钱学森

力学手稿

1

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XI'AN JIAOTONG UNIVERSITY PRESS

钱学森力学手稿 ①

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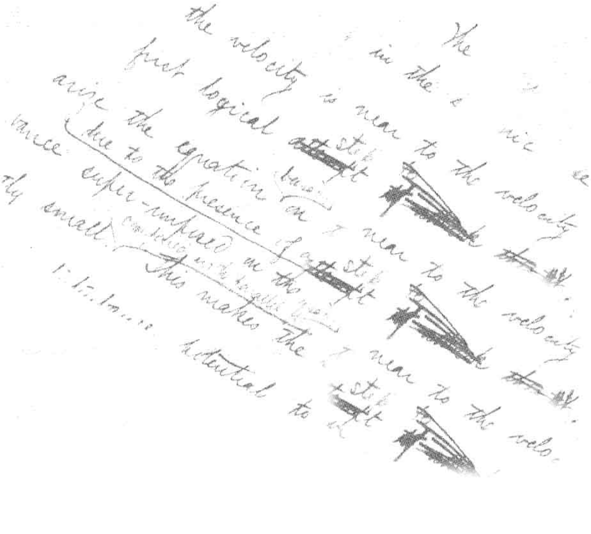

西安交通大学出版社
XI'AN JIAOTONG UNIVERSITY PRESS

图书在版编目(CIP)数据

钱学森力学手稿 1/钱学森著. —西安:西安交通大学出版社,2011.11

ISBN 978-7-5605-4116-7

Ⅰ.①钱… Ⅱ.①钱… Ⅲ.①钱学森(1911～2009)-力学-文集-英文 Ⅳ.①03-53

中国版本图书馆 CIP 数据核字(2011)第 231953 号

书　　名 钱学森力学手稿 1

著　　者 钱学森

责任编辑 桂　亮

出版发行 西安交通大学出版社

(西安市兴庆南路 10 号　邮政编码 710049)

网　　址 http://www.xjtupress.com

电　　话 (029)82668357　82667874(发行中心)

(029)82668315　82669096(总编办)

传　　真 (029)82668280

印　　刷 西安煤航信息产业有限公司

开　　本 787mm×1092mm　1/16　**印张** 21.75　**字数** 529 千字

版次印次 2011 年 12 月第 1 版　2011 年 12 月第 1 次印刷

书　　号 ISBN 978-7-5605-4116-7/O·380

定　　价 80.00 元

读者购书、书店添货、如发现印装质量问题,请与本社发行中心联系、调换。

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投稿热线:(029)82668134

读者信箱:jdlgy@yahoo.cn

出版前言

2011年12月11日是西安交通大学杰出校友钱学森先生的百年诞辰。为缅怀钱学森学长，学习他的科学思想和卓越风范，展示其丰功伟绩和人格魅力，西安交通大学举办了“纪念钱学森诞辰100周年”系列活动：作为制片方之一，参与西部电影集团摄制传记故事片《钱学森》；与中央电视台合作，出品纪录片《实验班的故事——沿着钱学森走过的路》；扩建钱学森生平业绩展馆，向校内外开放；举办钱学森科学与教育思想研讨会；出版发行《钱学森力学手稿》、《钱学森年谱（初编）》、《钱学森第六次产业革命思想探微丛书》等。

钱学森先生在美国深造和工作期间留下大量珍贵手稿，是钱老勇攀科学高峰和严谨治学的集中体现。这里，我们将部分原稿整理汇集成册，出版《钱学森力学手稿》，作为钱老百年诞辰的献礼。

