a}\right)^4 + \frac{14}{16}f_2\left(\frac{r}{a}\right)^6 - \frac{18}{32}f_2\left(\frac{r}{a}\right)^8 + \frac{22}{112}f_2\left(\frac{r}{a}\right)^{10} - \frac{26}{896}f_2\left(\frac{r}{a}\right)^{12} + 2p_2 + 6h_2\left(\frac{r}{a}\right)^2 \right\}$$

$$\hat{w} - \hat{w_0} = E\left(\frac{a}{R}\right)^2\left[\frac{1}{2}(1-\nu)g_0 + \frac{(1-3\nu)}{8}A\left(\frac{r}{a}\right)^2 + \frac{(1-5\nu)}{24}B\left(\frac{r}{a}\right)^4 + \frac{(1-7\nu)}{24}C\left(\frac{r}{a}\right)^6 + \frac{(1-9\nu)}{40}D\left(\frac{r}{a}\right)^8 + \frac{(1-11\nu)}{88}E\left(\frac{r}{a}\right)^{10}\right.$$
$$+ \frac{(1-13\nu)}{52}F\left(\frac{r}{a}\right)^{12}$$
$$\left. + \frac{(1-15\nu)}{56}G\left(\frac{r}{a}\right)^{14} + \frac{15(1-17\nu)}{2}H\left(\frac{r}{a}\right)^{16} - \frac{9(1-19\nu)}{4}I\left(\frac{r}{a}\right)^{18} + \frac{9(1-21\nu)}{22}J\left(\frac{r}{a}\right)^{20} - \frac{3(1-23\nu)}{88}K\left(\frac{r}{a}\right)^{22}\right.$$
$$+ \cos 2\theta\left\{ - \frac{(1-15\nu)}{96}(f_1 + 15f_2)\left(\frac{r}{a}\right)^4 + \frac{(1-14\nu)}{4}f_2\left(\frac{r}{a}\right)^6 - \frac{(3-45\nu)}{16}f_2\left(\frac{r}{a}\right)^8 + \frac{(2-33\nu)}{28}f_2\left(\frac{r}{a}\right)^{10} - \frac{(5-91\nu)}{448}f_2\left(\frac{r}{a}\right)^{12}\right.$$
$$\left.\left. - 2(1+\nu)p_2 - 12\nu l_2\left(\frac{r}{a}\right)^2\right\}\right]$$

$$\frac{1}{2}\left(\frac{\partial w}{\partial r}\right)^2 - \frac{1}{2}\left(\frac{\partial w_0}{\partial r}\right)^2 = \left(\frac{a}{R}\right)^2\left[-f_1\left[1-\left(\frac{r}{a}\right)^2\right]\left(\frac{r}{a}\right)^2 - 3f_2\left[1-\left(\frac{r}{a}\right)^2\right]^2\left(\frac{r}{a}\right)^2 + 2f_1^2\left[1-\left(\frac{r}{a}\right)^2\right]\left(\frac{r}{a}\right)^2 + 12f_1f_2\left[1-\left(\frac{r}{a}\right)^2\right]^2\left(\frac{r}{a}\right)^2\right.$$
$$\left. + 18f_2^2\left[1-\left(\frac{r}{a}\right)^2\right]^{10}\left(\frac{r}{a}\right)^2 + \cos 2\theta\left\{f_1\left[1-\left(\frac{r}{a}\right)^2\right]\left(\frac{r}{a}\right)^2 + 3f_2\left[1-\left(\frac{r}{a}\right)^2\right]^2\left(\frac{r}{a}\right)^2\right\}\right]$$
$$= \left(\frac{a}{R}\right)^2\left[-A\left(\frac{r}{a}\right)^2 - B\left(\frac{r}{a}\right)^4 - 2C\left(\frac{r}{a}\right)^6 - 2D\left(\frac{r}{a}\right)^8 - 15E\left(\frac{r}{a}\right)^{10} - 3F\left(\frac{r}{a}\right)^{12} - 4G\left(\frac{r}{a}\right)^{14} - 2160H\left(\frac{r}{a}\right)^{16}\right.$$
$$+ 810I\left(\frac{r}{a}\right)^{18} - 180J\left(\frac{r}{a}\right)^{20} + 18K\left(\frac{r}{a}\right)^{22}$$
$$\left. + \cos 2\theta\left\{(f_1+3f_2)\left(\frac{r}{a}\right)^2 - (f_1+15f_2)\left(\frac{r}{a}\right)^4 + 30f_2\left(\frac{r}{a}\right)^6 - 30f_2\left(\frac{r}{a}\right)^8 + 15f_2\left(\frac{r}{a}\right)^{10} - 3f_2\left(\frac{r}{a}\right)^{12}\right\}\right]$$