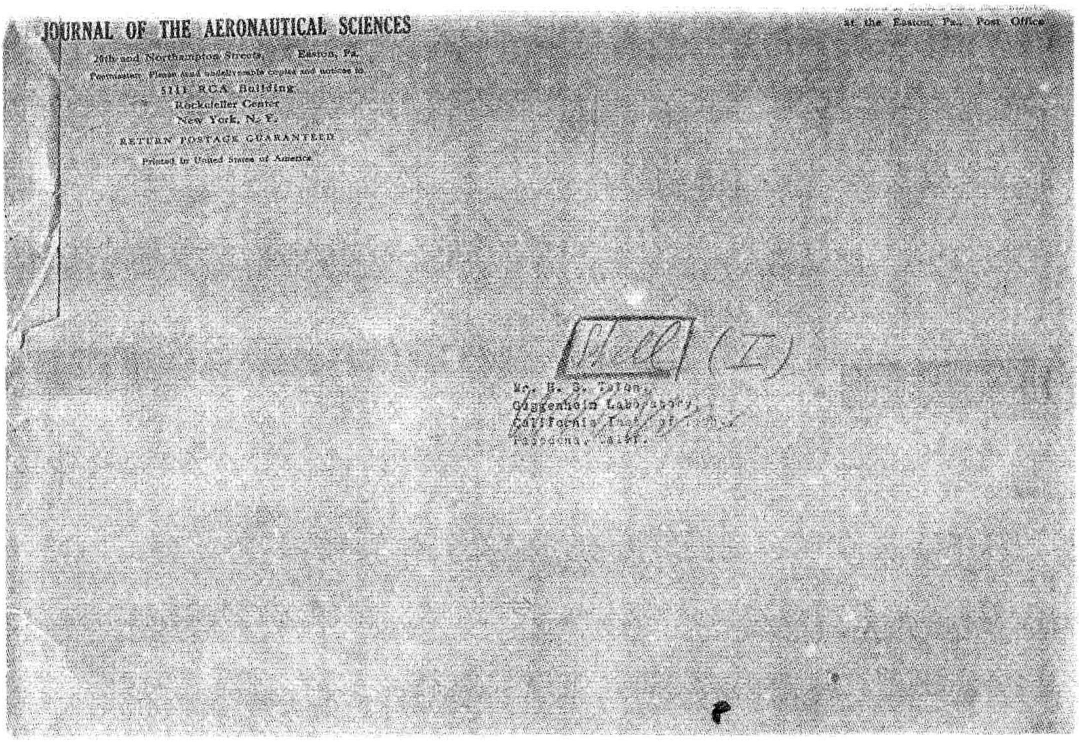
《钱学森力学手稿》共八册，其中《钱学森力学手稿1》包含四部分内容：Shell（I）Buckling of Cylindrical Shell without Shear；Shell（II）Collapse of Slightly Curved Circular Plate；Shell（III）Preliminary Calculation of Circular Cylinder；Buckling of Spherical Shell。其余七册将在之后陆续出版。

本手稿是在西安交通大学校领导的大力支持下，由西安交通大学航天航空学院沈亚鹏教授整理完成。图书出版过程中得到了西安交通大学党委宣传部、校友关系发展部、图书馆、航天航空学院等的积极协助，在此深表感谢。

Contents

Section 1

Shell (Ⅰ) Buckling of Cylindrical Shell without Shear


JOURNAL OF THE AERONAUTICAL SCIENCES

20th and Northampton Streets, Easton, Pa.

Postmaster: Please send undeliverable copies and notices to

5111 RCA Building
Rockefeller Center
New York, N. Y.

RETURN POSTAGE GUARANTEED

Printed in United States of America

at the Easton, Pa., Post Office

Shell (I)

Mr. H. S. Tsien,
Guggenheim Laboratory,
California Inst. of Tech.,
Pasadena, Calif.

<u>Buckling of Cylindrical Shell</u> 1)
<u>Without Shear</u>

$$\frac{\partial N_x}{\partial x} = 0$$

$$\frac{\partial N_y}{\partial \theta} + aN_x \frac{\partial^2 v}{\partial x^2} + \frac{\partial M_{xy}}{\partial x} - \frac{\partial M_y}{a\partial\theta} = 0$$

$$aN_x \frac{\partial^2 w}{\partial x^2} + N_y + a\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{yx}}{\partial x \partial \theta} + \frac{\partial^2 M_y}{a d\theta^2} = 0.$$

$$\frac{\partial^2 u}{\partial x^2} + \nu\left\{\frac{\partial^2 v}{a\partial x\partial\theta} - \frac{1}{a}\frac{\partial w}{\partial x}\right\} = 0.$$

$$\frac{Eh}{1-\nu^2}\left\{\frac{\partial^2 v}{a\partial\theta^2} - \frac{1}{a}\frac{\partial w}{\partial\theta} + \nu\frac{\partial^2 u}{\partial x\partial\theta}\right\} \blacksquare\, a N_x \frac{\partial^2 v}{\partial x^2}$$