$(\frac{1}{a})^2$	$(\frac{1}{a})^4$	$(\frac{1}{a})^6$	$(\frac{1}{a})^8$	$(\frac{1}{a})^{10}$	$(\frac{1}{a})^{12}$	$(\frac{1}{a})^{14}$	$(\frac{1}{a})^{16}$	$(\frac{1}{a})^{18}$	$(\frac{1}{a})^{20}$	$(\frac{1}{a})^{22}$
$-f_1$	$+f_1$									
$-3f_2$	$+15f_2$	$-30f_2$	$+30f_2$	$-15f_2$	$+3f_2$					
$+2f_1^2$	$-4f_1^2$	$+2f_1^2$								
$+12f_1f_2$	$-72f_1f_2$	$+180f_1f_2$	$-240f_1f_2$	$+180f_1f_2$	$-72f_1f_2$	$+12f_1f_2$				
$+18f_2^2$	$-180f_2^2$	$+810f_2^2$	$-2160f_2^2$	$+3780f_2^2$	$-4536f_1f_2$	$+3780f_2^2$	$-2160f_2^2$	$+810f_2^2$	$-180f_2^2$	$+18f_2^2$
$+f_1$	$-f_1$									
$+3f_2$	$-15f_2$	$+30f_2$	$-30f_2$	$+15f_2$	$-3f_2$					

$$\frac{\partial u}{\partial r} = \left(\frac{a}{R}\right)^2\left[\frac{1}{2}(1-\nu)g_0 + \frac{3(3-\nu)}{8}A\left(\frac{r}{a}\right)^2 + \frac{5(5-\nu)}{24}B\left(\frac{r}{a}\right)^4 + \frac{7(7-\nu)}{24}C\left(\frac{r}{a}\right)^6 + \frac{9(9-\nu)}{40}D\left(\frac{r}{a}\right)^8\right.$$

$$+ \frac{11(11-\nu)}{8}E\left(\frac{r}{a}\right)^{10} + \frac{13(13-\nu)}{56}F\left(\frac{r}{a}\right)^{12} + \frac{15(15-\nu)}{56}G\left(\frac{r}{a}\right)^{14} + \frac{15\times17(17-\nu)}{2}H\left(\frac{r}{a}\right)^{16} - \frac{9\times19(19-\nu)}{4}\left.\right]\left(\frac{r}{a}\right)^{18}$$

$$+ \frac{9\times21(21-\nu)}{22}J\left(\frac{r}{a}\right)^{20} - \frac{3\times23(23-\nu)}{88}K\left(\frac{r}{a}\right)^{22}$$

$$+\cos 2\theta\left\{-(f_1+3f_2)\left(\frac{r}{a}\right)^2 + \frac{195+15\nu}{96}(f_1+15f_2)\left(\frac{r}{a}\right)^4 - \frac{119+14\nu}{4}f_2\left(\frac{r}{a}\right)^6 + \frac{477+45\nu}{16}f_2\left(\frac{r}{a}\right)^8\right.$$

$$\left.- \frac{(418+33\nu)}{28}f_2\left(\frac{r}{a}\right)^{10} + \frac{(1339+9\nu)}{448}f_2\left(\frac{r}{a}\right)^{12} - 3(1+\nu)f_2 - 12\nu h_2\left(\frac{r}{a}\right)^2\right\}\Bigg]$$

$$\frac{u}{R} = \left(\frac{a}{R}\right)^3\left[\frac{1}{2}(1-\nu)g_0\left(\frac{r}{a}\right) + \frac{(3-\nu)}{8}A\left(\frac{r}{a}\right)^3 + \frac{(5-\nu)}{24}B\left(\frac{r}{a}\right)^5 + \frac{(7-\nu)}{24}C\left(\frac{r}{a}\right)^7 + \frac{(9-\nu)}{40}D\left(\frac{r}{a}\right)^9\right.$$