$$+ D(1-\nu)\frac{1}{a}\left\{\frac{\partial^2 v}{\partial x^2} + \frac{\partial^3 w}{\partial x^2\partial\theta}\right\} + \frac{D}{a}\left\{\frac{1}{a^2}\left(\frac{\partial^2 v}{\partial\theta^2} + \frac{\partial^3 w}{\partial\theta^3}\right) + \nu\frac{\partial^3 w}{\partial x^2\partial\theta}\right\} = 0.$$

$$\frac{\partial^2 v}{a\partial\theta^2} - \frac{1}{a}\frac{\partial w}{\partial\theta} + \nu\frac{\partial^2 u}{\partial x\partial\theta} + \alpha\left\{(1-\nu)a\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^3 w}{\partial x^2\partial\theta}\right)\right.$$

$$\left. + \frac{1}{a}\left(\frac{\partial^2 v}{\partial\theta^2} + \frac{\partial^3 w}{\partial\theta^3}\right) + \nu a\frac{\partial^3 w}{\partial x^2\partial\theta}\right\} - a\phi\frac{\partial^2 v}{\partial x^2} = 0.$$

or

$$\frac{\partial^2 v}{a\partial\theta^2} - \frac{1}{a}\frac{\partial w}{\partial\theta} + \nu\frac{\partial^2 u}{\partial x\partial\theta} + \alpha\left\{a(1-\nu)\frac{\partial^2 v}{\partial x^2} + a\frac{\partial^3 w}{\partial x^2\partial\theta} + \frac{\partial^2 v}{a d\theta^2} + \frac{\partial^3 w}{a\partial\theta^3}\right\} - a\phi\frac{\partial^2 v}{\partial x^2} = 0$$

2)

$$\frac{\partial^2 u}{\partial x^2} + \nu\left\{\frac{\partial^2 v}{a\,\partial x\,\partial\theta} - \frac{1}{a}\frac{\partial w}{\partial x}\right\} = 0.$$

$$\frac{\partial^2 v}{a\,\partial\theta^2} - \frac{\partial w}{a\,d\theta} + \nu\frac{\partial^2 u}{\partial x\,\partial\theta} + \alpha\left\{a(1-\nu)\frac{\partial^2 v}{\partial x^2} + a\frac{\partial^3 w}{\partial x^2\partial\theta} + \frac{\partial^2 v}{a\,d\theta^2} + \frac{\partial^3 w}{a\,\partial\theta^3}\right\} - a\phi\frac{\partial^2 v}{\partial x^2} = 0.$$

$$-a\phi\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial u}{\partial x} + \frac{\partial v}{a\,d\theta} - \frac{w}{a} - \alpha\left[\frac{\partial^3 v}{a\,\partial\theta^3} + (2-\nu)a\frac{\partial^3 v}{\partial x^2\partial\theta} + a^3\frac{\partial^4 w}{\partial x^4}\right.$$

$$\left. + \frac{\partial^4 w}{a\,\partial\theta^4} + 2a\frac{\partial^4 w}{\partial x^2\partial\theta^2}\right] = 0.$$

$$-A\left(\frac{m\pi}{l}\right)^2 + \nu\left\{-B\frac{n}{a}\left(\frac{m\pi}{l}\right) - \frac{C}{a}\frac{m\pi}{l}\right\} = 0.$$

$$A\lambda^2 + B\cdot\nu n\lambda + C\nu\lambda = 0$$

or $\cancel{\lambda A + \nu n\lambda B}$

$$\underline{\lambda A + \nu n B + \nu C = 0}$$

$$-\cancel{\frac{B}{an^2}}\ \cancel{C}\quad -\frac{n^2 B}{a} - \frac{nC}{a} - \nu\,A\cdot n\left(\frac{m\pi}{l}\right)$$

$$+\alpha\left\{a(1-\nu)(-1)\,B\cdot\left(\frac{m\pi}{l}\right)^2 - anC\left(\frac{m\pi}{l}\right)^2 - \frac{1}{a}n^2 B - \frac{n^3 C}{a}\right\} + Ba\phi\left(\frac{m\pi}{l}\right)^2 = 0.$$

$$n^2 B + nC + \nu A n\lambda + \alpha\left\{(1-\nu)B\lambda^2 + nC\lambda^2 + n^2 B + n^3 C\right\}$$

$$-B\phi\lambda^2 = 0.$$

$$\nu n\lambda A + (n^2 + \alpha n^2 + \alpha(1-\nu)\lambda^2 - \phi\lambda^2)B + (n + \alpha n\lambda^2 + \alpha n^3)C = 0.$$

3)

$$\lambda A + \nu n B + \nu C = 0$$

$$\nu n\lambda A + \{(1+\alpha)n^2 + \alpha(1-\nu)\lambda^2 - \lambda^2\phi\}B + \{n + \alpha n(\lambda^2+n^2)\}C = 0$$

~~Axx~~

$$\nu\lambda A + n\{1+\alpha[n^2+(2-\nu)\lambda^2]\}B + [1-\lambda^2\phi + \alpha(\lambda^2+n^2)^2]C = 0.$$

Determinant to be zero.

$$\lambda\{(1+\alpha)n^2 + \alpha(1-\nu)\lambda^2 - \lambda^2\phi\}\{1 - \lambda^2\phi + \alpha(\lambda^2+n^2)^2\}$$

$$+ \nu n\{n+\alpha n(\lambda^2+n^2)\}\nu\lambda + \nu^2 n^2\lambda\{1+\alpha[n^2+(2-\nu)\lambda^2]\}$$

$$- \nu^2\lambda\{(1+\alpha)n^2+\alpha(1-\nu)\lambda^2 - \lambda^2\phi\} - \nu^2 n^2\lambda[1-\lambda^2\phi+\alpha(\lambda^2+n^2)^2]$$

$$- n\lambda\{1+\alpha[n^2+(2-\nu)\lambda^2]\}\{n+\alpha n(\lambda^2+n^2)\} = 0.$$

$$(1-\nu^2)\lambda\{(1+\alpha)n^2 + \alpha(1-\nu)\lambda^2 - \lambda^2\phi\}$$

$$- \lambda^3\phi\{(1+\alpha)n^2 + \alpha(1-\nu)\lambda^2\}$$

$$+ \alpha(\lambda^2+n^2)^2\lambda\{n^2 - \lambda^2\phi - \nu^2 n^2\}$$

$$- n\lambda(1-\nu^2)\{n + \alpha n(\lambda^2+n^2)\} - n^2\lambda\alpha[n^2+(2-\nu)\lambda^2](1-\nu^2)$$

$$- \qquad + \nu^2 n^2\lambda^3\phi = 0 \qquad = 0.$$

$$(1-\nu^2)\lambda\left\{\alpha n^2+\alpha(1-\nu)\lambda^2-\lambda^2\phi-\alpha n^2(\lambda^2+n^2)\right\} \quad 4)$$

$$-\lambda^3\phi\left\{(1+\alpha)n^2+\alpha(1-\nu)\lambda^2\right\}$$

$$+\alpha(\lambda^2+n^2)^2\lambda\left\{(1-\nu^2)n^2-\lambda^2\phi\right\}+\nu^2n^2\lambda^3\phi$$

$$-n^2\lambda\alpha(1-\nu^2)\left[n^2+(2-\nu)\lambda^2\right]=0.$$

$$\phi\left\{\lambda^3(1-\nu^2)+\lambda^3\left[(1+\alpha)n^2+\alpha(1-\nu)\lambda^2\right]\right.$$
$$\left.+\alpha\lambda^3(\lambda^2+n^2)^2-\nu^2n^2\lambda^3\right\}$$

$$=(1-\nu^2)\lambda\left\{\alpha n^2+\alpha(1-\nu)\lambda^2-\alpha n^2(\lambda^2+n^2)\right\}$$
$$+\alpha(\lambda^2+n^2)^2\lambda(1-\nu^2)n^2-n^2\lambda\alpha(1-\nu^2)\left[n^2+(2-\nu)\lambda^2\right]$$

$$\lambda^2\phi\left[(1-\nu^2)+(1+\alpha)n^2+\alpha(1-\nu)\lambda^2+\alpha(\lambda^2+n^2)^2-\nu^2n^2\right]$$

$$=\alpha(1-\nu^2)\left\{n^2+(1-\nu)\lambda^2-\cancel{n^2(\lambda^2+n^2)+n^2(\lambda^2+n^2)}\right.$$
$$\left.-n^4-n^2(2-\nu)\lambda^2\right\}$$

$$\phi = \frac{\alpha(1-\nu^2)\left\{n^2(1-n^2) + \lambda^2\left[(1-\nu) - n^2(2-\nu)\right]\right\}}{\lambda^2\left\{(1-\nu^2)(1+n^2) + \alpha\left[n^2 + (1-\nu)\lambda^2 + (\lambda^2+n^2)^2\right]\right\}}$$