$$+ \frac{(11-\nu)}{8}E\left(\frac{r}{a}\right)^{11} + \frac{(13-\nu)}{56}F\left(\frac{r}{a}\right)^{13} + \frac{(15-\nu)}{56}G\left(\frac{r}{a}\right)^{15} + \frac{15(17-\nu)}{2}H\left(\frac{r}{a}\right)^{17} - \frac{9(19-\nu)}{4}\left.\right]\left(\frac{r}{a}\right)^{19}$$

$$+ \frac{9(21-\nu)}{22}J\left(\frac{r}{a}\right)^{21} - \frac{3(23-\nu)}{88}K\left(\frac{r}{a}\right)^{23}$$

$$+ \cos 3\theta\left\{-\frac{1}{3}(f_1+3f_2)\left(\frac{r}{a}\right)^3 + \frac{(19+3\nu)}{96}(f_1+15f_2)\left(\frac{r}{a}\right)^5 - \frac{(17+3\nu)}{4}f_2\left(\frac{r}{a}\right)^7 + \frac{(53+5\nu)}{16}f_2\left(\frac{r}{a}\right)^9\right.$$

$$\left.- \frac{(38+3\nu)}{28}f_2\left(\frac{r}{a}\right)^{11} + \frac{(103+7\nu)}{448}f_2\left(\frac{r}{a}\right)^{13} - 3(1+\nu)f_2\left(\frac{r}{a}\right) - 4\nu h_2\left(\frac{r}{a}\right)^3\right\}\Bigg]$$

The non-uniform part of $\delta\theta - \nu\hat{R}$ is

$$E\left(\frac{a}{R}\right)^2 \cos 2\theta \left\{ -\frac{(15-\nu)}{96}(f_1+15f_2)\left(\frac{r}{a}\right)^4 + \frac{(14-\nu)}{4}f_2\left(\frac{r}{a}\right)^6 - \frac{(45-3\nu)}{16}f_2\left(\frac{r}{a}\right)^8 + \frac{(33-2\nu)}{28}f_2\left(\frac{r}{a}\right)^{10} - \frac{(91-5\nu)}{488}f_2\left(\frac{r}{a}\right)^{12} + 2(1+\nu)f_2 + 12f_2\left(\frac{r}{a}\right)^2 \right\}$$

$$\frac{1}{2}\frac{\partial v}{\partial\theta} = \left(\frac{a}{R}\right)^2 \cos 2\theta \left\{ \frac{1}{3}(f_1+3f_2)\left(\frac{r}{a}\right)^2 - \frac{(34+2\nu)}{96}(f_1+15f_2)\left(\frac{r}{a}\right)^4 + \frac{(31+\nu)}{4}f_2\left(\frac{r}{a}\right)^6 - \frac{(98+2\nu)}{16}f_2\left(\frac{r}{a}\right)^8 + \frac{(71+\nu)}{24}f_2\left(\frac{r}{a}\right)^{10} - \frac{(194+2\nu)}{448}f_2\left(\frac{r}{a}\right)^{12} + 4(1+\nu)f_2 + 4(3+\nu)\left(\frac{r}{a}\right)^2 \right\}$$

$$\frac{w}{R} = \left(\frac{a}{R}\right)^3 \sin 2\theta \left\{ \frac{1}{6}(f_1+3f_2)\left(\frac{r}{a}\right)^3 - \frac{(17+\nu)}{96}(f_1+15f_2)\left(\frac{r}{a}\right)^5 + \frac{(31+\nu)}{8}f_2\left(\frac{r}{a}\right)^7 - \frac{(49+\nu)}{16}f_2\left(\frac{r}{a}\right)^9 + \frac{(71+\nu)}{56}f_2\left(\frac{r}{a}\right)^{11} - \frac{(97+\nu)}{448}f_2\left(\frac{r}{a}\right)^{13} + 2(1+\nu)f_2 + 2(3+\nu)\left(\frac{r}{a}\right)^2 \right\}$$

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how the non-uniform parts

$$\widehat{nn}_a = \cos 2\theta \left\{ -\frac{1}{96} f_1 - \frac{15}{448} f_2 - 3 p_2 \right\} \quad \text{——} \quad \left\{ \frac{1}{2} - 6 q_2 - 4 s_2 \right\}$$

$$\widehat{n\theta}_a = \cos 2\theta \left\{ -\frac{5}{32} f_1 - \frac{305}{448} f_2 + 7 p_2 + 12 r_2 \right\} \quad \text{——} \quad \left\{ 6 q_2 - \frac{1}{2} \right\}$$

$$\widehat{n\theta}_a = \sin 2\theta \left\{ -\frac{5}{96} f_1 - \frac{135}{448} f_2 + 2 p_2 + 6 r_2 \right\} \quad \text{——} \quad \left\{ -\frac{1}{2} - 6 q_2 - 2 s_2 \right\}$$

$$\left(\frac{u}{R}\right)_a = \cos 2\theta \left\{ -\frac{(13-3\nu)}{96} f_1 - \frac{43-85\nu}{448} f_2 - 2(1+\nu) p_2 - 4 r_2 \right\} - \left\{ \frac{1}{2}(1+\nu) + 2(1+\nu) q_2 + 4 s_2 \right\}$$

$$\left(\frac{v}{R}\right)_a = \sin 2\theta \left\{ -\frac{(11+\nu)}{96} f_1 - \frac{131+35\nu}{448} f_2 + 2(1+\nu) p_2 + 2(3+\nu) r_2 \right\} - \left\{ 2(1+\nu) q_2 - \frac{1}{2}(1+\nu) \right\}$$

$\frac{112.5}{1164.8}$ $\frac{141.5}{1164.8}$

$$q_2 + 0.66666667 s_2 - 0.08333333 = \eta\{ + 0.33333333 p_2 + 0 + 0.00173611 f_1 + 0.00558036 f_2\} \quad (4.42)$$

$$q_2 + 0 - 0.08333333 = \eta\{ + 0.33333333 p_2 + 2 r_2 - 0.02604167 f_1 - 0.11346726 f_2\}$$

$$q_2 + 0.33333333 s_2 + 0.08333333 = \eta\{ - 0.33333333 p_2 - r_2 + 0.00868056 f_1 + 0.05022321 f_2\}$$

$$q_2 + 1.53846154 s_2 + 0.25000 = \eta\{ - p_2 - 0.46153846 r_2 - 0.04447756 f_1 - 0.01502407 f_2\}$$

$$q_2 + 0 - 0.25000 = \eta\{ + p_2 + 2.53846154 r_2 - 0.00520833 f_1 - 0.12148008 f_2\}$$

$$0.66666667 s_2 + 0 = \eta\{ 0 - 2 r_2 + 0.02777778 f_1 + 0.11954762 f_2\}$$

$$0.33333333 s_2 + 0.16666667 = \eta\{ - 0.66666667 p_2 - 3 r_2 + 0.03472222 f_1 + 0.16369047 f_2\}$$

$$1.20512821 s_2 + 0.16666667 = \eta\{ - 0.66666667 p_2 + 0.53846154 r_2 - 0.05715812 f_1 - 0.06524215 f_2\}$$

$$1.53846154 s_2 + 0.50000 = \eta\{ - 2 p_2 - 3 r_2 - 0.04326923 f_1 + 0.10645604 f_2\}$$

$$s_2 + 0 = \eta\{ 0 - 3 r_2 + 0.04166667 f_1 + 0.17857143 f_2\}$$

$$s_2 + 0.50000 = \eta\{ - 2 p_2 - 9 r_2 + 0.10416667 f_1 + 0.49107141 f_2\}$$

$$s_2 + 0.13829787 = \eta\{ - 0.55319149 p_2 + 0.44680851 r_2 - 0.04742908 f_1 - 0.05414133 f_2\}$$

$$s_2 + 0.325000 = \eta\{ - 1.3 p_2 - 1.95 r_2 - 0.02812500 f_1 + 0.06917643 f_2\}$$

$$0.50000000 = \eta\{ - 2 p_2 - 6 r_2 + 0.06250000 f_1 + 0.31250000 f_2\}$$

$$0.36170213 = \eta\{ - 1.44680851 p_2 - 9.44680851 r_2 + 0.15159575 f_1 + 0.54521274 f_2\}$$

$$0.18670213 = \eta\{ - 0.74680851 p_2 - 2.39680851 r_2 + 0.01930408 f_1 + 0.12333771 f_2\}$$