$$P = 0.$$

$$\sigma_{cr} \approx E\left(\frac{t}{a}\right)^2$$

$$(1-\nu^2)\cancel{\chi}\left\{(1+\alpha)n^2+\alpha(1-\nu)\lambda^2-\lambda^2\phi\right\}-\lambda^2\phi\left\{(1+\alpha)n^2+\alpha(1-\nu)\lambda^2-\lambda^2\phi\right\} \qquad 6)$$

$$+\alpha\cancel{\chi}(\lambda^2+n^2)^2\left\{(1+\alpha)n^2+\alpha(1-\nu)\lambda^2-\lambda^2\phi\right\}$$

$$+\nu^2n^2\cancel{\chi}\left\{\alpha\left[n^2+(2-\nu)\lambda^2\right]+\lambda^2\phi-\alpha(\lambda^2+n^2)^2\right\}$$

$$-n\cancel{\chi}(1-\nu^2)\left\{n+\alpha n(\lambda^2+n^2)\right\}-n^2\cancel{\chi}\alpha\left[1+\alpha(\lambda^2+n^2)\right]\left[n^2+(2-\nu)\lambda^2\right]$$

$$=0.$$

$$(1-\nu^2)\left\{(1+\alpha)n^2+\alpha(1-\nu)\lambda^2-n^2-\alpha n^2(\lambda^2+n^2)\right\}$$

$$+\alpha(\lambda^2+n^2)^2\left\{(1+\alpha)n^2+\alpha(1-\nu)\lambda^2-\nu^2n^2\right\}$$

$$+\nu^2n^2\alpha\left[n^2+(2-\nu)\lambda^2\right]-n^2\alpha\left[1+\alpha(\lambda^2+n^2)\right]\left[n^2+(2-\nu)\lambda^2\right]$$

Coeff. for $(\lambda^2\phi)$

$$-(1-\nu^2)-(1+\alpha)n^2-\alpha(1-\nu)\lambda^2$$

$$-\alpha(\lambda^2+n^2)^2+\nu^2n^2$$

Coeff. $(\lambda^2\phi)^2$ $\quad 1$

$$(\lambda^2\phi)^2-B(\lambda^2\phi)+C=0.$$

7)

$$B = (1-\nu^2)(n^2+1) + \alpha\left\{(\lambda^2+n^2)(1+\lambda^2+n^2) - \nu\lambda^2\right\}$$

$$C = \alpha\Big[(1-\nu^2)\left\{n^2[1+(1-\nu)\lambda^2] + (1-\nu)\lambda^2 - \alpha n^2\lambda^2(\lambda^2+n^2)\right\}$$

$$+ (\lambda^2+n^2)^2\left\{(1-\nu^2)n^2 + \alpha(1-\nu)\lambda^2\right\}\Big]$$

$$B^2 - 4C = (1-\nu^2)^2(n^2+1)^2 + 2\alpha(1-\nu^2)(n^2+1)\left\{(\lambda^2+n^2)(1+\lambda^2+n^2) - \nu\lambda^2\right\}$$

$$+ \alpha^2\left\{(\lambda^2+n^2)(1+\lambda^2+n^2) - \nu\lambda^2\right\}^2$$

$$-4\alpha\Big[(1-\nu^2)\left\{n^2[1+(1-\nu)\lambda^2] + (1-\nu)\lambda^2 + n^2(\lambda^2+n^2)^2\right\}$$

$$-4\alpha^2\Big[-(1-\nu^2)n^2\lambda^2(\lambda^2+n^2) + (\lambda^2+n^2)^2\lambda^2(1-\nu)\Big]$$

III

$$B^2 - 4C = (1-\nu^2)^2(n^2+1)^2$$

$$+2\alpha(1-\nu^2)\Big[(n^2+1)\left\{(\lambda^2+n^2)(1+\lambda^2+n^2) - \nu\lambda^2\right\} - 2(n^2+1)(1-\nu)\lambda^2$$

$$-2n^2 - 2n^2(\lambda^2+n^2)^2\Big]$$

$$+\alpha^2\Big[\left\{(\lambda^2+n^2)(1+\lambda^2+n^2) - \nu\lambda^2\right\}^2 + 2\left\{(1-\nu^2)n^2\lambda^2(\lambda^2+n^2) - (1-\nu)\lambda^2(n^2+\lambda^2)^2\right\}$$

$$B^2-4C=(1-\nu^2)^2(n^2+1)^2$$

$$+2\alpha(1-\nu^2)\left[(n^2+1)\left\{(\lambda^2+n^2)(1+\lambda^2+n^2)-(2-\nu)\lambda^2\right\}-2n^2\left\{1+(\lambda^2+n^2)^2\right\}\right]$$

$$+\alpha^2\left[\left\{(\lambda^2+n^2)(1+\lambda^2+n^2)-\nu\lambda^2\right\}^2+2\lambda^2(\lambda^2+n^2)\left\{(1-\nu^2)n^2-(1-\nu)(\lambda^2+n^2)\right\}\right]$$

$$=(1-\nu^2)^2(n^2+1)^2$$

$$+2\alpha(1-\nu^2)\left[(n^2-1)(\lambda^2+n^2)(1-\lambda^2-n^2)+\lambda^2\left\{\nu-n^2(2-\nu)\right\}\right]$$

$$+\alpha^2\left[\left\{(\lambda^2+n^2)(1+\lambda^2+n^2)-\nu\lambda^2\right\}^2+2\lambda^2(\lambda^2+n^2)(1-\nu)(\nu n^2-\lambda^2)\right]$$

$$\approx \cancel{(1-\nu^2)^2(n^2+1)^2}$$

$$\cancel{+2\alpha(1-\nu^2)\left[-n^2(\lambda^2+n^2)^2-\lambda^2n^2(2-\nu)\right]}$$

$$\cancel{+\alpha^2\left[\{(\lambda^2+n^2)^4\right]}$$

$$\phi=\frac{1}{2\lambda^2}\left\{B\pm\sqrt{B^2-4C}\right\}$$

If n and λ are large compared with unity, we have 9)

$$B = n^2(1-\nu^2) + \alpha\{(\lambda^2+n^2)^2\}$$

$$C = \alpha\left[(1-\nu^2)\{(n^2+1)(1-\nu)\lambda^2+n^2\}+(\lambda^2+n^2)^2(1-\nu^2)n^2\right]$$

$$+\alpha^2\left[-(1-\nu^2)n^2\lambda^2(\lambda^2+n^2)+(1-\nu)\lambda^2(\lambda^2+n^2)^2\right]$$

$$\approx \alpha\left[\cancel{(1-\nu^2)(1-\nu)n^2\lambda^2}\ (1-\nu^2)n^2(\lambda^2+n^2)^2\right]$$

$$+\ \cancel{\alpha^2\left[(1+\nu)n^2\lambda^2\right.}\ \alpha^2\{(\lambda^2+n^2)-(1+\nu)n^2\}\lambda^2(\lambda^2+n^2)(1-\nu)$$

$$\approx \alpha(1-\nu^2)n^2(\lambda^2+n^2)^2 + \alpha^2\lambda^2(1-\nu)(\lambda^2+n^2)(\lambda^2-\nu n^2)$$

$$B^2-4C = n^4(1-\nu^2)^2 + 2\alpha n^2(1-\nu^2)(\lambda^2+n^2)^2 + \alpha^2(\lambda^2+n^2)^4$$

$$-4\alpha(1-\nu^2)n^2(\lambda^2+n^2)^2 - 4\alpha^2\lambda^2(1-\nu)(\lambda^2+n^2)(\lambda^2-\nu n^2)$$

$$\cong \{n^{\cancel{4}2}(1-\nu^2)-\alpha(\lambda^2+n^2)^2\}^2 -$$

$$\phi \doteqdot \frac{1}{2\lambda^2}\ 2\alpha(\lambda^2+n^2)^2$$

$$= \alpha\left(\frac{\lambda^2+n^2}{\lambda^2}\right) = \alpha\frac{(\lambda^2+n^2)^2}{\lambda^2}$$

$$\cancel{\frac{(\lambda^2+n^2)^2}{\lambda^2}\ \ \frac{2(\lambda^2+n^2)}{\lambda^2}}$$

$$\tau_{cr} = \frac{E}{12(1-\nu^2)}\left(\frac{t}{R}\right)^2 \frac{(\lambda^2+n^2)^2}{\lambda^2}$$

If we put $\lambda^2 = n^2$

$$\sigma = \frac{E}{12(1-\nu^2)}\left(\frac{t}{R}\right)^2 4n^2 = \frac{E\,n^2}{3(1-\nu^2)}$$

$$= \frac{n^2}{3(1-\nu^2)} E\left(\frac{t}{R}\right)^2$$